Design & Analysis of Physical Design Algorithms

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Outline

- Introduction
- Problem Transformation: Upper-Bound
- Practical Implication of Lower-Bound Analysis
- Greedy Algorithms and their proofs
- Greedy vs Global
- Approximation Algorithms
- Probabilistic Algorithms
- Conclusions

- Main message:
  - we need more/better algorithms
  - Better understanding (less hacks)

Introduction

Some points to consider

- Physical Design Problems are getting harder
  - Size, Concurrent optimization, DSM, …
- Novel/effective algorithms are essential in coping with the complexity
  - (mincut vs congestion)
- Analysis of these algorithms are of fundamental importance
  - We can then concentrate on other issues/parameters
- Novel algorithmic paradigms need to be tried

In this talk

- Talk about paradigms that have been (can be) used
  - Upper-bound transformation
- Seemingly unimportant concepts can be very powerful
  - Proof of a greedy algorithm
- There are LOTS of things that we still do not understand (and yet seem important in making progress)
  - congestion
- "We" need to make research progress (abstract/long term in addition to the usual hacks).

Problem Transformation: Upper Bound

Formal Definition

- *A is X(n)-transformable to B:*
  - *The input to A is converted to a suitable input to B*
  - *Problem B is solved*
  - *The output of B is transformed into a correct solution to problem A*
    - *Steps 1 & 3 takes X(n) time*
- *Upper Bound via Transformability:*

If B can be solved in T(n) time and A is X(n)-transformable to B, then A can be solved in T(n) + O(X(n)) time.

- Quality of the solution to A?
  - Bad example: finding min of a list via sorting O(n) transform
  - Good example: element uniqueness via sorting O(n) transform

Multi-way Partitioning Using Bi-partition Heuristics [Wang et al]

Input:

- A target graph to be partitioned.
- k: number of target partitions.

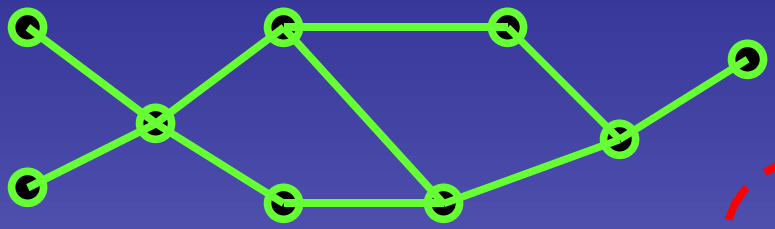
Output:

Each vertex in the target graph gets assigned to one of the target partitions.
Numbers of vertices among target partitions are “the same” (balanced).

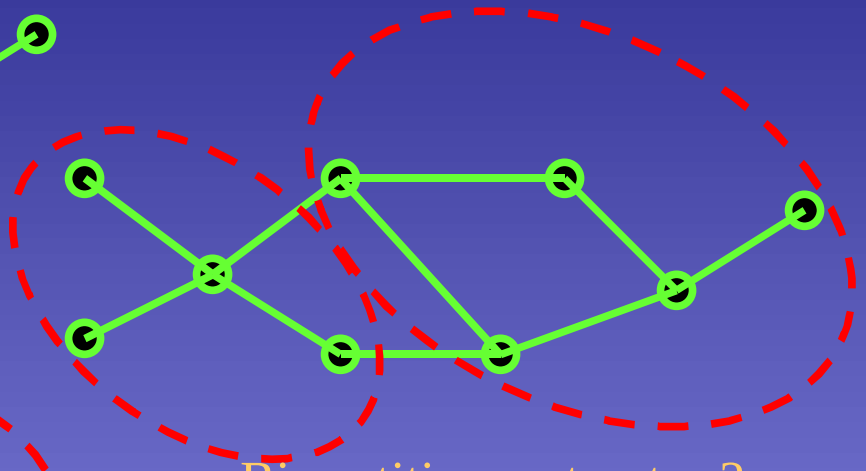
Objective:

The number of edges between target partitions (net-cut) is minimized.

