SMT-Based Layout Synthesis Approaches for Quantum Circuits

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Outline

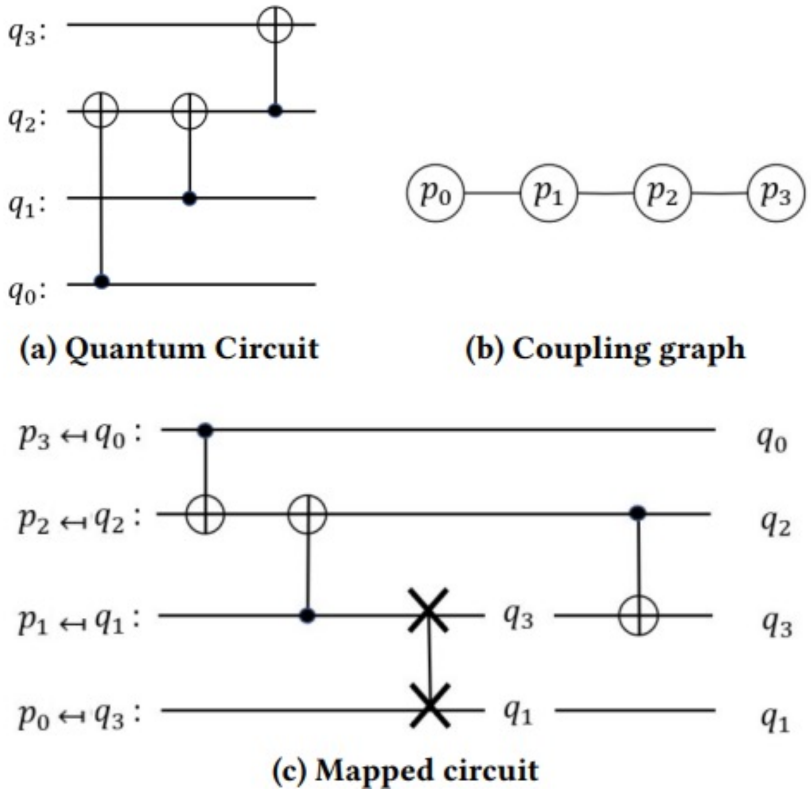
- Introduction
- Approaches
- Experimental Results
- Conclusion

Introduction (1/4)

- A quantum circuit consists of quantum gates and logical qubits.
- In layout synthesis, we need to map the logical qubits of a quantum circuit to the physical qubits on a quantum computer.
- A coupling graph specifies which pairs of physical qubits are connected in a quantum computer.
- In order to execute a two-qubit gate, its two logical qubits must be mapped to physical qubits which are connected in the coupling graph.
- If the two logical qubits of a two-qubit gate are mapped to unconnected physical qubits, SWAP gate(s) can be inserted at some additional cost to exchange the mapping such that the two logical qubits can reach two connected physical qubits.

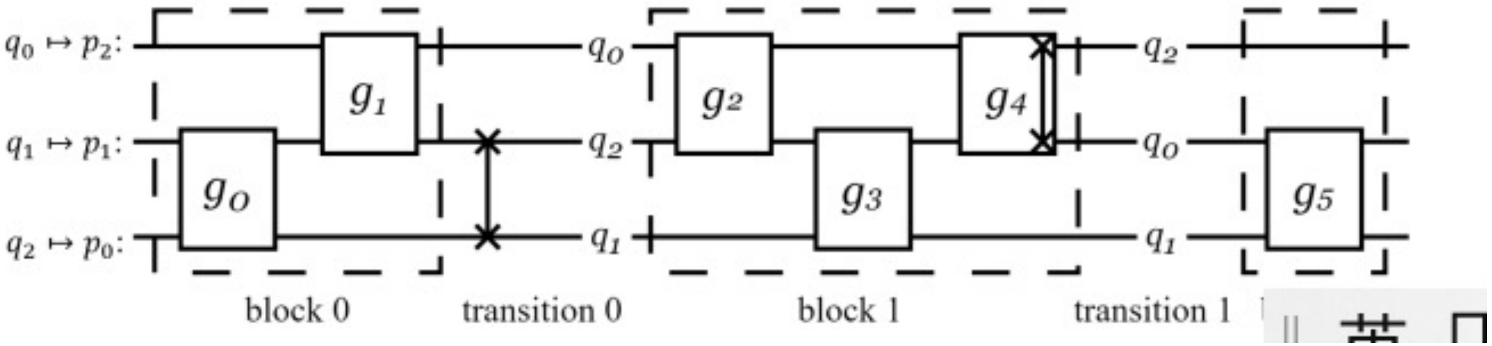
Introduction (2/4)

- Layout synthesis can be divided into two parts: initial mapping and insertion of SWAP gates.



