Toward Accurate, Large-scale Electromigration Analysis and Optimization in Integrated Systems

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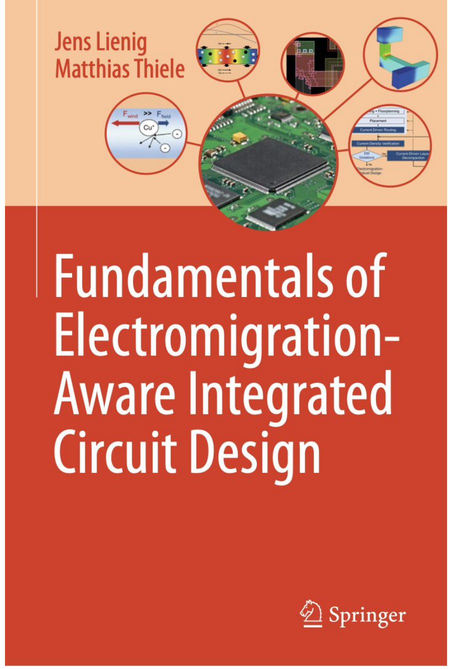
Updated version of the paper at https://arxiv.org/abs/2603.14318

Acknowledgments: (alphabetically) Nestor Evmorfopoulos, Palkesh Jain, Vivek Mishra, Gracieli Posser, Susann Rothe, and Mohammad Shohel

Kudos to Jens Lienig for his work on this topic, including…

- Jens Lienig, “Introduction to electromigration-aware physical design,” ISPD 2006.
- Jens Lienig, “Electromigration and its impact on physical design in future technologies,” ISPD 2013.
- Jens Lienig and Matthias Thiele, “The Pressing Need for Electromigration-Aware Physical Design,” ISPD 2018.



Jens Lienig
Matthias Thiele

Fundamentals of Electromigration-Aware Integrated Circuit Design

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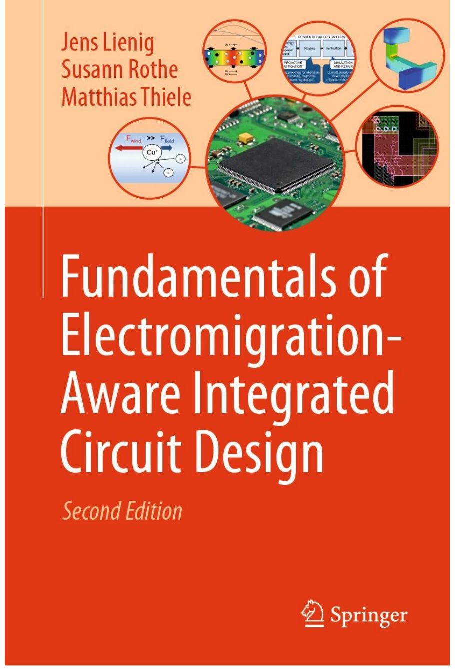

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Fundamentals of Electromigration-Aware Integrated Circuit Design

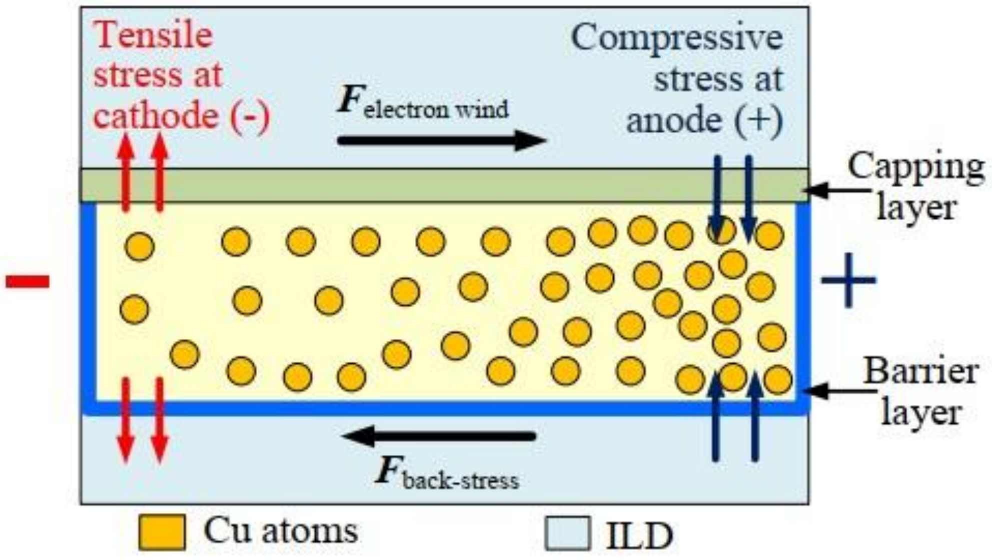
Second Edition

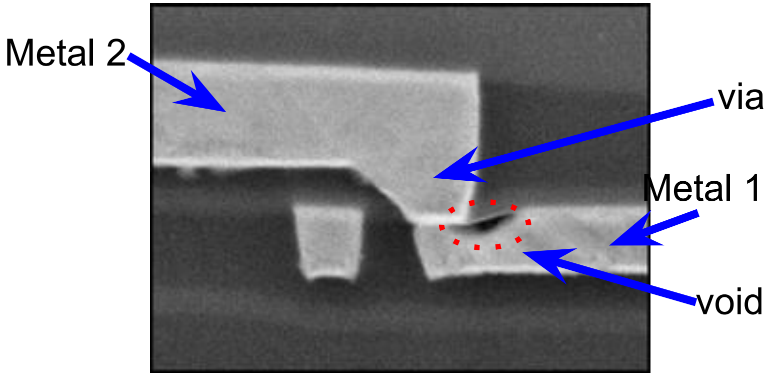
Springer

What is electromigration?

- Interaction between two forces in a current-carrying wire
  - Electron wind moves atoms away from cathode
  - Back-stress due to diffusion opposes this motion
- Tensile force at cathode may form a void and result in functional failure
- Blech criterion: when the forces balance out, the wire is “immortal”

Each metal layer can be analyzed independently





Cross-section TEM image

Empirical EM analysis

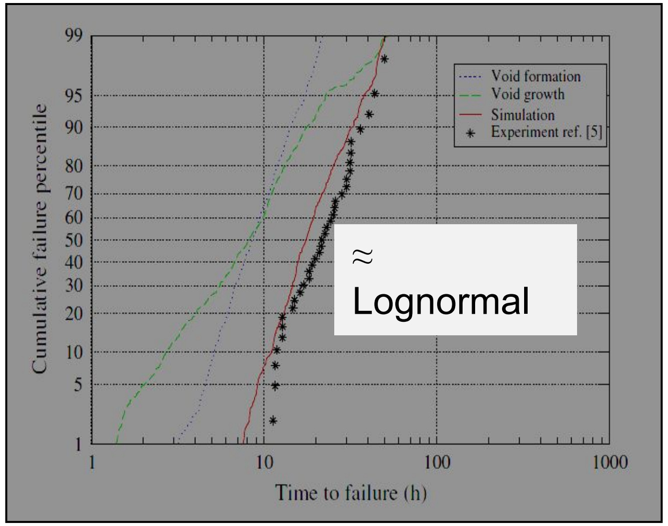
- Black's equation

$$t_{50} = \frac{A}{j^n} \exp \frac{E_a}{kT}$$

- Blech criterion to check immortality

$$jL < (jL)_{crit}$$

- Time to failure is lognormally distributed

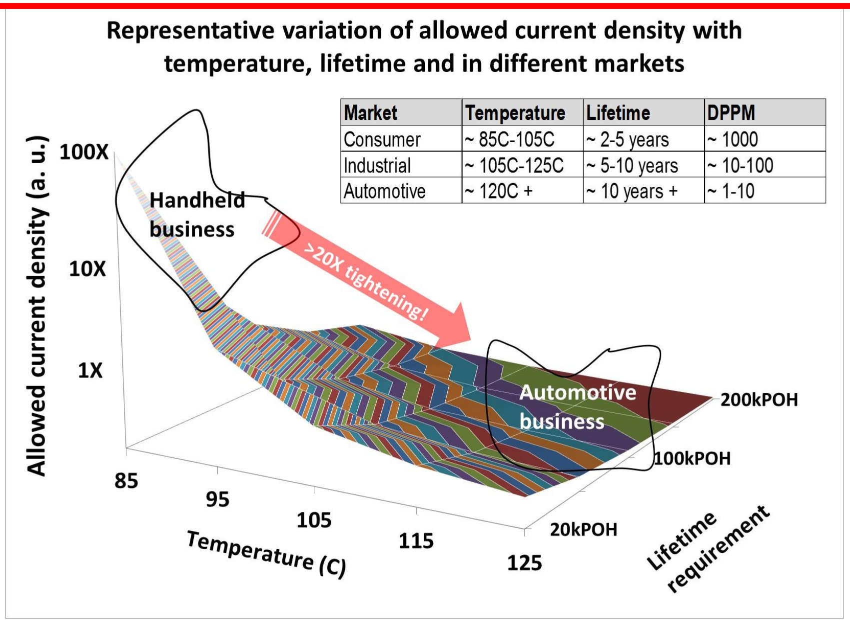


[de Orio, 2011]