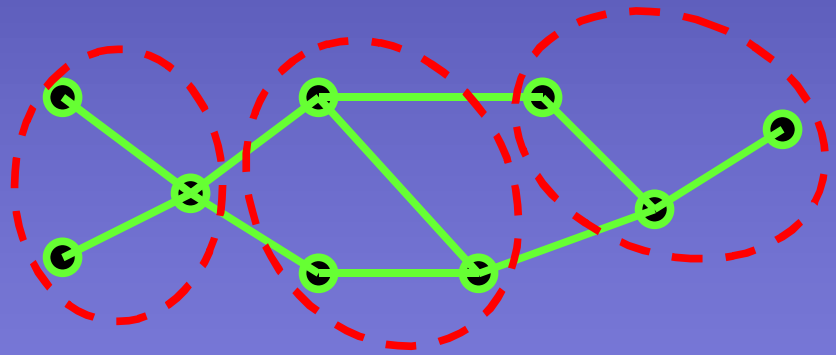
Example



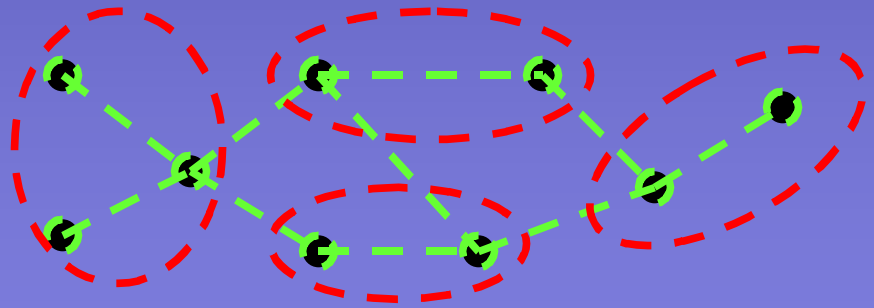
Target Graph



Bi-partition, net-cut = 2

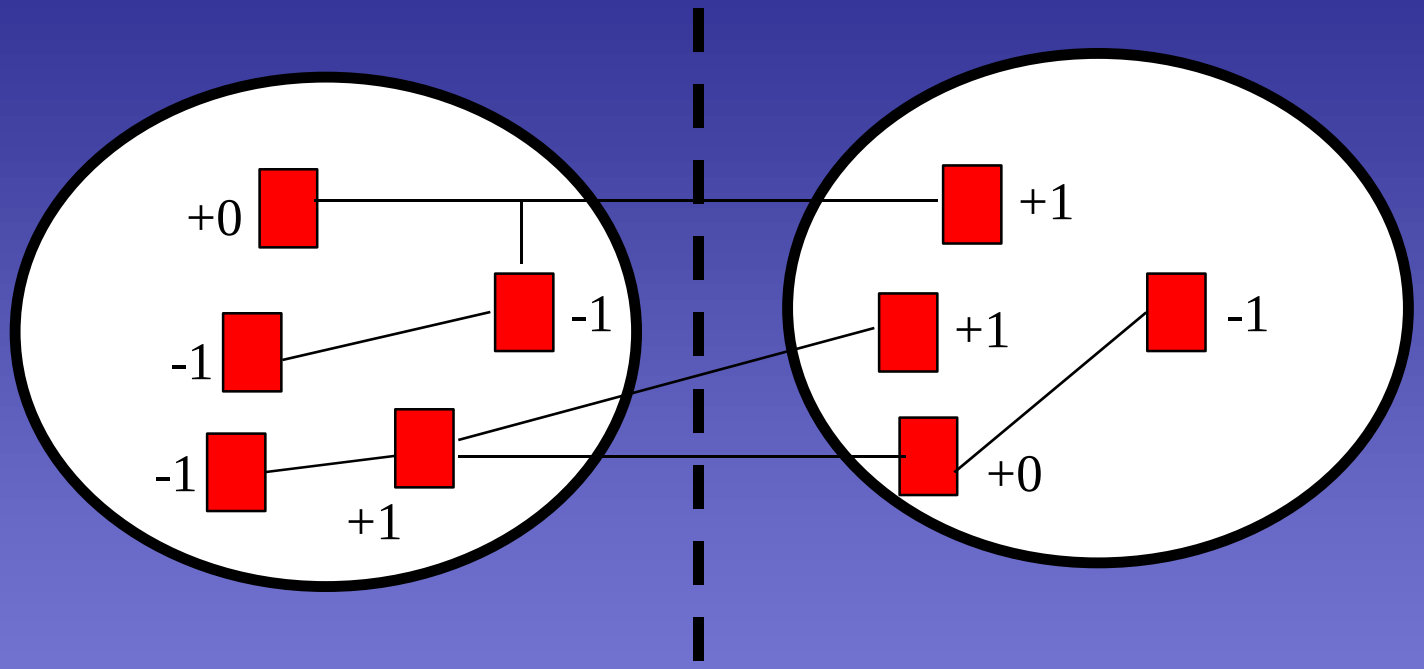


3-way partition, net-cut = 4



4-way partition, net-cut = 5

FM (and FM++): The Industry Standard for Bi-Partitioning



- Each vertex has a “gain” associated with it.
- The vertex with the biggest gain will be moved.

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Possible Approaches For Multi-Way Partitioning

- Direct extension of FM. Each target vertex has k-1 possible moving destinations.
- Using bi-partitioning heuristics (problem transf):
  - Hierarchical approach: Recursively bi-partition the target graph. (1+2+4 bipartition to do 8-way)
  - Flat approach: Iteratively improve the k-way partition result by performing local bi-partitioning (C(8,2) = 28 or more bipartition to do 8-way)
  - Question: which one is more powerful?