Introduction (3/4)

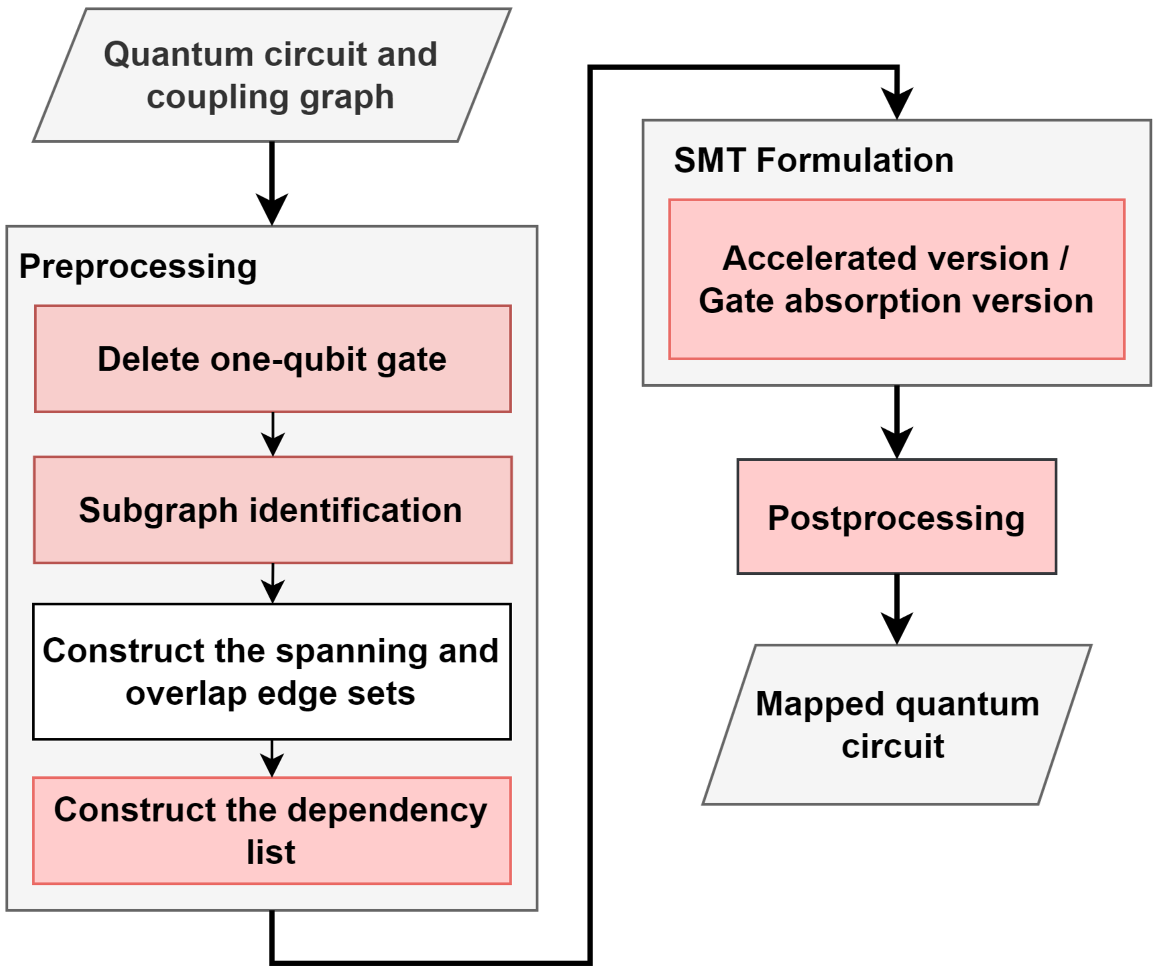
- In Ref. [22], two SMT-based layout synthesis approaches, OLSQ and TB-OLSQ, were proposed.
  - OLSQ finds an optimal solution in a specified depth, while TB-OLSQ employs a transition-based SMT formulation to speed up OLSQ but may sacrifice the solution quality.
- In Ref. [18], a SMT-based approach, OLSQ-GA, which considers gate absorption to reduce the number of explicit SWAP gates, was proposed.



Introduction (4/4)

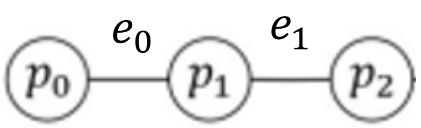
- In this paper, our focus is on minimizing the number of SWAP gates for layout synthesis.
- We present how to modify TB-OLSQ to obtain an accelerated version for runtime reduction.
- In addition, we extend the accelerated version by considering gate absorption for better solution quality.