- Given a failure fraction, FF, map this back to failure time t_f using CDF $\Phi(z)$

$$\mathrm{FF} = \Phi(z); \quad z = \frac{\ln t_f - \ln t_{50}}{\sigma}$$

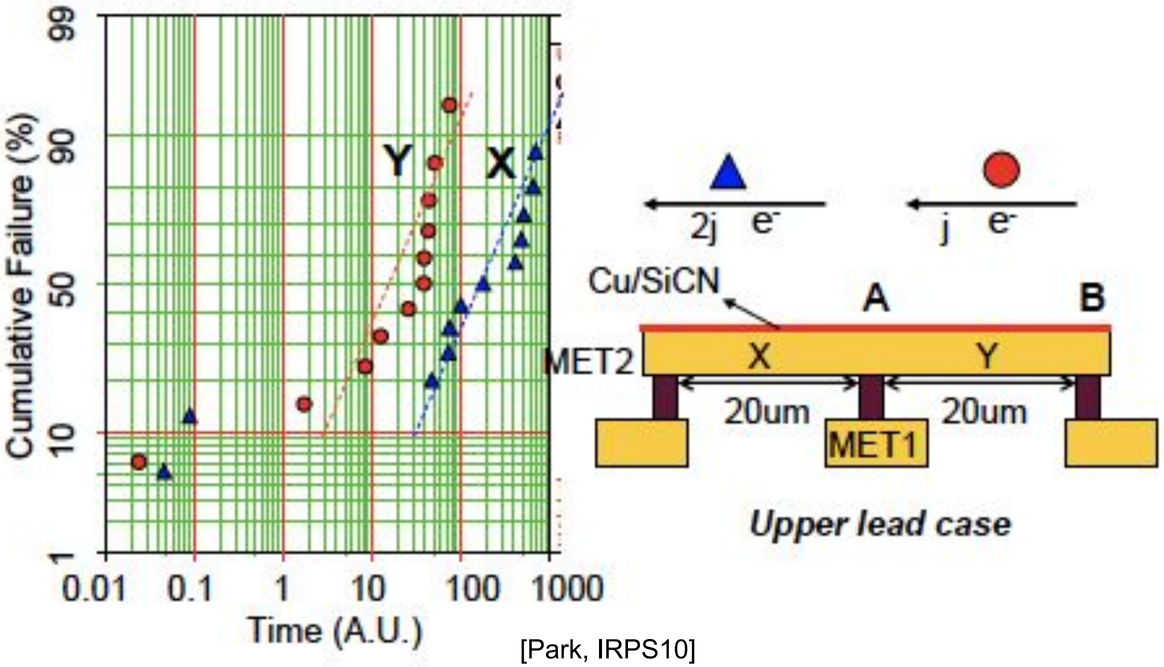
- Constraint on t_f @ z □ t_{50} □ j


Representative variation of allowed current density with temperature, lifetime and in different markets

Market	Temperature	Lifetime	DPPM
Consumer	~ 85C-105C	~ 2-5 years	~ 1000
Industrial	~ 105C-125C	~ 5-10 years	~ 10-100
Automotive	~ 120C +	~ 10 years +	~ 1-10

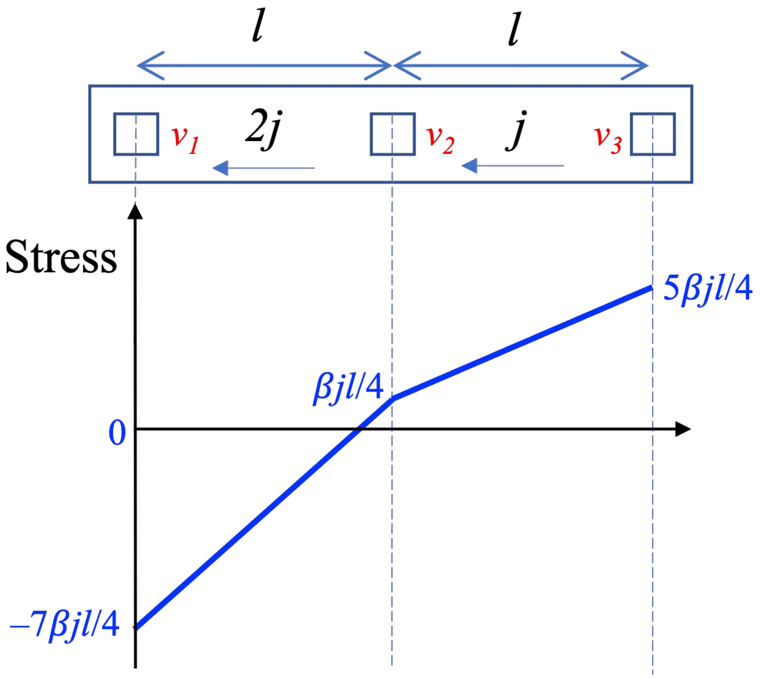
[Jain, TVLSI 2016]

Limitations of this analysis



[Park, IRPS10]

- The wire with <u>lower</u> current density dies first
- Easily explained by physics-based methods

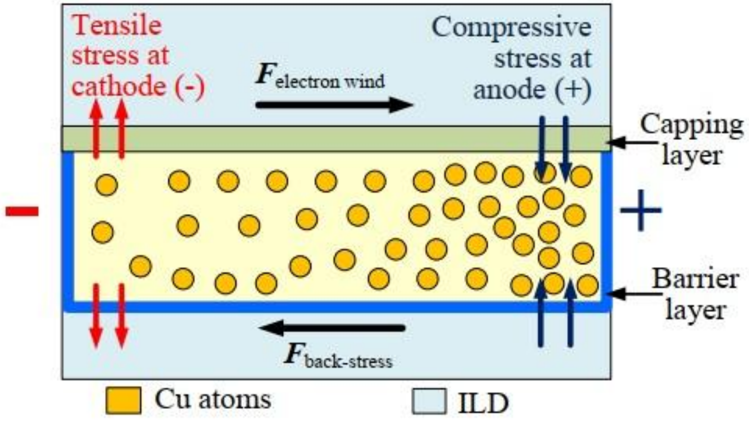


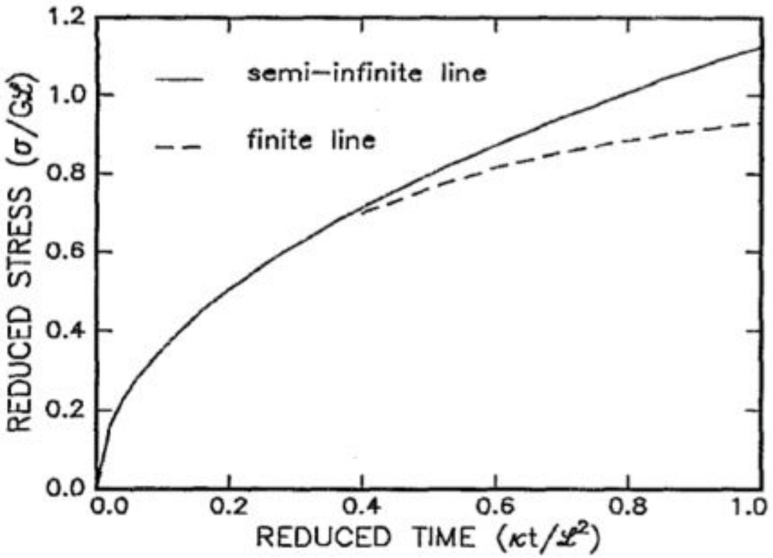
Physics-based EM analysis

- Korhonen model
  - Void nucleation

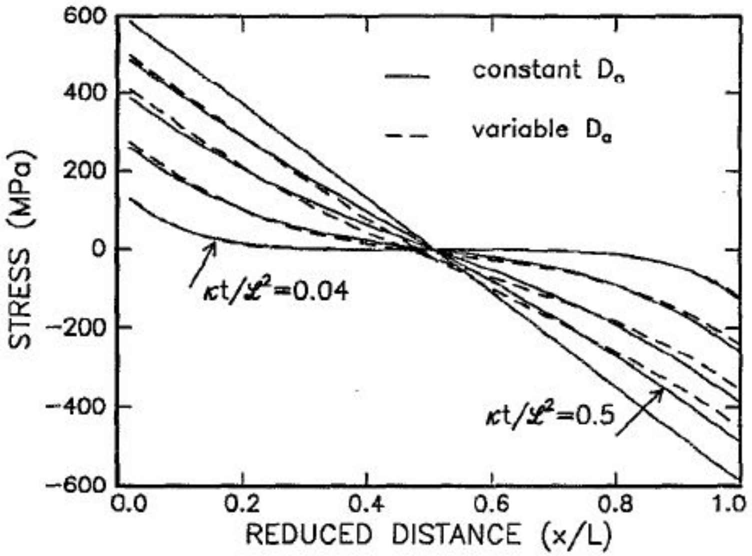
$$\frac{\partial\sigma}{\partial t} = \frac{\partial}{\partial x}\left[\kappa\left(\frac{\partial\sigma}{\partial x}+\beta j\right)\right]$$

$$\frac{\partial\sigma}{\partial t} = \frac{\partial}{\partial x}\left[\kappa\left(F_{\text{back-stress}}+F_{\text{electron wind}}\right)\right]$$





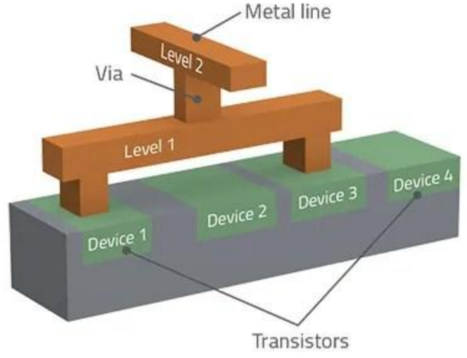
Stress at a blocking boundary (cathode)