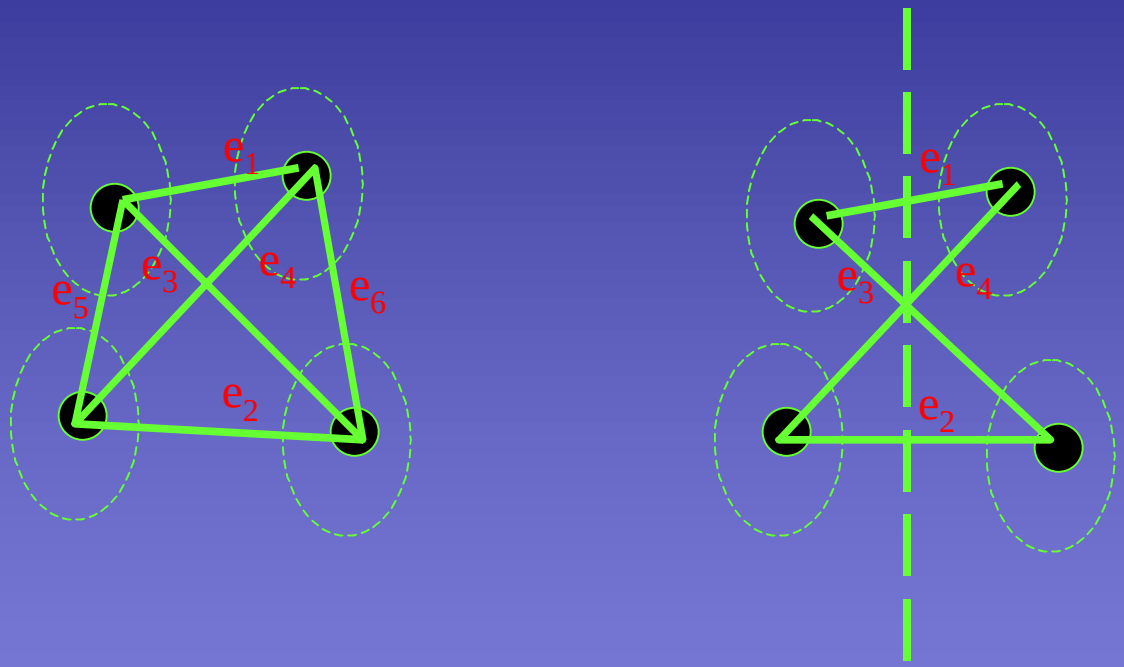
Why problem transformation?

- The direct extension of FM approach does not yield good partitioning results in general.
- The state-of-the-art bi-partitioning tools are very effective.
- It is straightforward (little R&D) to solve multi-way partitioning problem using existing bi-partitioning tools via hierarchical or flat approaches.

Bi-partition Algorithms

- A δ-approximation algorithm: the bi-partition result is less than or equal to δC_{opt}.
- An α-balanced bi-section problem: the number of nodes in each partition is between αn and $(1-\alpha)n$. (A perfectly balanced bi-section problem is a 0.5-balanced problem.)

The First Cut



$$Cut_1 = e_1 + e_2 + e_3 + e_4 <= e_1 + e_2 + e_3 + e_4 + e_5 + e_6 <= \delta\ OPT$$

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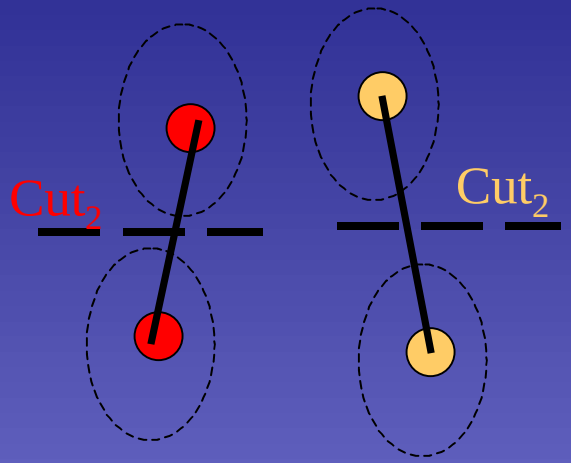
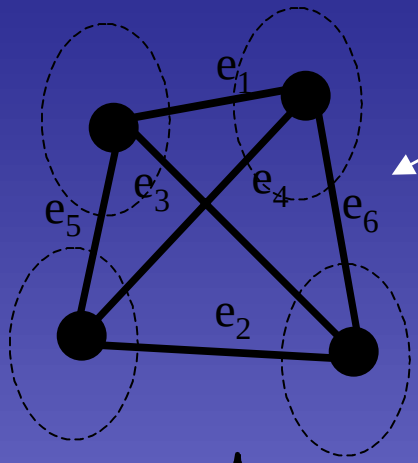
The Second Cut