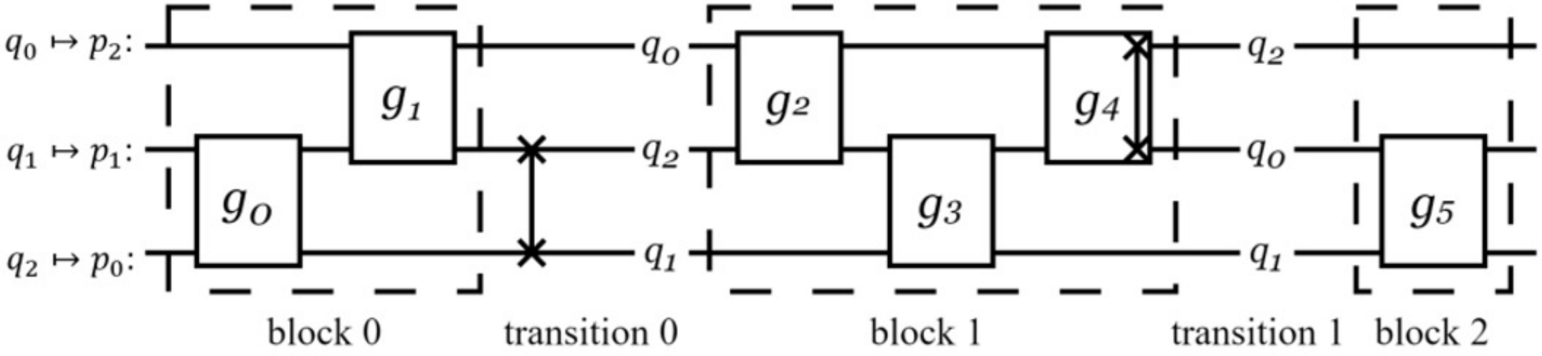
Overall Flow



Accelerated Version

- Differences from TB-OLSQ
  - Difference mapping
  - Mapping implied by time coordinates
    - No space variables
  - Dependencies
  - Swap gate insertion





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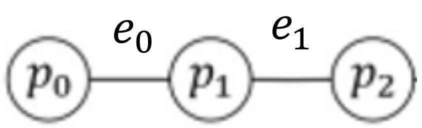
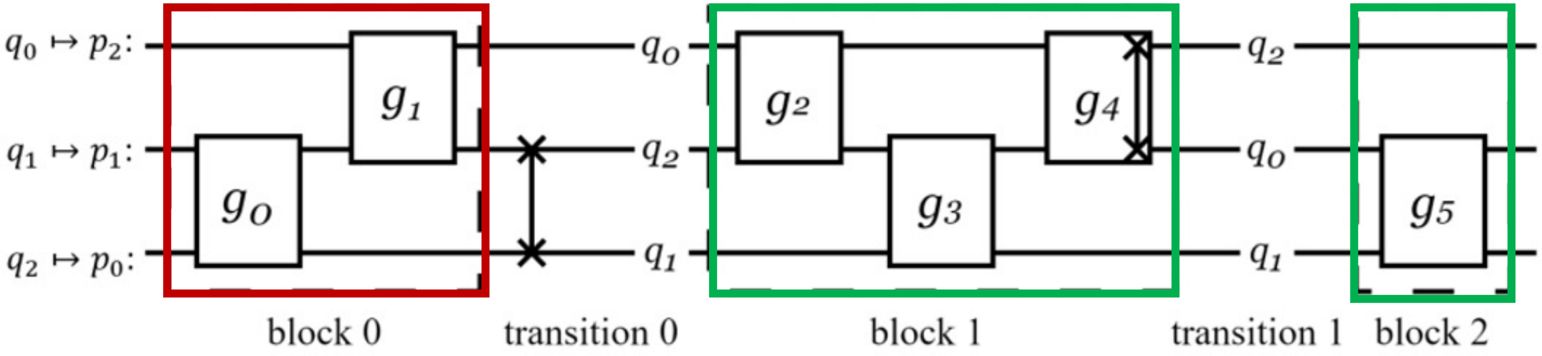