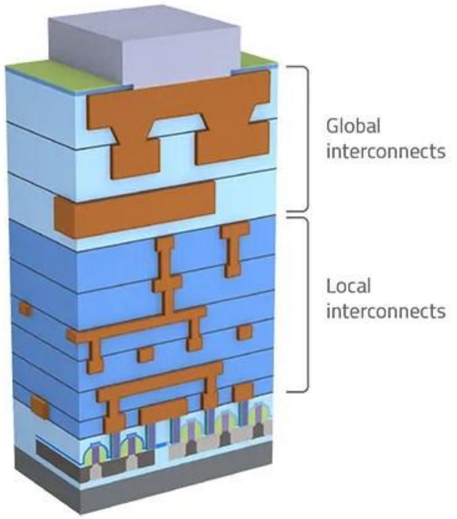


Stress evolution along the wire

[Korhonen, JAP 1993]

Physics-based EM for real lines

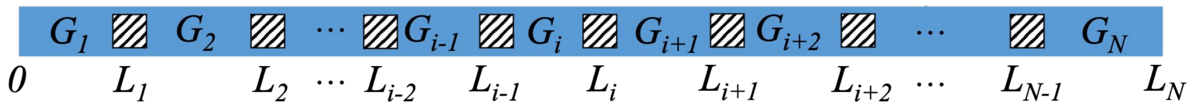




[https://semiengineering.com/all-about-interconnects/]

Each metal layer can be analyzed independently

- Real interconnect in one layer



$$\frac{\partial \sigma}{\partial t} = \frac{\partial}{\partial x}\left[\kappa\left(\frac{\partial \sigma}{\partial x} + \beta j\right)\right]$$

- Boundary conditions
  - Flux from left and right is the same

$$\frac{\partial \sigma_1(0,t)}{\partial x} + G_1 = 0\ ; \quad \frac{\partial \sigma_N(L_N,t)}{\partial x} + G_N = 0$$

$$\frac{\partial \sigma_i(L_i,t)}{\partial x} + G_i = \frac{\partial \sigma_{i+1}(L_i,t)}{\partial x} + G_{i+1}$$

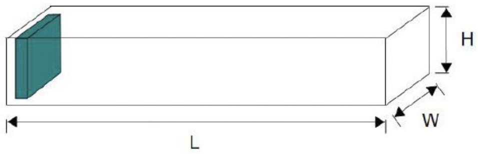
  - Stress is continuous

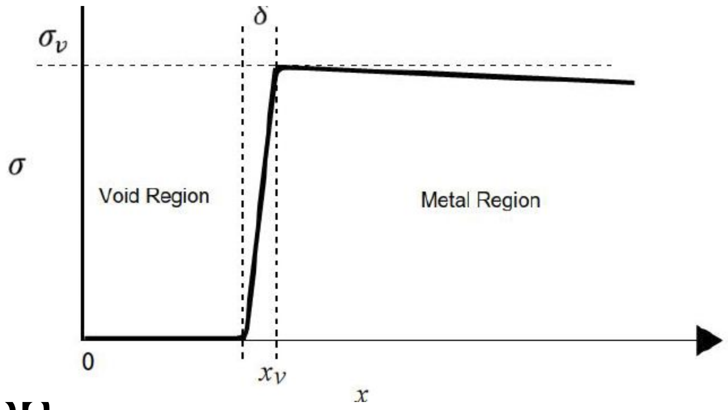
$$\sigma_i(L_i,t) = \sigma_{i+1}(L_i,t)$$

A few other issues

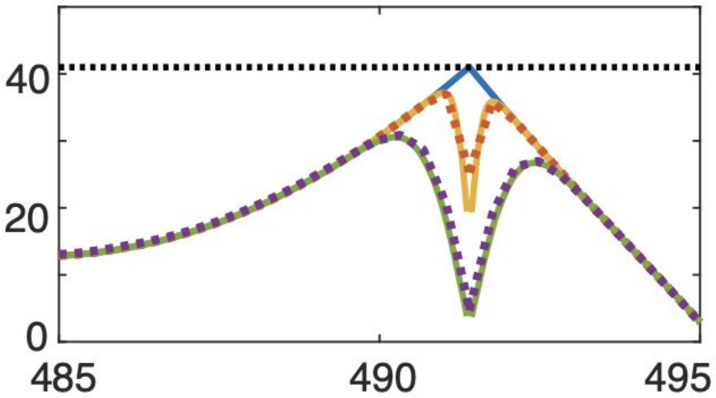
- Postvoiding BC

$$\frac{\partial \sigma(x_v,t)}{\partial x} = \frac{\sigma(x_v,t)}{\delta}$$

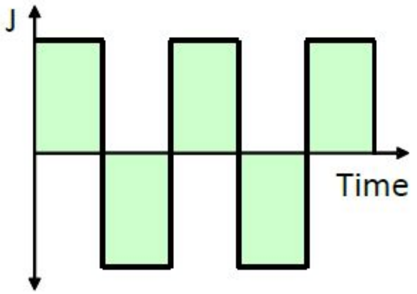


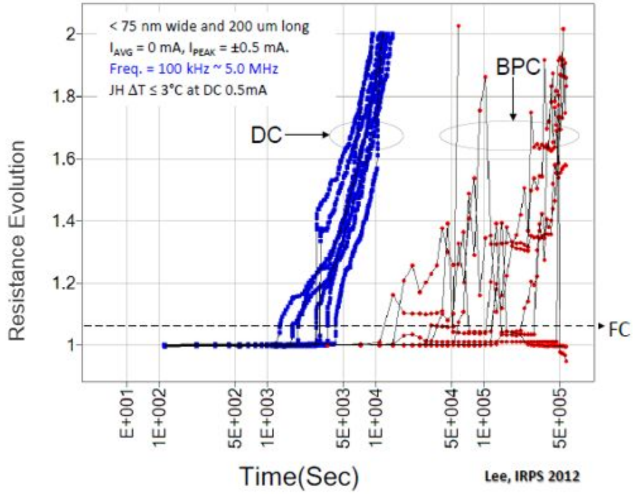


- Stress evolution



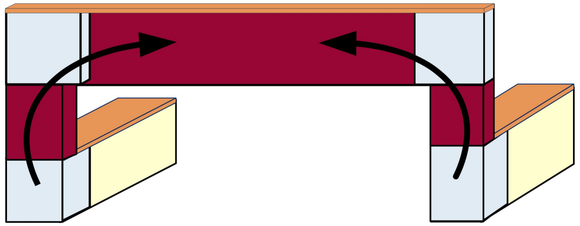
"AC EM"





Lee, IRPS 2012

$$J_{avg} = J_{avg}^{+} - r.J_{avg}^{-}$$



🟦 Potential void sites

(See Caveat 1 in the paper)

What has changed in the past few years

The good

- **Linear-time solutions** are now available for physics-based formulations for the DC and transient cases
- **Physical insights** are available, accessible to EEs, for stress evolution in a wire

The bad

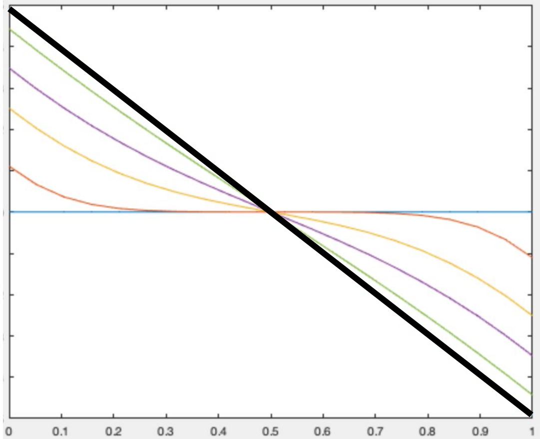
- Foundries are still geared up for empirical EM analysis
- Characterization processes for physics-based models don't exist

Steady-state solutions for immortality checks

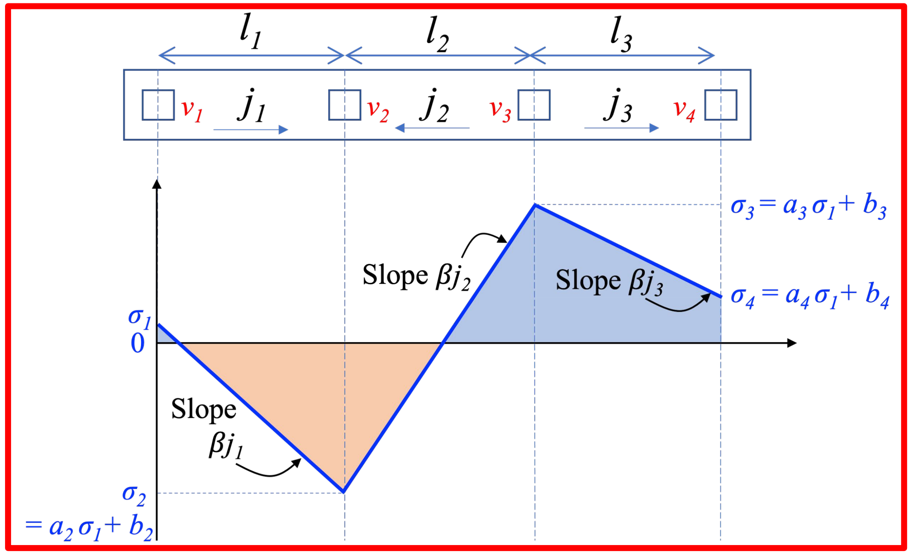
- Blech's criterion works for one segment