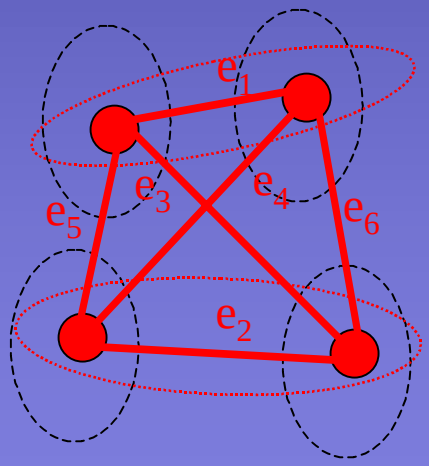


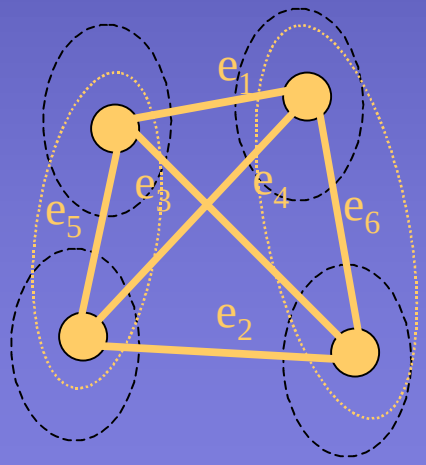
Optimal solution

$Cut_2 <= \delta (e_1 + e_2 + e_3 + e_4 + e_5 + e_6)$

$Cut_2 <= \delta (e_1 + e_2 + e_3 + e_4 + e_5 + e_6)$

$Cut_2 <= \delta OPT$





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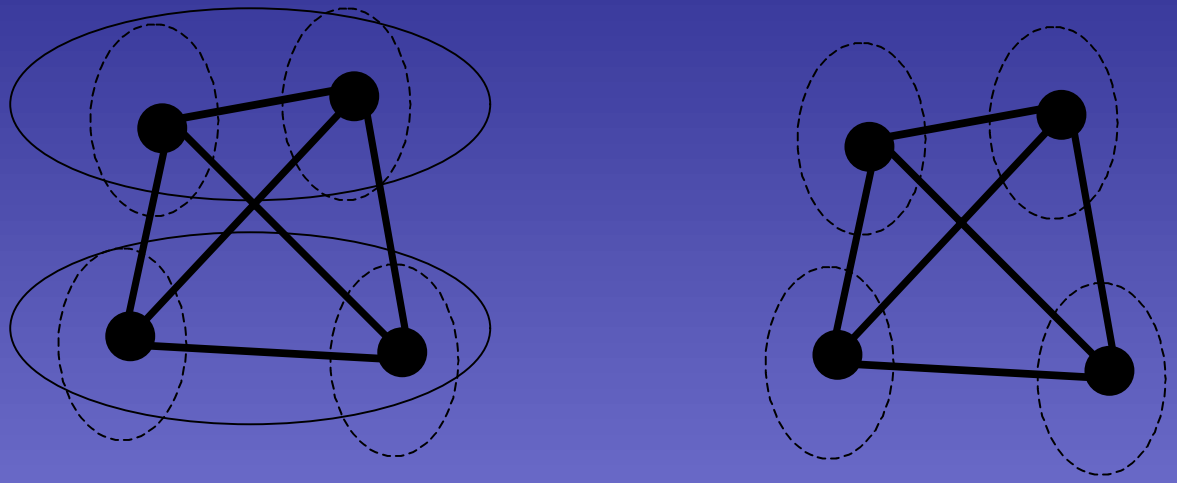
Conclusion

- For hierarchical approach:

$$C_{hie} = O(\delta \log k) C_{opt}$$

The Flat Approach



$$C_{hie} = O(\delta k) C_{opt}$$

Variations of Flat Approaches

- Random: randomly pick two partitions at a time.
- Exhaustive: use a specific sequence to exhaustively pick all possible paris of partitions.
- Cut-based: pick a pair or two most tightly or loosely connected partitions.
- Gain-based: pick a pair of two partitions between which the cutsize reduction is maximum or minimum during last pass.

Multi-pass Flat vs. Hierarchical

circuit	best of allway		hierarchical		hie. vs. all-way	
	net-cut	runtime(s)	net-cut	runtime(s)	%improv.	speedup
fract	64	23.5	56	1.21	12.5%	19.4
struct	133	627.6	128	5.74	3.8%	109
p1	145	411.7	150	5.17	-3.4%	79.6
p2	455	1979	450	12.5	1.1%	158
biomed	305	1995	213	13.18	30.2%	151
in1	203	1249	175	9.68	13.8%	129
in2	979	5006	898	44.9	8.3%	111
in3	2210	5512	2070	80.1	6.3%	68.8
avqs	532	4485	532	42.6	0%	105
avql	532	24315	543	47.8	-2.1%	509
avg.					7.1%	144

The hierarchical approach is more effective than the all-way approach.

Conclusion

- The hierarchical approach is the correct way to solve the multi-way partition problem using a state-of-the-art bi-partitioner.
- Problem transformation can be very powerful (or very weak)
- A little analysis goes a long way (and results can be counter intuitive)

Practical Implications of Lower Bound Analysis

Formal Definition

- *A is X(n)-transformable to B:*
  - *The input to A is converted to a suitable input to B*
  - *Problem B is solved*
  - *The output of B is transformed into a correct solution to problem a*

- *LOWER Bound via Transformability:*

If A is known to require T(n) time and A is X(n)-transformable to B, then B requires at least T(n) - O(X(n)) time.

Proofs of NP-Completeness are important because

- Knowledge of NP-Completeness points out that research must be directed towards finding efficient heuristics and not optimal algorithms

- **The proof itself gives important information about the problem. It can give useful directions to algorithm designers.**