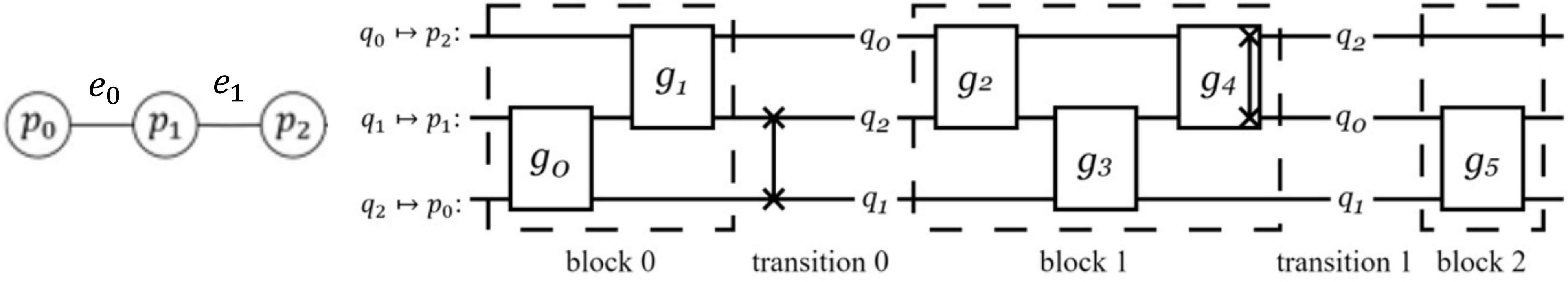
Difference mapping



Accelerated Version

- Differences from TB-OLSQ
  - Difference mapping
  - Mapping implied by time coordinates
    - No space variables
  - Dependencies
  - $(time_g == t \wedge space_g == e) \Rightarrow (\pi^t_{g.q} == e.p \wedge \pi^t_{g.\hat{q}} == e.\hat{p}) \vee (\pi^t_{g.q} == e.\hat{p} \wedge \pi^t_{g.\hat{q}} == e.p),\ \forall e \in E, \forall g \in G, 0 \leq t \leq T-1$

$$[(\pi^t_q == p) \wedge (\bigwedge_{e_k \in E_p} \sigma^t_{e_k} == 0)] \Rightarrow (\pi^{t+1}_q == p),\ \forall q \in Q, \forall p \in P, 0 \leq t \leq T-2$$



Accelerated Version

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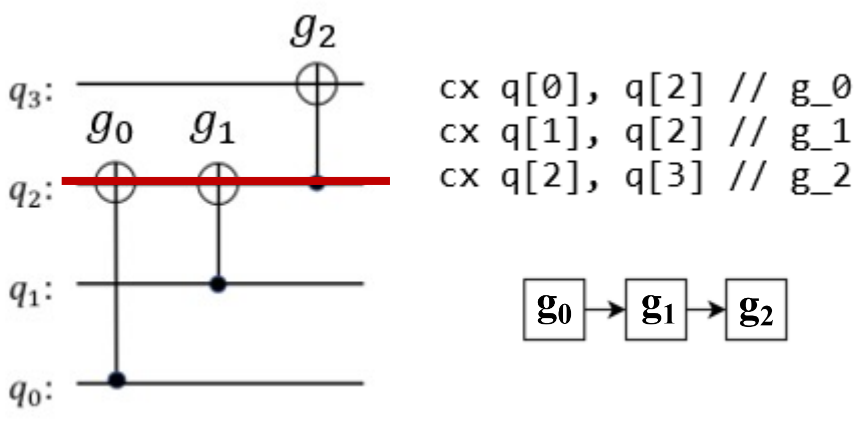
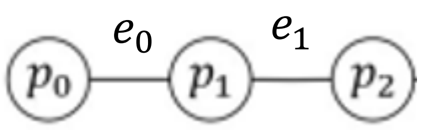
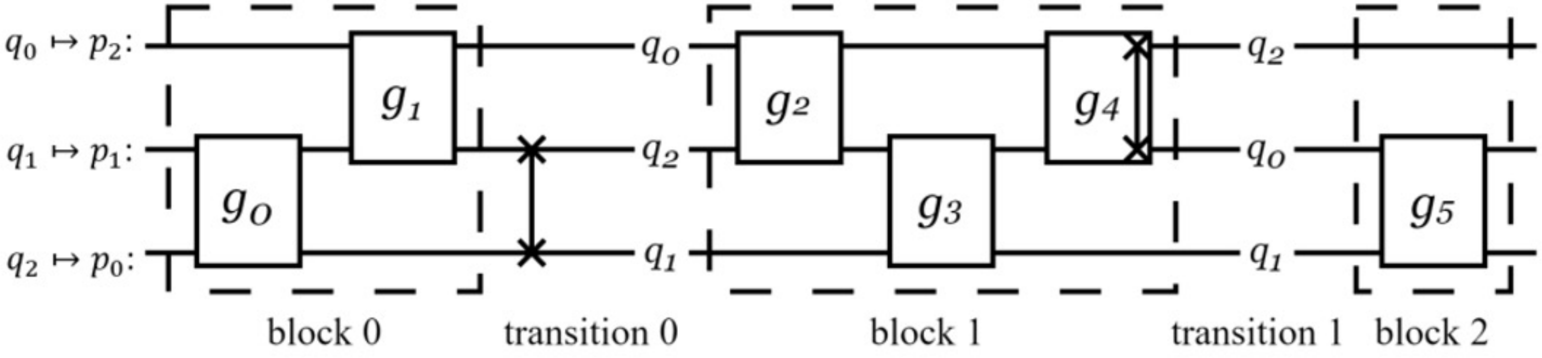
  - Swap gate insertion