Stress evolution:

$$d\sigma_i/dx = -\beta j_i$$



- For multiple segments,



- This is a simple geometry problem
  - The slope for each segment is known

$$d\sigma_i/dx = -\beta j_i$$

  - Given σ_1 at x=0, we can find all stresses
  - Finding σ_1
    - Conservation of mass criterion
    - Area above x-axis = area below x-axis
- Relation between Σ jL and Σ IR

$$j_i = \frac{V_b - V_a}{R_i(w_i h_i)} = \frac{V_b - V_a}{\rho l_i}$$

$$\sigma^b - \sigma^a = \boxed{-\beta j_i l_i = -\frac{\beta}{\rho}(V_b - V_a)}$$

- Minimizing IR drop is *correlated* with improving EM reliability
  - (not identical: Caveats 2, 3 in the paper)

Transient stress analysis

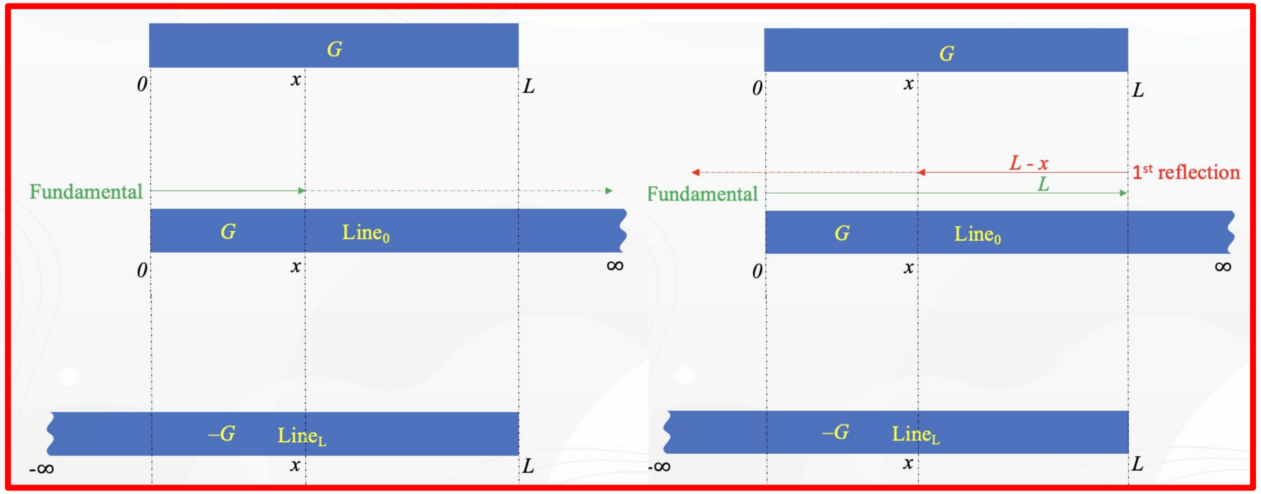
- Numerical solution in the time-domain can be expensive

$$\frac{\partial \sigma}{\partial t} = \frac{\partial}{\partial x}\left[\kappa\left(\frac{\partial \sigma}{\partial x} + G\right)\right]$$

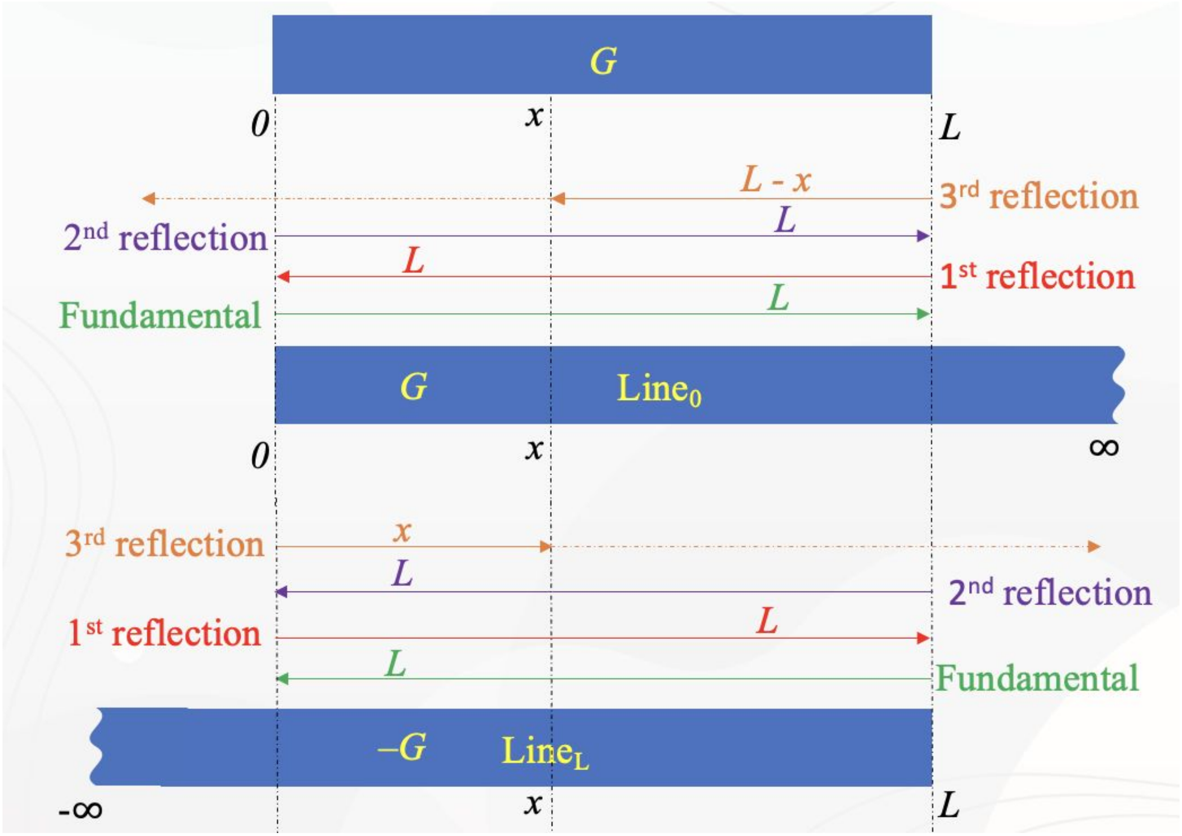
- Infinite series solution
  - Works under restricted assumptions
  - For a single line segment [Korhonen, 1993]

$$\sigma = GL\left(1/2 - \xi - 4\sum_{n=0\to\infty} m_n^{-2}\exp(-m_n^2\tau)\cos(m_n\xi)\right):$$

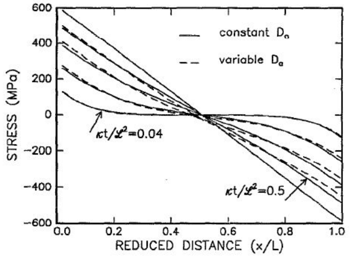
- Can provide meaning to this



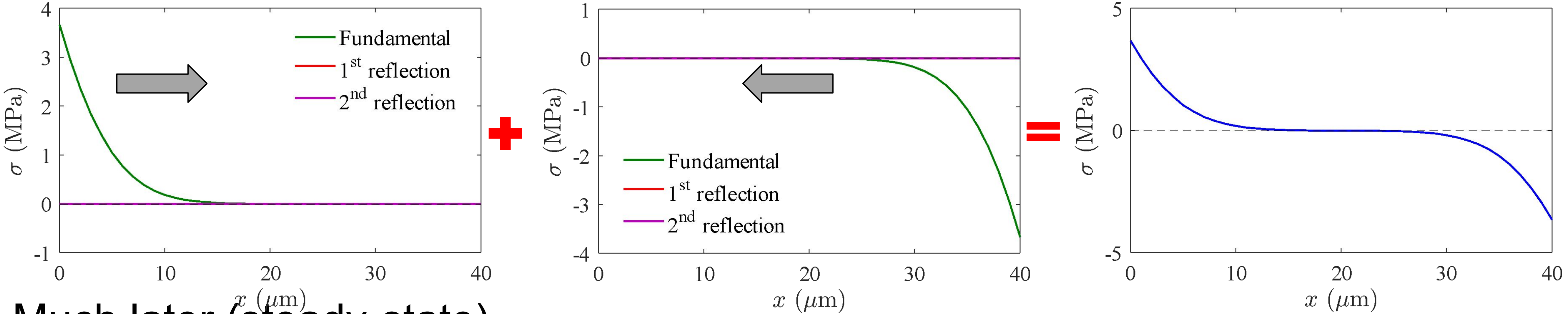
- Another stress wave from the right
  - Net result: superposition of the two stress waves and their reflections