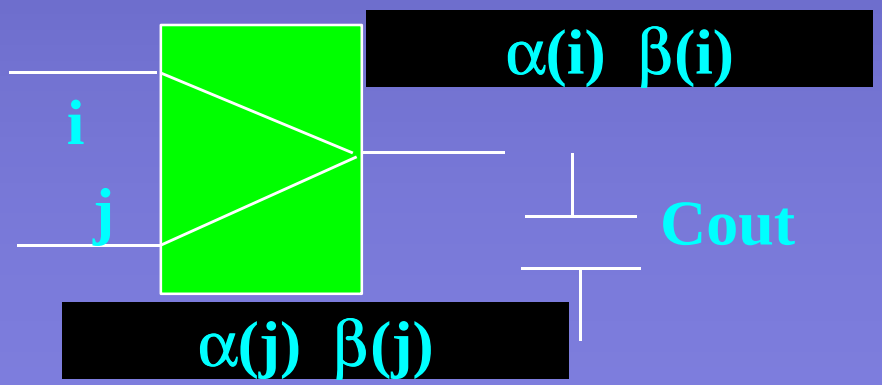
Case Study: Global Buffer Insertion

- Problem NP-Complete in the Load Dependent Delay Model

$$\delta(i) = \alpha(i) + \beta(i)^* \text{ Cout}$$

$\alpha(i)$: Internal Delay from pin i to output

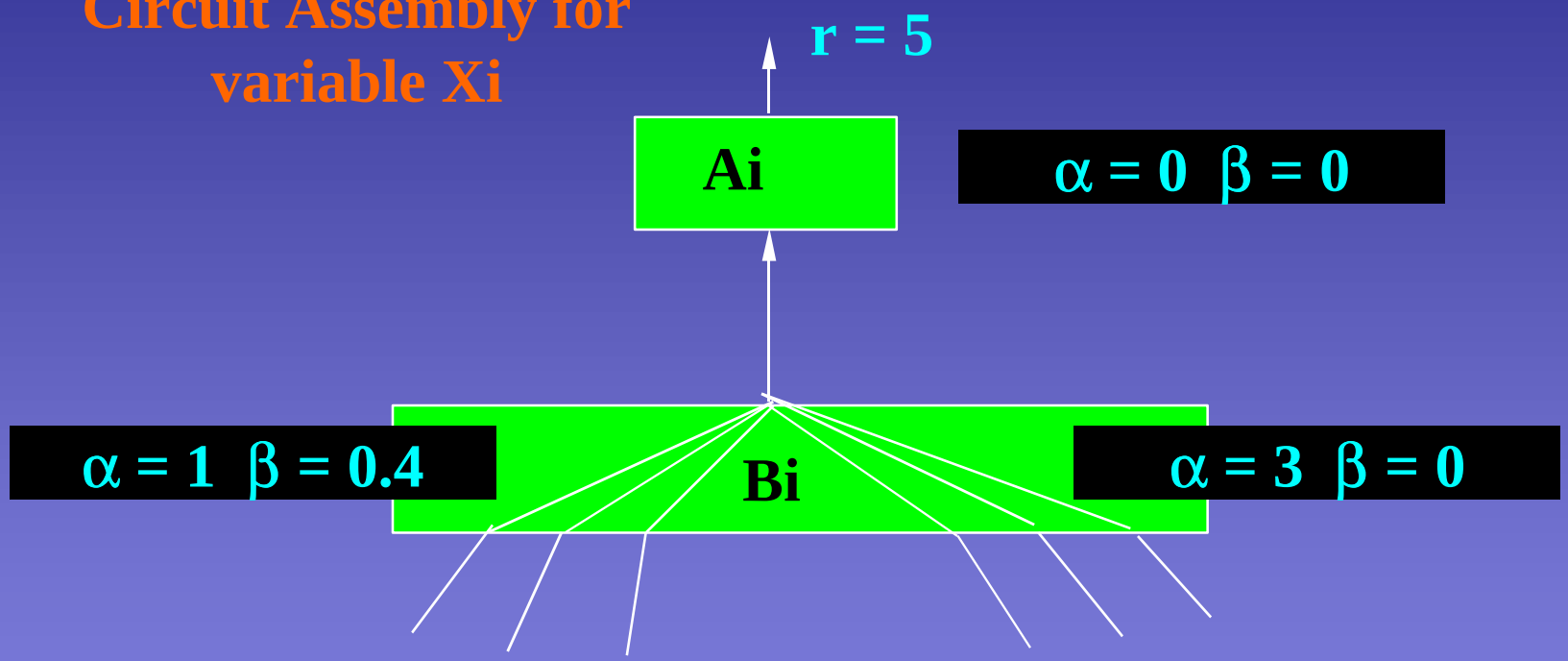
$\beta(i)$: Load Sensitivity



Transforming 3SAT to Buffer Insertion

3SAT Problem : C = (Xi + Xj + Xk). (Xi' + Xl + Xm')……..

Circuit Assembly for variable Xi



Connected to Clauses Where Xi appears as positive phase

Connected to Clauses Where Xi appears as negative phase

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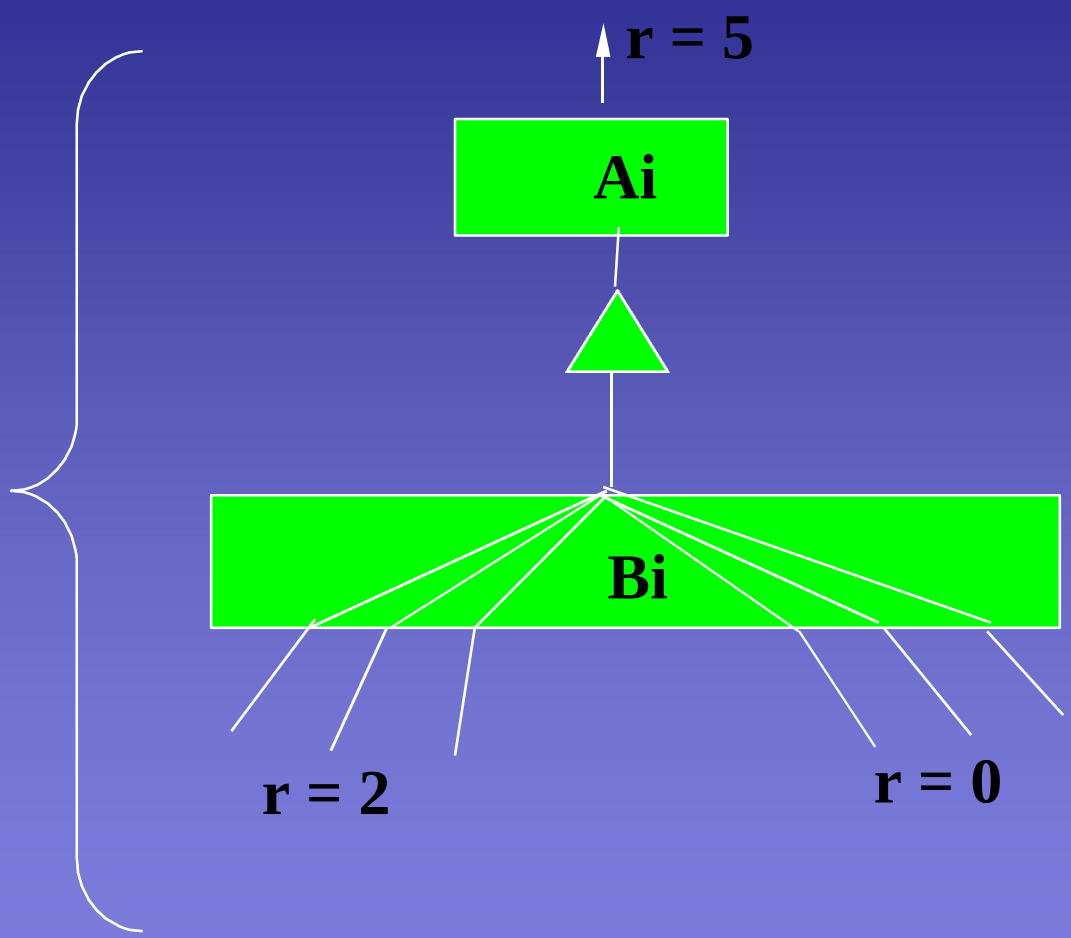
Transforming 3SAT.....