Dependency list





Accelerated Version

- Differences from TB-OLSQ
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    - No space variables
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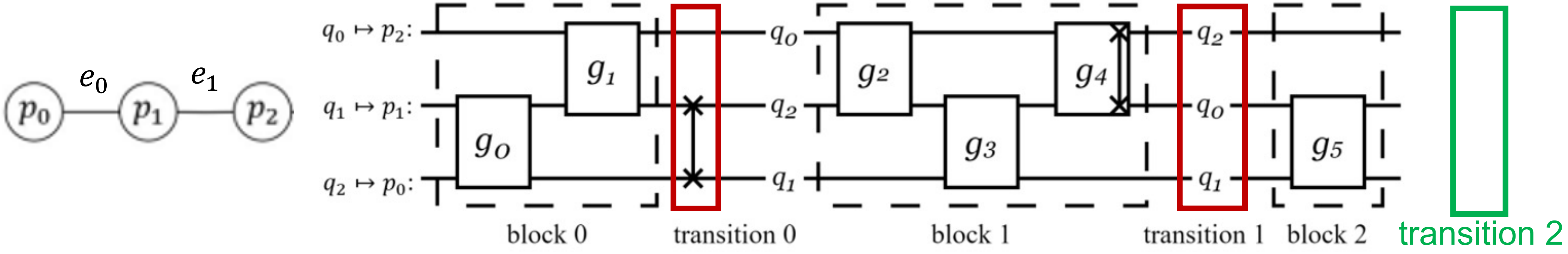
  - Swap gate insertion



Gate Absorption Version

- **Constraints in Accelerated Version**
  - Difference mapping
  - Mapping implied by time coordinates
  - Dependencies
  - Swap gate insertion
  - Mapping transformation

- **Constraints in Gate Absorption Version**
  - Difference mapping
  - Mapping implied by space and time coordinates
  - Dependencies
  - Swap gate insertion
    - Explicit Swap gate insertion
    - Absorption SWAP gate insertion
  - Mapping transformation
    - Mapping transformation
    - Update $change_e^t$

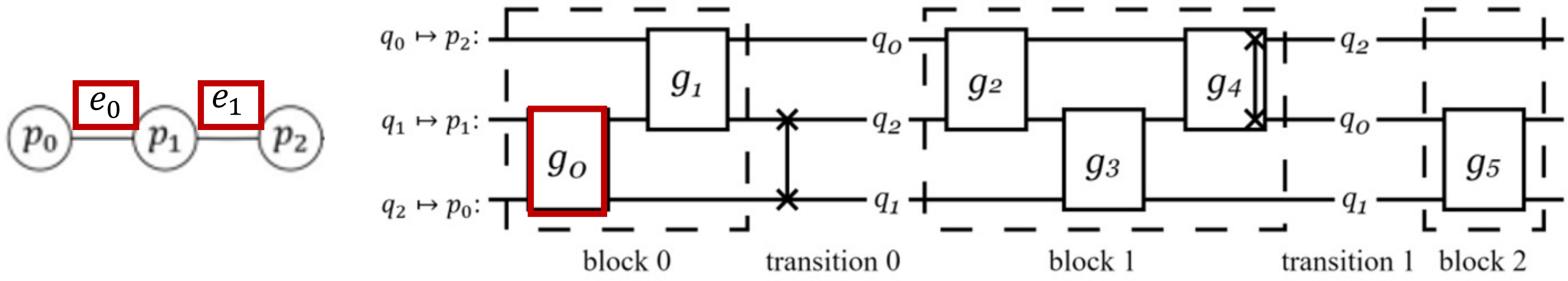
Gate Absorption Version

- Constraints
  - Mapping implied by space and time coordinates

$$space_g \in E, \forall g \in G$$

$$(time_g == t \wedge space_g == e) \Rightarrow (\pi^t_{g.q} == e.p \wedge \pi^t_{g.\hat{q}} == e.\hat{p}) \vee (\pi^t_{g.q} == e.\hat{p} \wedge \pi^t_{g.\hat{q}} == e.p), \ \forall e \in E, \forall g \in G, 0 \le t \le T-1$$

$$(space_{g_0} = e_0 \wedge time_{g_0} = 0) \Rightarrow (\pi^0_{q_1} = p_1 \wedge \pi^0_{q_2} = p_0) \vee (\pi^0_{q_1} = p_0 \wedge \pi^0_{q_2} = p_1)$$