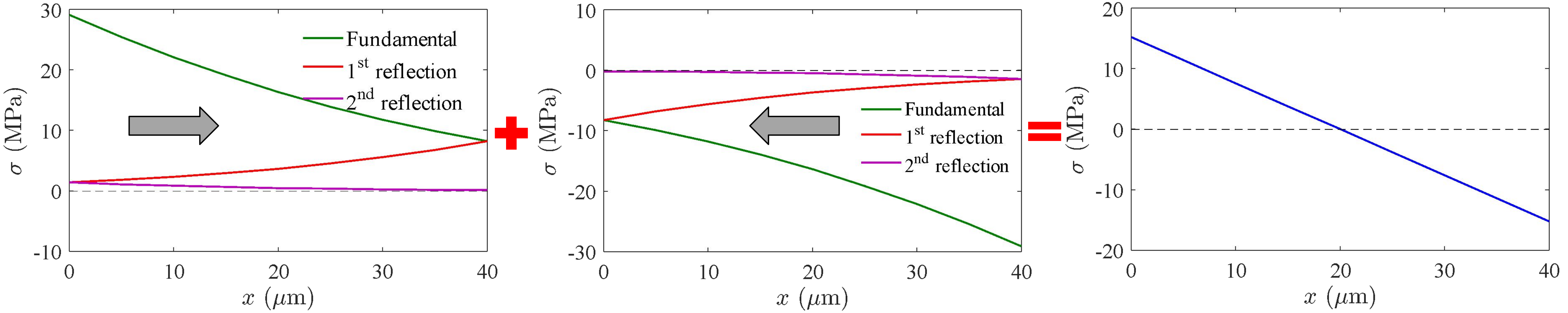
Illustrations



- Early stage

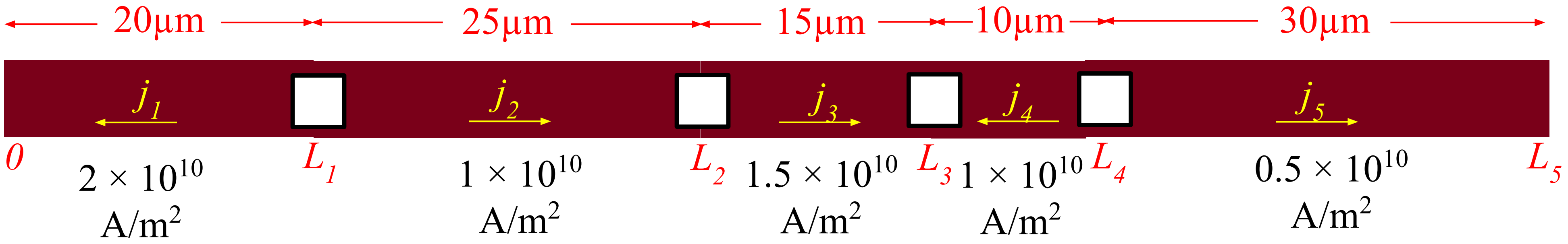


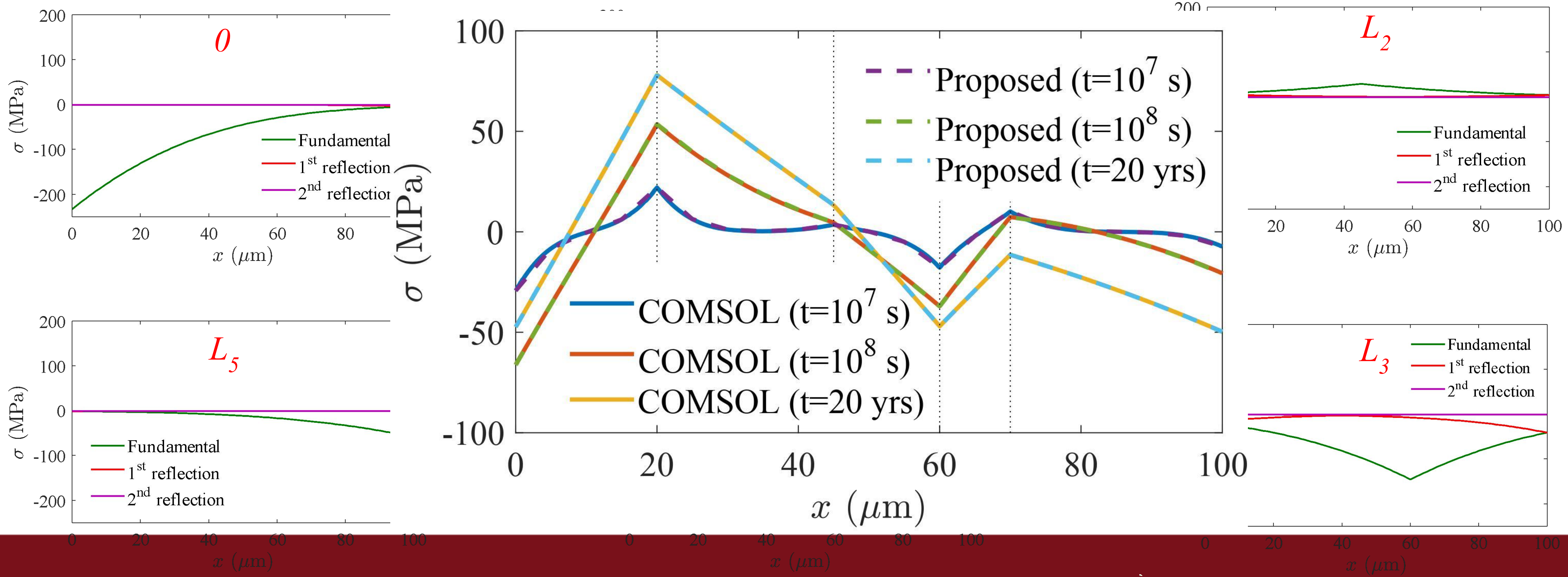
- Much later (steady-state)



The idea works for multiple segments

Stress wave emanates from each point of discontinuity (end point or via)





Another approach: An equivalent RC model

Heart equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

↔

Transmission Line Theory

$$\frac{\partial V}{\partial t} = \frac{1}{RC} \frac{\partial^2 V}{\partial x^2}$$

Korhonen's equation

$$\frac{\partial \sigma}{\partial t} = \kappa \frac{\partial^2 \sigma}{\partial x^2}$$

↔

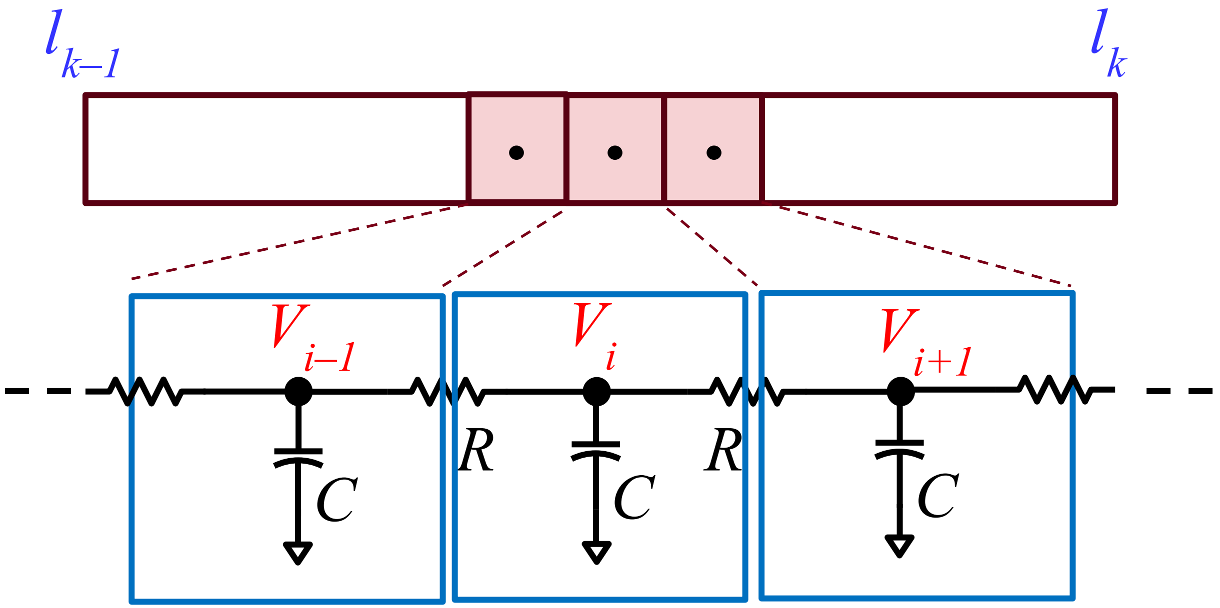
Transmission Line Theory

$$\frac{\partial V}{\partial t} = \frac{1}{RC} \frac{\partial^2 V}{\partial x^2}$$