If the net BiAi is buffered then the required time at one set of fanins is 2 and at other set of fanins is 0



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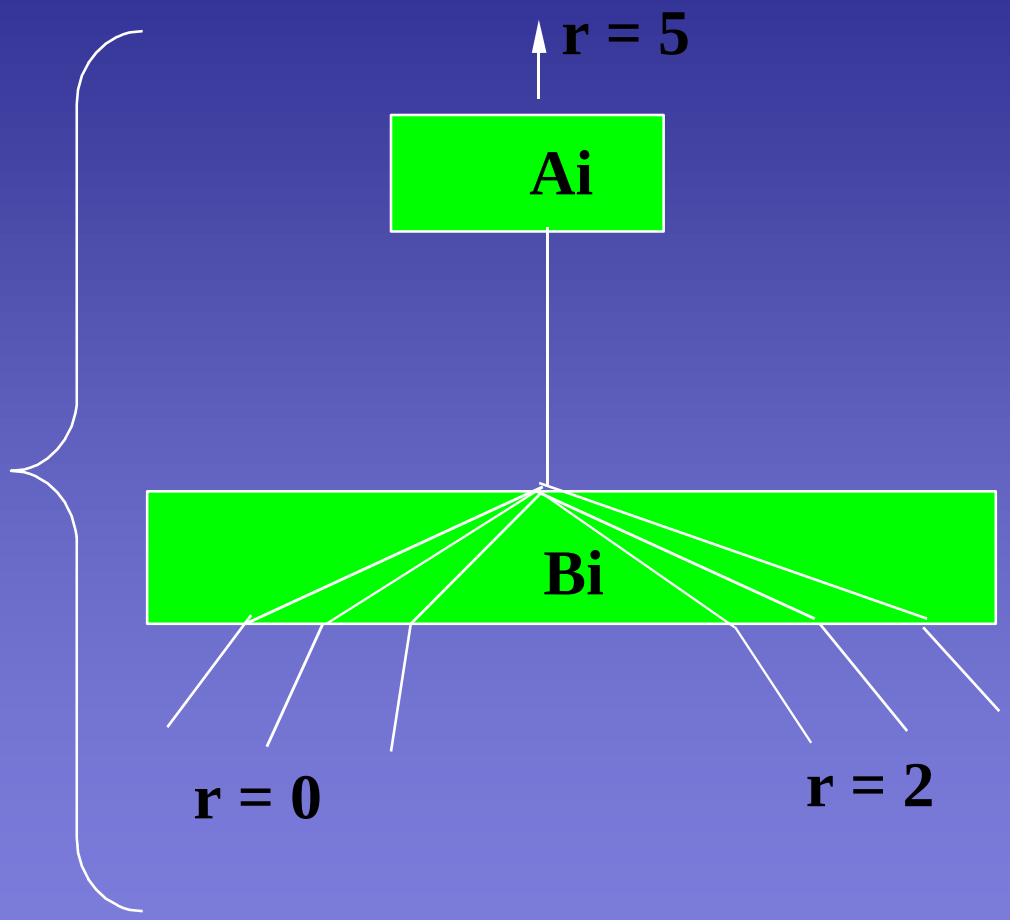
Transforming 3SAT.....