Gate Absorption Version

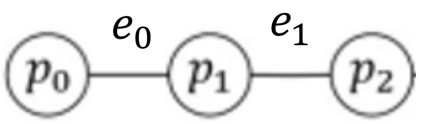
- Constraints
  - Absorption SWAP gate insertion

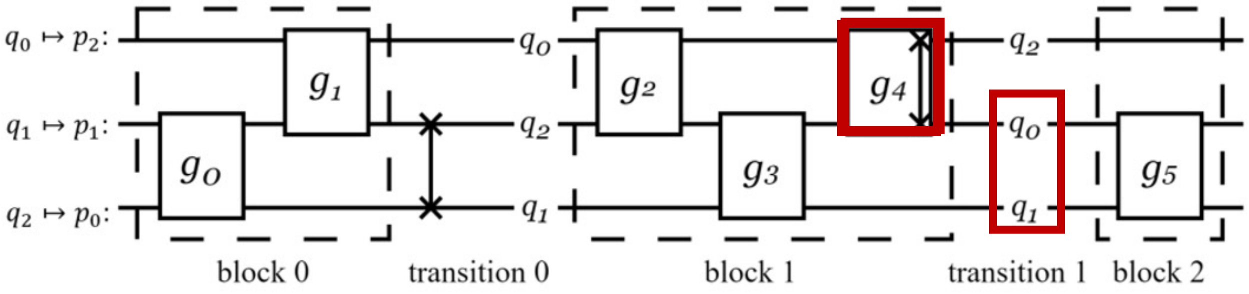
$$(time_g == t \wedge space_g == \hat{e} \wedge \lambda_g == 1) \Rightarrow (\sigma_e^t == 0) \;\; \forall e \in E, \forall \hat{e} \in O(e), \forall g \in G, 0 \leq t \leq T-2$$

$$(time_g == t \wedge space_g == e \wedge time_{\hat{g}} == t \wedge space_{\hat{g}} == e) \Rightarrow (\lambda_g == 0)$$

$$(time_g == t \wedge space_g == e \wedge time_{\hat{g}} == t \wedge space_{\hat{g}} == \hat{e}) \Rightarrow (\lambda_g == 0)$$

$$\forall (g, \hat{g}) \in D, \forall e \in E, \forall \hat{e} \in O(e), 0 \leq t \leq T-2$$

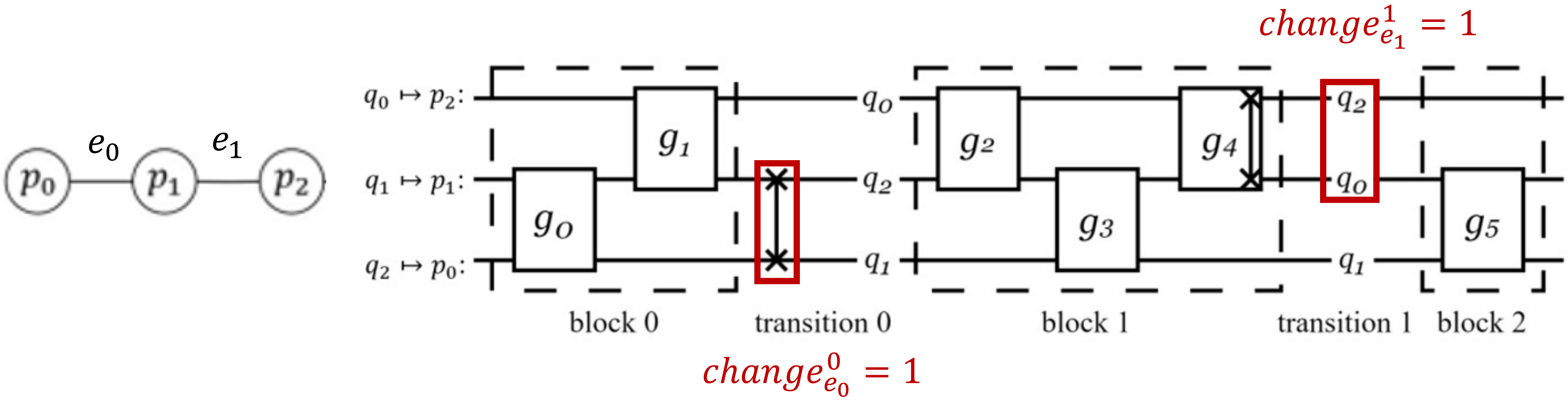




Gate Absorption Version

- Constraints
  - Update $change_e^t$

$$change_e^t == [(\sigma_e^t == 1) \vee \bigvee_{g \in G} (\lambda_g \wedge space_g == e \wedge time_g == t)] \; \forall e \in E, 0 \leq t \leq T-2$$