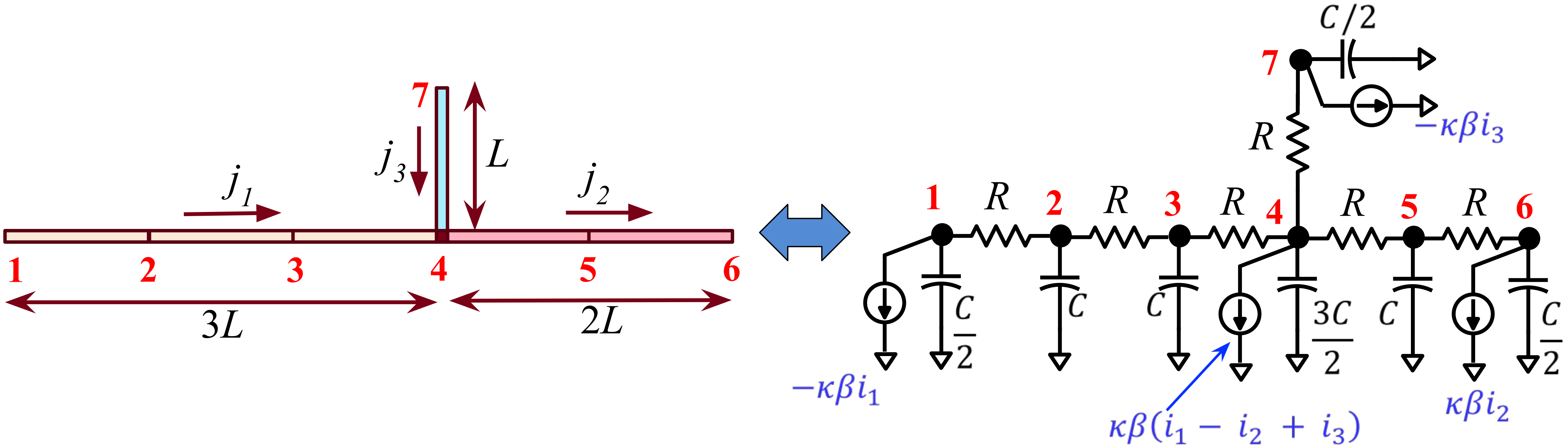
$\sigma \leftrightarrow V$

$(\Delta x \cdot w \cdot h) \leftrightarrow C$

$\frac{1}{\kappa}\left(\frac{\Delta x}{w \cdot h}\right) \leftrightarrow R$



Stress-electrical analogy: Boundary conditions



Stress Continuity ↔ Voltage Continuity

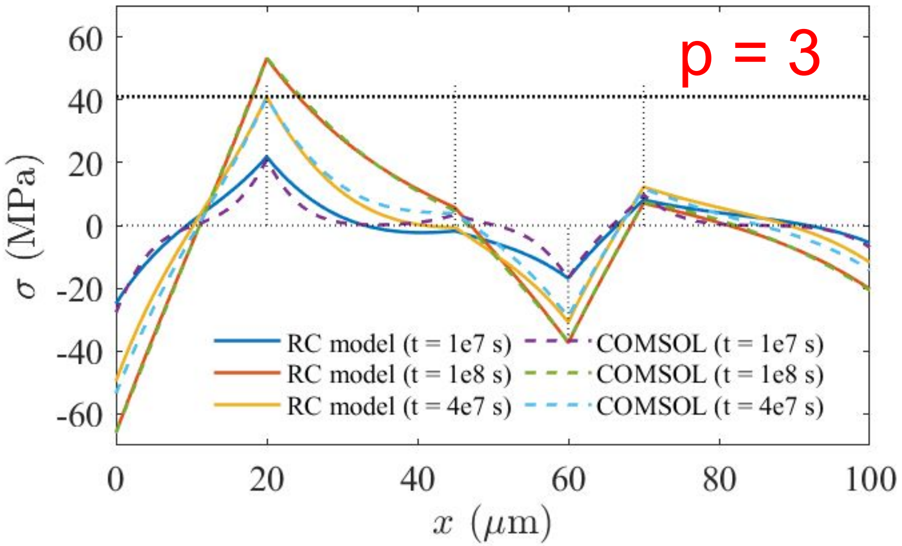
Flux Continuity ↔ KCL

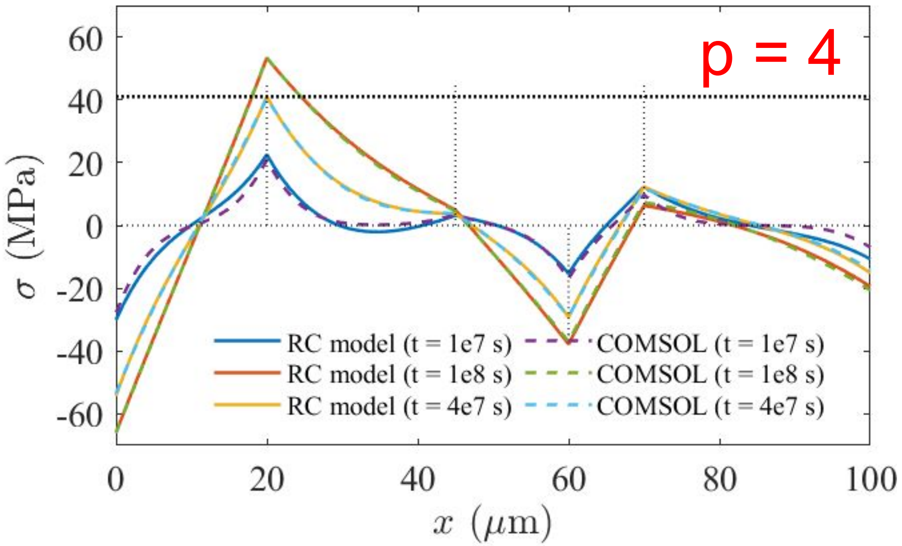
Mass Balance ↔ Charge Balance

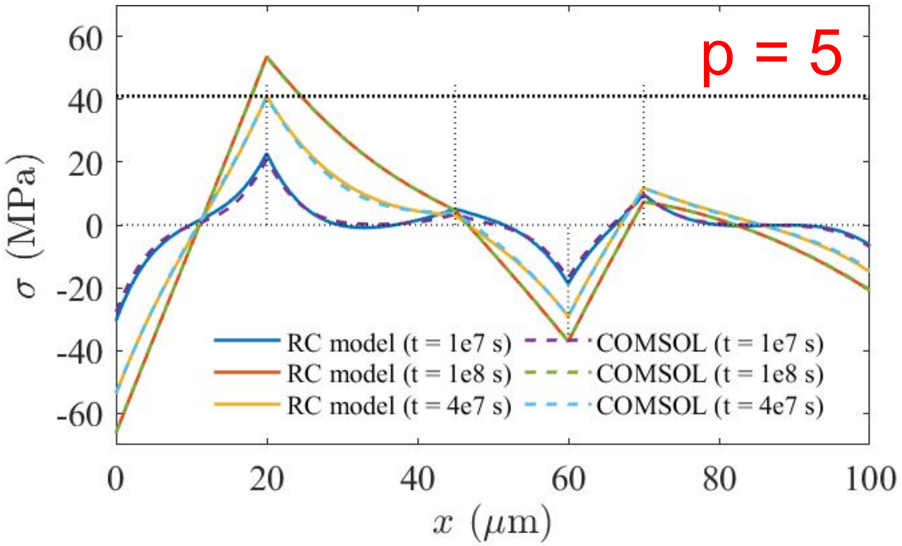
$$\sum_k C_k V_k = 0$$

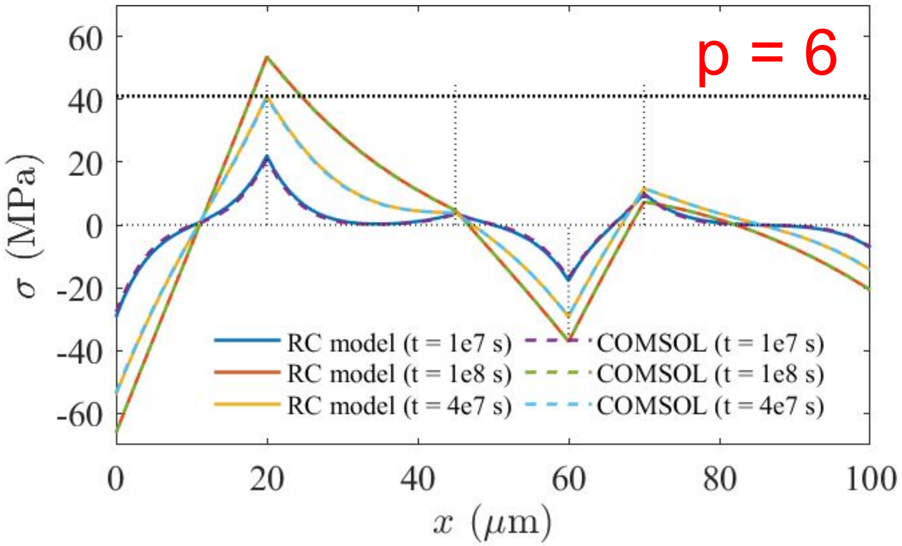
Solve this system using model-order reduction
Linear-time tree traversals!