If the net BiAi is not buffered then the required time behavior becomes opposite



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Observations from Transformation

- Buffer Insertion on net BiAi helped one set of fanins and was not beneficial for the other. No buffer insertion had the opposite effect.
- This happens primarily because the load sensitivity (β) of gate Bi is different for different sets of input pins
- The problem became NP-Complete primarily because the load sensitivity (β) for different input pins of a gate is different

Observation from Transformation

- Since each input pin has different load sensitivity, an optimal buffering solution w.r.t. a particular input pin of the net connected at the output of the gate, may not be optimal w.r.t. other pins. Hence the problem is NP-Complete

All these observations were made directly from the transformation and the proof of NP-Completeness

Practical Implications

- If all gates of a circuit have the same load sensitivity then the problem is optimally solvable (if the local problem is optimally solvable).
- Fanout Optimization on substructures which have similar value of β will be more effective
- This also suggests methodologies for partitioning the circuit

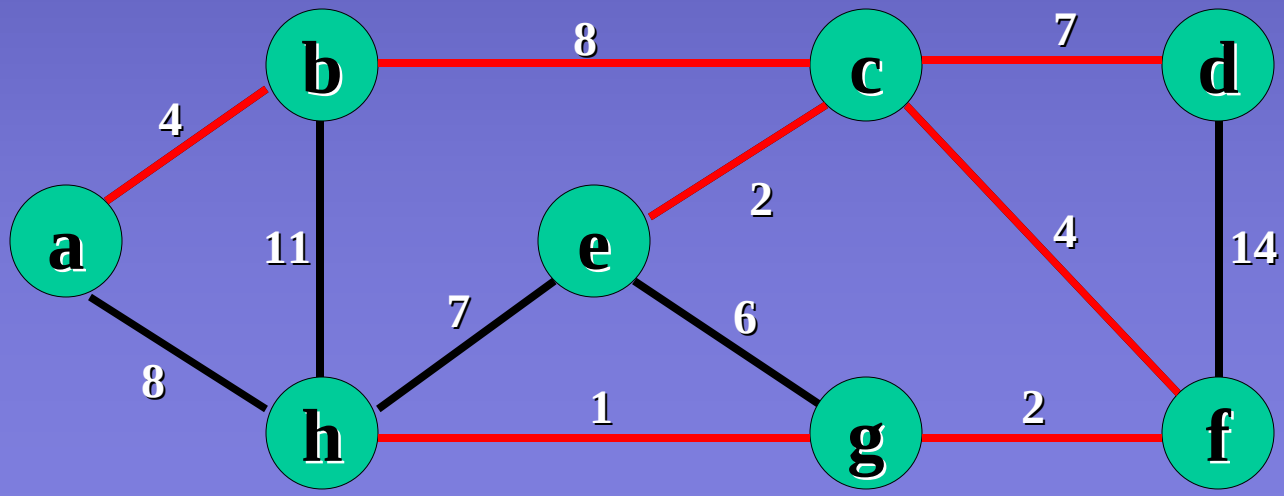
Conclusion

- Proofs of NP-Completeness can give a deep insight into the optimization problem. They also assist algorithm designers in finding useful ways of solving the problem.
- Who will prove these things?

Proof of Greedy Algorithms

Greedy Algorithms

- Make a locally optimal choice to achieve a globally optimal solution
- Correct greedy algorithm often the simplest
- Example: Prim's Minimum Spanning Tree algorithm
  - Iteratively grow the spanning tree
  - Select edge which adds minimum value at each step



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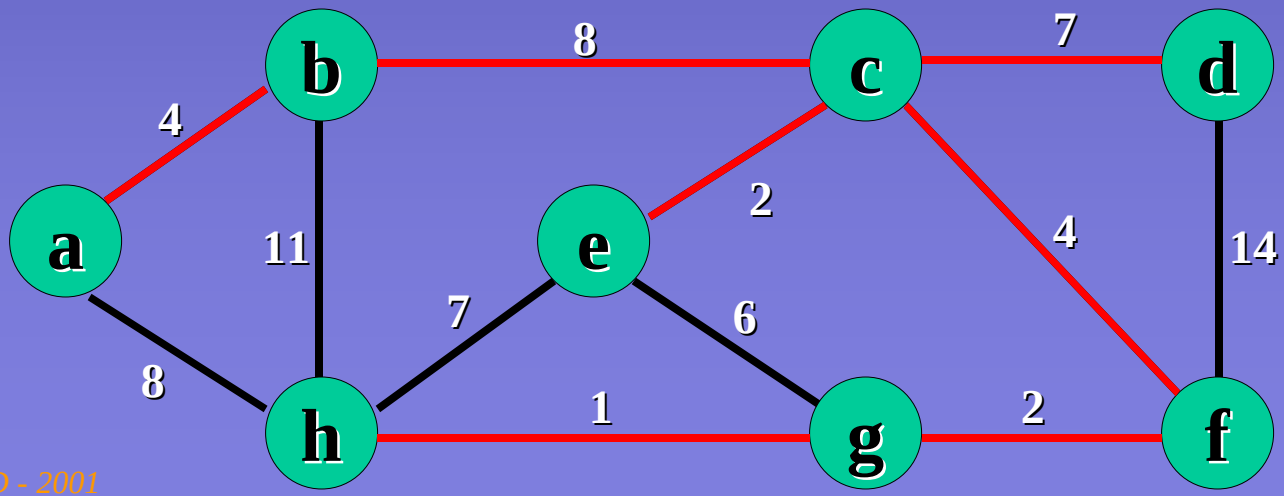
Proof of Greedy Algorithm Correctness

Correct greedy algorithms exhibit two properties:

- Optimal substructure
  - Optimal solution to the problem contains optimal solutions to sub-problems
  - Stopping Prim's MST algorithm at any instant gives optimal MST for the currently "spanned" nodes
- Greedy choice property
  - Globally optimal solution can be built using a series of locally optimal choices



Optimal Substructure

MST for {a,b,c}

Greedy Choice

Next edge is minimum wieghted connected edge (c,e)