Experimental Results:
Comparison among [25], TB-OLSQ, and TB-OLSQ-ACC

Benchmark	Coupling graph	#Q	Qubit Mapping Checker [25]		TB-OLSQ [22]				TB-OLSQ-ACC					
			#SWAPs	time (s)	#SWAPs	#V	#C	time (s)	#SWAPs	time (s)	#SWAPs Reduction Rate cp. to [25]	#SWAPs Reduction Rate cp. to [22]	# V	# C
Benchmarks Set 1														
16QBT_queko_100_0	Aspen-4	16	0	0.283	0	2308	78069	30.696	0	1.529	0	0	337	1734
16QBT_queko_100_1	Aspen-4	16	0	0.237	0	2308	78069	29.474	0	1.598	0	0	337	1735
16QBT_queko_900_0	Aspen-4	16	0	0.318	0	20484	5471728	837.749	0	12.778	0	0	2897	14535
16QBT_queko_900_1	Aspen-4	16	0	0.248	0	20484	5471113	1447.432	0	13.403	0	0	2897	14535
20QBT_queko_100_0	Tokyo	20	0	1.552	0	2905	107904	306.878	0	5.288	0	0	421	2208
20QBT_queko_100_1	Tokyo	20	0	1.611	0	2905	107901	356.149	0	4.693	0	0	421	2210
20QBT_queko_500_0	Tokyo	20	0	3.595	0	14265	2193917	6418.524	0	21.919	0	0	2021	10211
20QBT_queko_500_1	Tokyo	20	0	2.416	0	14265	2194181	6824.817	0	21.29	0	0	2021	10207
54QBT_queko_05_0	Sycamore	54	0	79.72	0	573	11403	13594.426	0	10.876	0	0	109	1756
54QBT_queko_05_1	Sycamore	54	0	66.15	0	573	11398	7800.068	0	15.415	0	0	109	1757
54QBT_queko_900_0	Sycamore	54	0	292.323	–	–	–	>24 hrs	0	1119.115	0	0	9774	50082
54QBT_queko_900_1	Sycamore	54	0	104.312	–	–	–	>24 hrs	0	402.101	0	0	9774	50082
Geometric Mean				0.246X		6.586X	64.934X	121.227X		1X			1X	1X

Experimental Results:
Comparison among [25], TB-OLSQ, and TB-OLSQ-ACC

Benchmark	Coupling graph	#Q	Qubit Mapping Checker [25]		TB-OLSQ [22]				TB-OLSQ-ACC					
			#SWAPs	time (s)	#SWAPs	#V	#C	time (s)	#SWAPs	time (s)	#SWAPs Reduction Rate cp. to [25]	#SWAPs Reduction Rate cp. to [22]	# V	# C
Benchmark Set 2														
or	IBM_QX2	3	0	0.016	0	25	124	0.025	0	0.009	0	0	10	37
adder	IBM_QX2	4	2	0.771	1	42	348	0.11	1	0.051	50.0%	0	23	128
qaoa5	IBM_QX2	5	0	0.017	0	29	149	0.032	0	0.02	0	0	14	55
mod5mils_65	IBM_QX2	5	2	2.228	2	67	781	0.447	2	0.259	0	0	44	338
4gt13_92	IBM_QX2	5	0	0.016	0	73	741	0.012	0	0.056	0	0	36	166
4mod5v1_22	IBM_QX2	5	1	1.649	1	46	412	0.122	1	0.084	0	0	28	186
queko_05_0	Aspen-4	16	0	0.152	0	66	577	0.454	0	0.121	0	0	32	213
queko_10_3	Aspen-4	16	0	0.583	0	94	963	0.949	0	0.317	0	0	46	275
queko_15_1	Aspen-4	16	0	0.38	0	124	1442	1.584	0	0.335	0	0	61	355
or	Melbourne	3	2	0.823	2	99	1039	1.239	2	0.041	0	0	20	95
adder	Melbourne	4	0	0.017	0	70	512	0.109	0	0.016	0	0	15	60
qaoa5	Melbourne	5	0	0.019	0	69	432	0.09	0	0.017	0	0	14	55
mod5mils_65	Melbourne	5	6	1.652	6	233	4741	21.306	6	2.203	0	0	82	714
4gt13_92	Melbourne	5	12	3.266	10	387	11107	1260	10	16.921	16.7%	0	136	1336
4mod5v1_22	Melbourne	5	3	0.901	3	136	2160	3.88	3	0.31	0	0	47	364
tof_4	Melbourne	7	1	0.754	1	162	2356	1.323	1	0.211	0	0	45	346
barenco_tof_4	Melbourne	7	5	10.412	5	296	7986	57.634	5	2.42	0	0	117	1293
tof_5	Melbourne	9	1	1.892	1	206	3188	2.308	1	0.412	0	0	60	541
barenco_tof_5	Melbourne	9	7	52.139	6	399	12914	311.0648	6	11.854	14.3%	0	180	2484
mod_mult_55	Melbourne	9	13	136.153	8	244	7870	1165.525	8	73.99	38.5%	0	150	2021
vbe_adder_3	Melbourne	10	8	75.488	8	270	9536	135.464	8	6.957	0	0	176	2576
qft_10	Melbourne	10	41	450.833	–	–	–	>24 hrs	20	377.006	51.2%	0	285	4457
rc_adder_6	Melbourne	14	11	161.505	9	304	11349	1953.76	9	238.758	18.2%	0	214	3801
Geometric Mean				2.26X		2.416X	10.430X	6.032X		1X	27.5%	0	1X	1X
Benchmarks Sets 1 and 2														
Geometric Mean				1.058X		3.305X	4.542X	15.406X		1X	27.5%	0	1X	1X