Results: Multisegment Line – Prenucleation



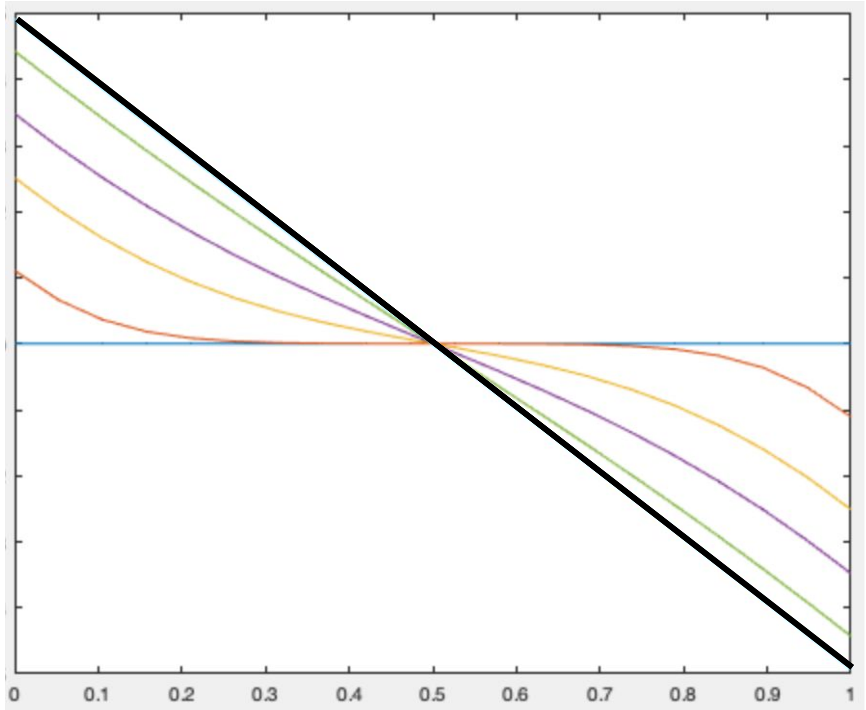




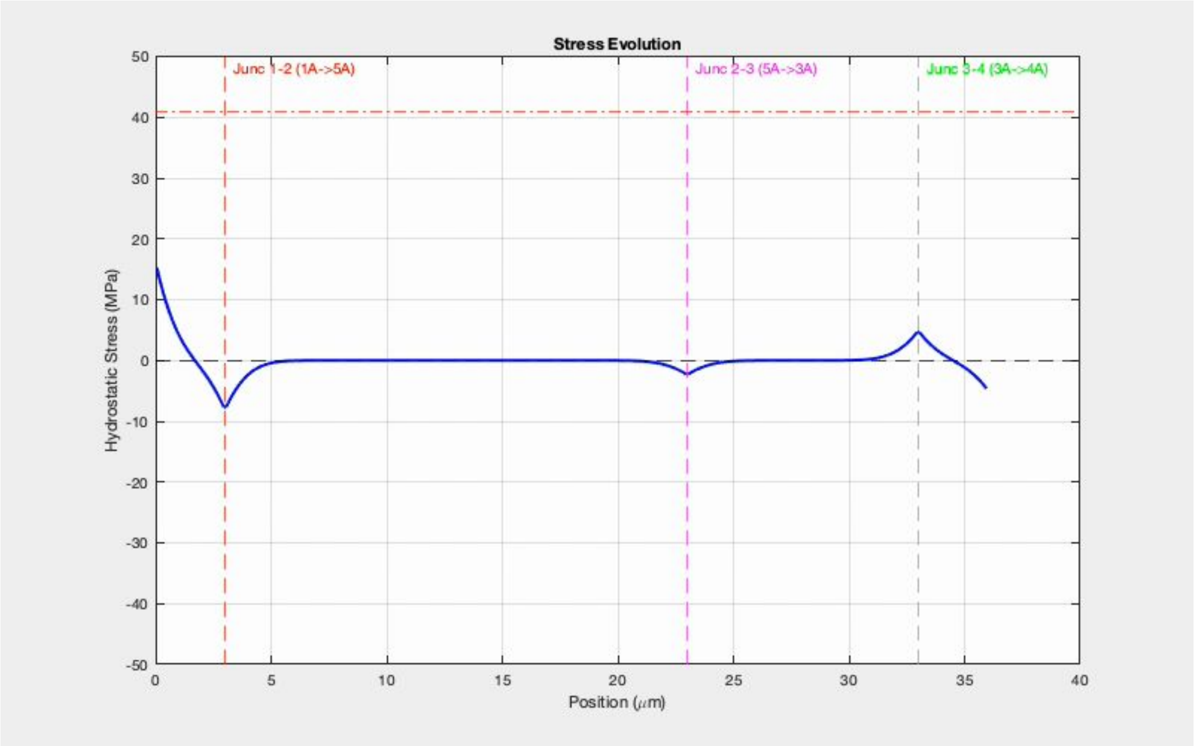


Transient stress and (non)monononicity

- Stress evolution on a single-segment line is monotonic



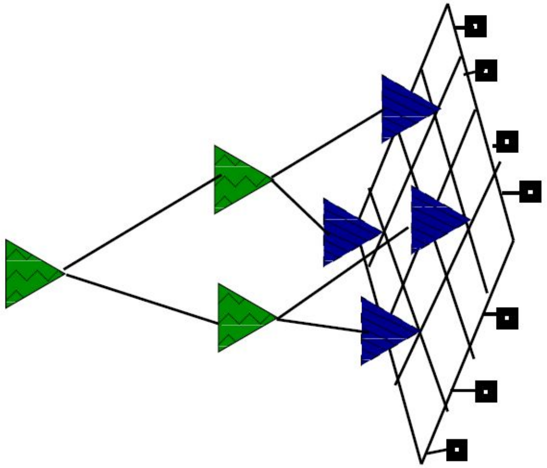
- Caveat 4: Does not have to be so on a multisegment line
- Steady-state stress indicates immortality, but the line is mortal!



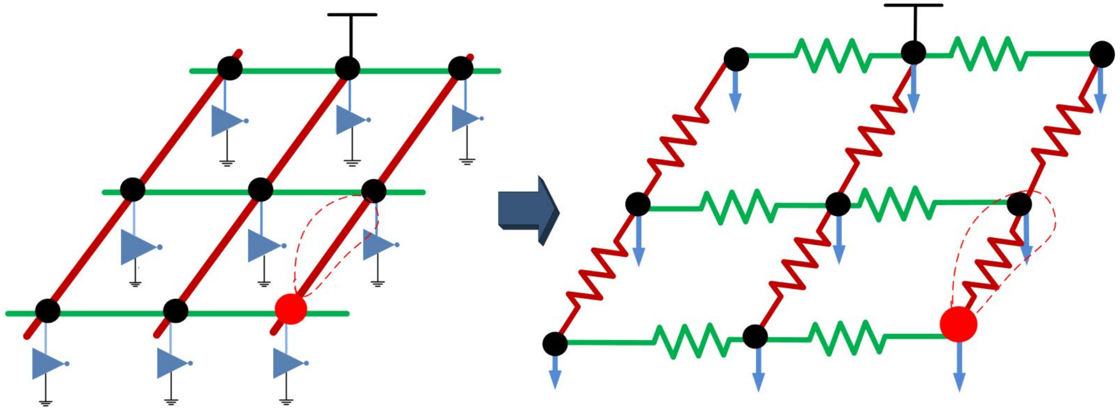
[Thank you, Gemini!]

From wires to systems

- Traditional “weakest link” methodology
  - P = failure probability of one wire
  - $(1–P)^n$ = probability that *n* wires are failure-free
  - System failure probability = $1 – (1–P)^n$
- Assumes that
  - Failures are uncorrelated
  - A single wire failure causes system failure
- Counterexample: clock mesh

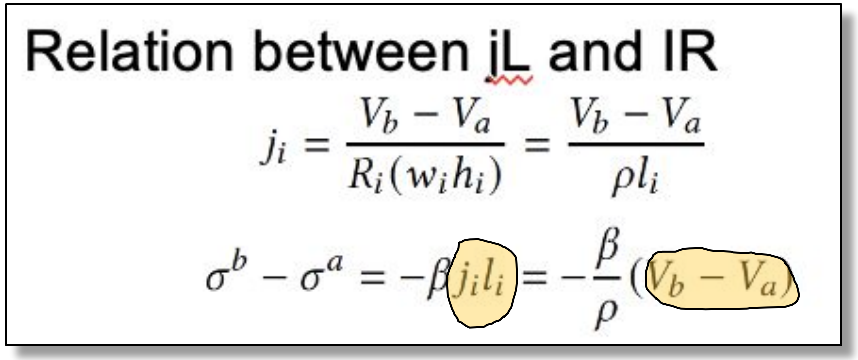


- Counterexample: power grid



Open problems in optimization

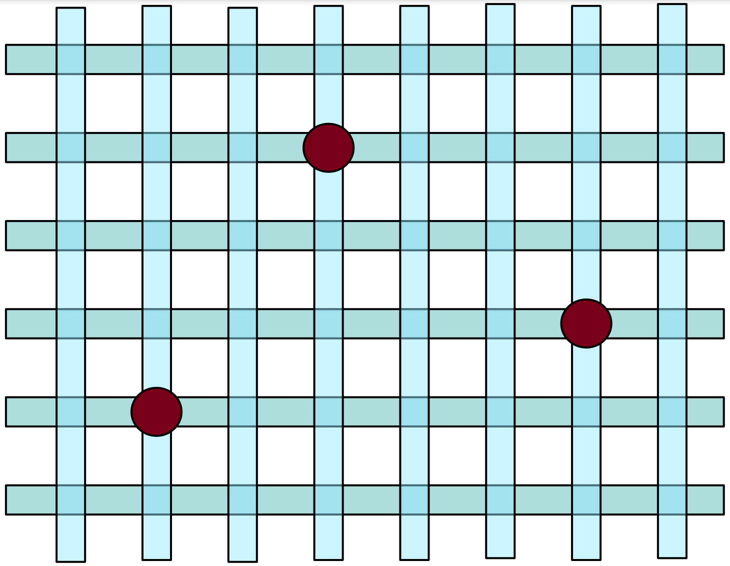
- EM in PDN optimization
  - Exploit the relationship between jL and IR in PDN optimization (see Caveats 2,3 in paper)



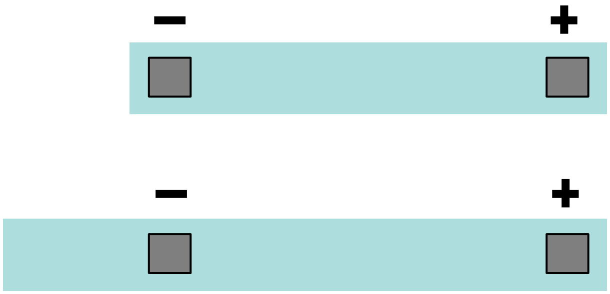
Relation between jL and IR

$$j_i = \frac{V_b - V_a}{R_i(w_i h_i)} = \frac{V_b - V_a}{\rho l_i}$$

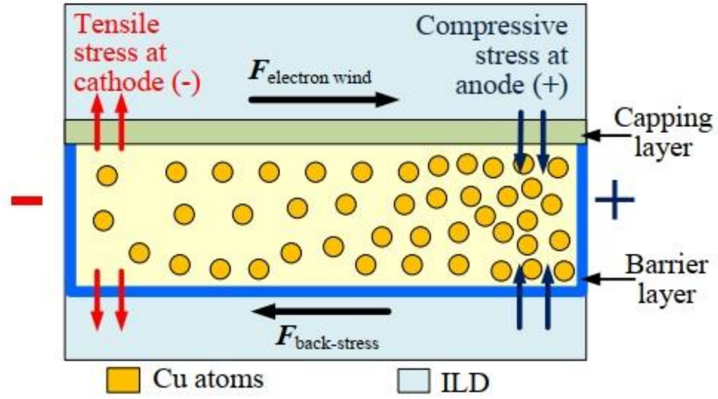
$$\sigma^b - \sigma^a = -\beta j_i l_i = -\frac{\beta}{\rho}(V_b - V_a)$$