Experimental Results:
Comparison among OLSQ-GA, TB-OLSQ, and TB-OLSQ-GA

Benchmark	Coupling graph	#Q	OLSQ-GA [18]				TB-OLSQ [22]		TB-OLSQ-GA				
			#SWAPs	time (s)	#V	#C	#SWAPs	time (s)	#SWAPs	time (s)	#V	#C	#SWAPs Reduction Rate cp. to [22]
Benchmarks Set 3													
or_GA	IBM_QX2	3	0	0.411	98	2773	0	0.025	0	0.01	10	37	0
adder_GA	IBM_QX2	4	1	6.56	176	8316	1	0.11	1	0.12	37	662	0
qaoa5_GA	IBM_QX2	5	0	0.82	148	5426	0	0.032	0	0.021	14	55	0
mod5mils_65_GA	IBM_QX2	5	0	3.377	300	21898	2	0.447	1	0.928	72	5422	50.0%
4gt13_92_GA	IBM_QX2	5	0	11.077	498	65833	0	0.012	0	0.059	36	166	0
4mod5v1_22_GA	IBM_QX2	5	0	1.395	188	9236	1	0.122	1	0.232	45	1424	0
queko_05_0_GA	Aspen-4	16	0	20.501	274	15094	0	0.454	0	0.126	32	213	0
queko_10_3_GA	Aspen-4	16	0	222.082	458	47131	0	0.949	0	0.329	46	275	0
queko_15_1_GA	Aspen-4	16	0	1810.191	644	99347	0	1.584	0	0.363	61	355	0
or_GA	Melbourne	3	0	1.329	230	8440	2	1.239	1	0.076	30	189	50.0%
adder_GA	Melbourne	4	0	2.169	244	13124	0	0.109	0	0.017	15	60	0
qaoa5_GA	Melbourne	5	0	2.828	328	16457	0	0.09	0	0.019	14	55	0
mod5mils_65_GA	Melbourne	5	0	17.038	672	67465	6	21.306	5	4.492	128	9864	16.7%
4gt13_92_GA	Melbourne	5	0	47.3	1110	204346	10	1260	8	25.468	216	54966	20.0%
4mod5v1_22_GA	Melbourne	5	0	7.302	416	28205	3	3.88	3	0.807	73	2557	0
tof_4_GA	Melbourne	7	0	37.293	931	123853	1	1.323	0	0.579	75	6250	100.0%
barenco_tof_4_GA	Melbourne	7	4	1186.176	1514	309678	5	57.634	3	11.897	191	70149	40%
tof_5_GA	Melbourne	9	0	113.575	1304	227458	1	2.308	0	1.255	101	16872	100.0%
barenco_tof_5_GA	Melbourne	9	5	7254.341	2244	645012	6	311.0648	5	39.865	296	270750	16.7%
mod_mult_55_GA	Melbourne	9	4	13145.59	1234	274054	8	1165.525	4	127.306	245	144956	50.0%
vbe_adder_3_GA	Melbourne	10	2	5679.879	1602	437593	8	135.464	4	34.768	291	275841	50.0%
qft_10	Melbourne	10	–	>24 hrs	–	–	–	>24 hrs	0	1798.748	479	1424221	0
rc_adder_6_GA	Melbourne	14	–	>24 hrs	–	–	9	1953.76	7	392.802	357	646210	22.2%
Geometric Mean				56.993X	8.126X	22.399X		3.198X		1X	1X	1X	38.9%

Conclusion

- We accelerated the prior work TB-OLSQ through removing redundant constraints, modifying constraints, reducing the dependency list, and identifying the specific subgraph.

- We extended the accelerated version to develop a transition-based SMT formulation for considering gate absorption.

- Encouraging experimental results were shown to support our approaches.

Thank you for listening.