- Using reservoirs



- Leveraging EM recovery

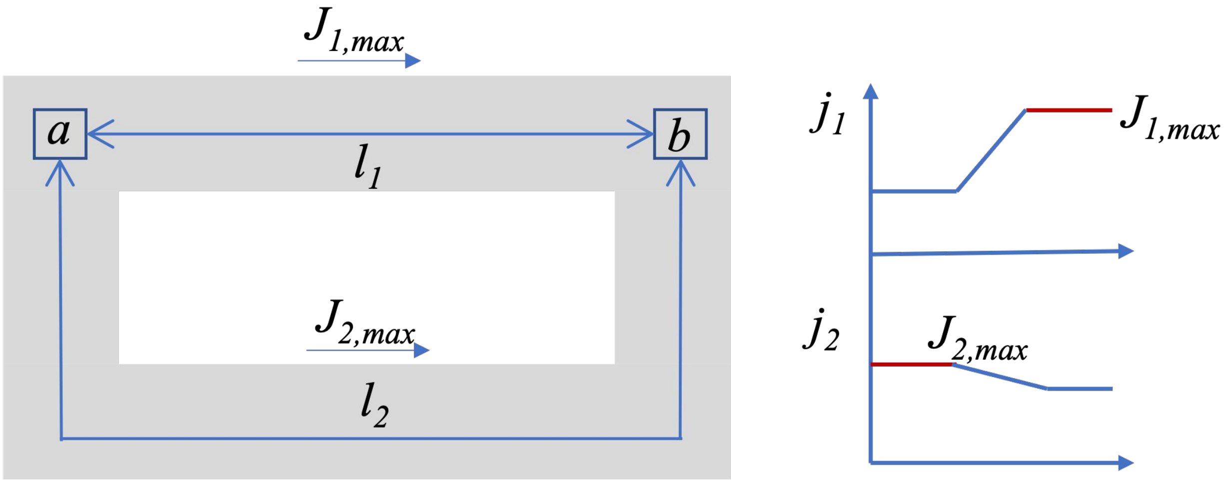


  - When power is switched off, there is no electron wind □ back-stress aids recovery

Open problems in analysis: Impact of variations

- Temperature
  - Hard to predict worst-case condition
  - Joule heating is “easy”

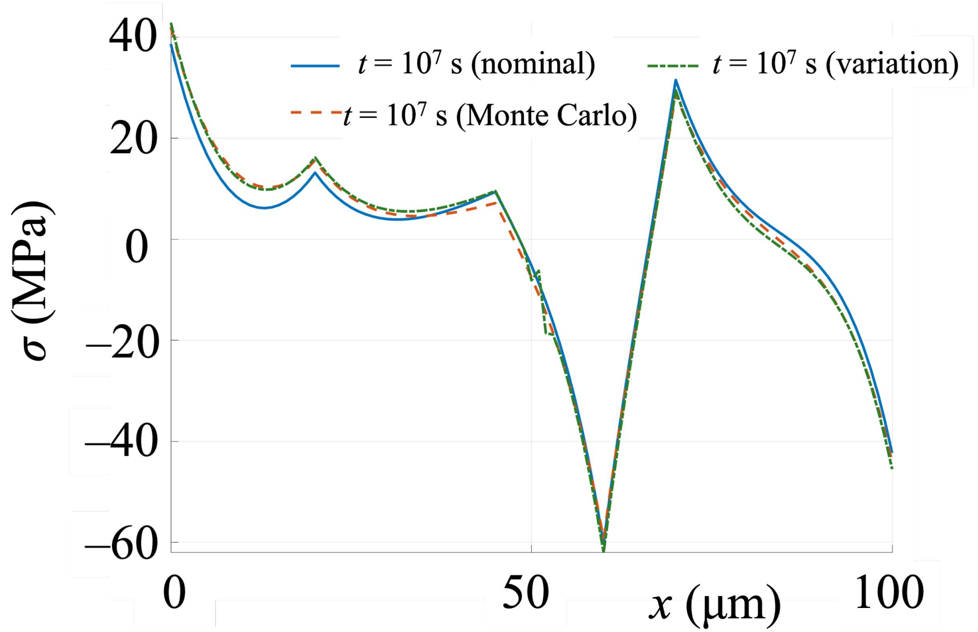
- Worst-case current impacts “jL ≡ IR”
  - May not appear at the same time, and so “KVL” may not be satisfied (Caveat 2)
  - “jL ≡ IR” neglects conservation of mass (Caveat 3) ☐ surmountable using a linear inequality



Open problems in analysis: Impact of process variations

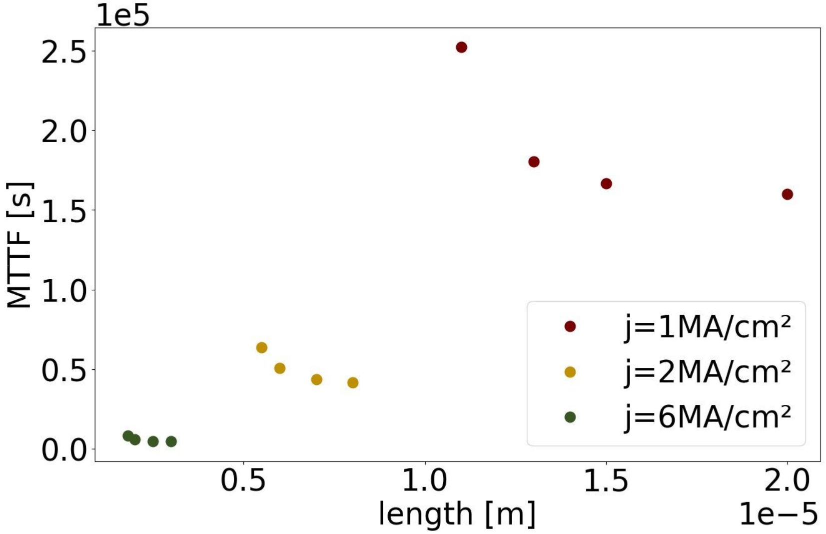
- Activation energy E_a depends on diffusion pathways, which depend on grain structure
- Exponential relationship ⇒ lognormal distribution
- Monte Carlo [Chatterjee, 2018]
- Direct variational analysis
  - Uses circuit interpretation previously shown
  - Paper shows outline of a proof that the moments of mean stress $\mathbf{m}_i$ and nominal stress $\mathbf{M}_i$ are related by a constant factor (new result!)
  - $\mathbf{m}_i = c\,\mathbf{M}_i$
  - Linear-time variational analysis for MTTF

Result on a five-segment interconnect:



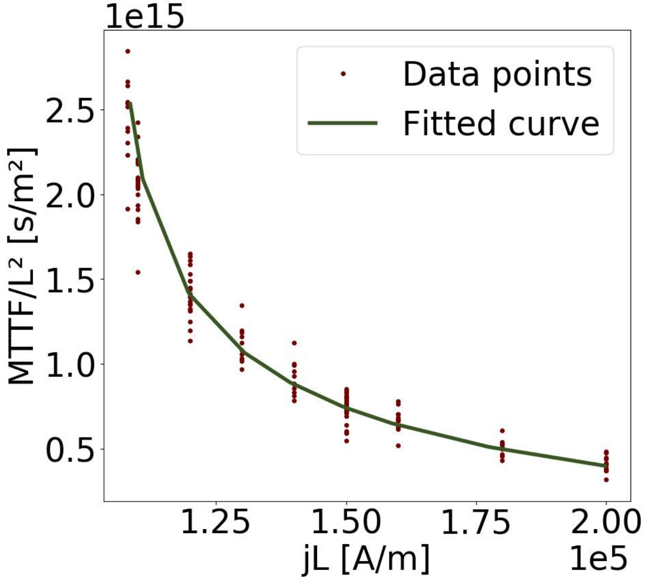
Calibrating physics-based models

- Test lines (length *L*, current density *j*)



- Korhonen's expression at $x=0$

$$jL = \frac{\sigma_{\text{crit}}}{\beta}\left(0.5 - 4\sum_{m=0}^{\infty}\frac{\exp\left(-(2m\pi+\pi)^2\kappa\left(\frac{t_{\text{life}}}{L^2}\right)\right)}{(2m\pi+\pi)^2}\right)^{-1}$$



- Curve-fit to obtain $\kappa, (\sigma_{crit}/\beta)$

$$\begin{array}{c} \dfrac{\partial\sigma}{\partial t} = \dfrac{\partial}{\partial x}\left[\kappa\left(\dfrac{\partial\sigma}{\partial x}+\beta j\right)\right] \\ \sigma \le \sigma_{crit} \end{array} \xrightarrow{\sigma' = \sigma/\beta} \begin{array}{c} \dfrac{\partial\sigma'}{\partial t} = \dfrac{\partial}{\partial x}\left[\kappa\left(\dfrac{\partial\sigma'}{\partial x}+j\right)\right] \\ \sigma' \le (\sigma_{crit}/\beta) \end{array}$$

[Content courtesy of Susann Rothe and Jens Lienig]

Conclusion

- Overview of recent advances in EM and some open challenges
- Tried to provide intuition, a list of caveats, and learnings over ~14 years of work on this topic [9 years of stumbling around and 5 years of results]
- This is an attempt at a “what they don’t tell you in all the papers” paper
- The manuscript has more information: update at https://arxiv.org/abs/2603.14318