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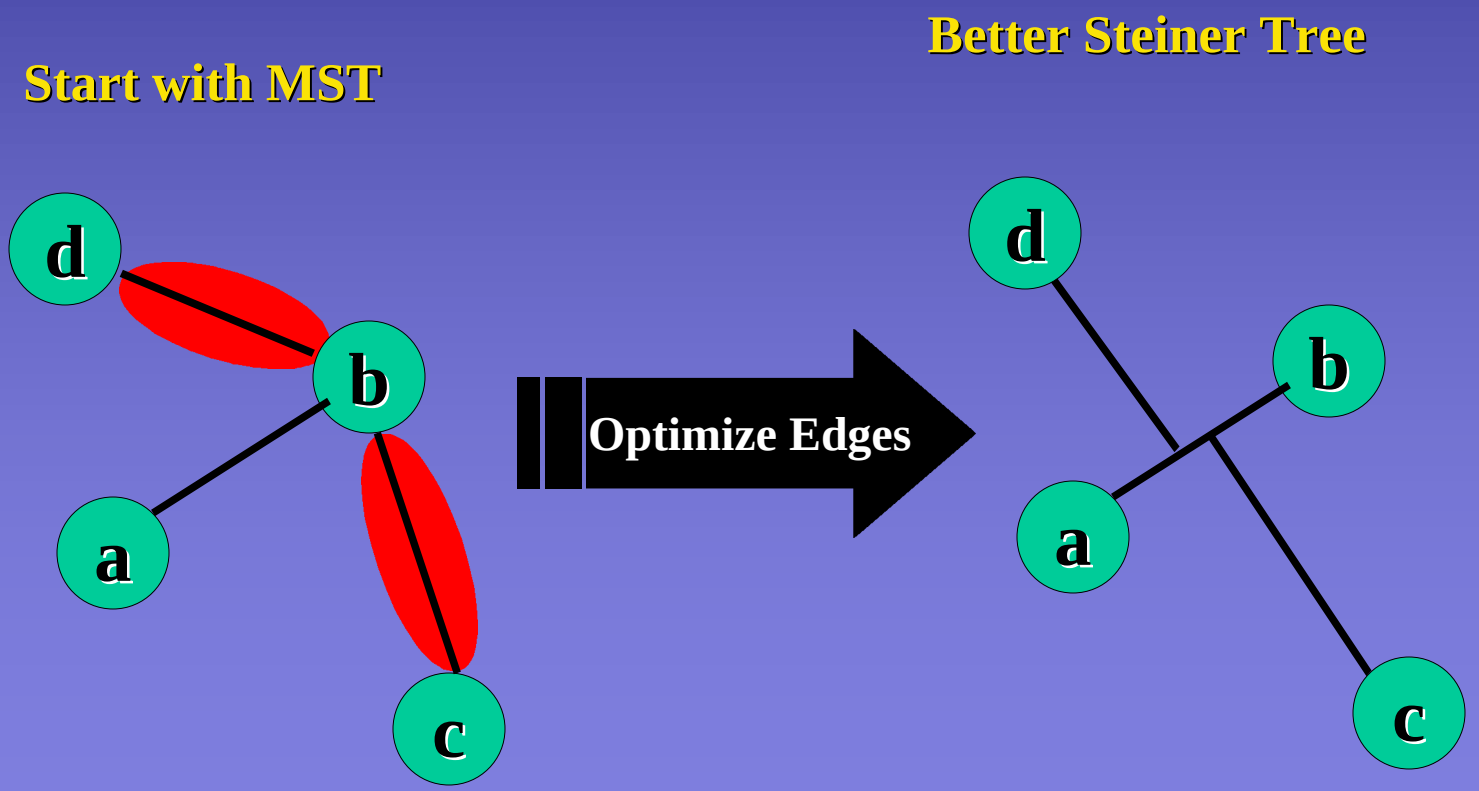
Greedily Solving NP-Complete Problems

- Use greedy solution as an optimization starting point
  Add more complex optimizations to greedy algorithm



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Use of Greedy Algorithms

- Simple
- Analysis may reveal interesting properties
- Can be used as heuristics in solving related problems

Greedy vs Global

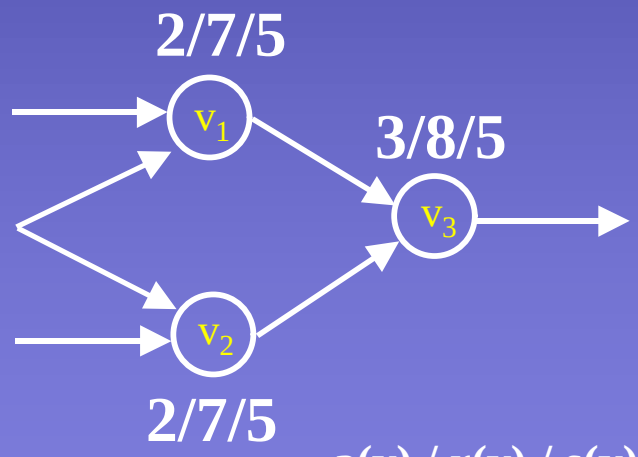
Slack Assignment Problem

- Combinational network ⟶ Directed-acyclic graph
- Given a delay model and timing requirement, calculate node slack: $s(v) = r(v) - a(v), \quad v \in \mathbf{V}$
- Slack distribution: $\mathbf{S(V)} = [s(v_1), s(v_2), \ldots, s(v_n)]$
  total slack: $|\mathbf{S(V)}| = \Sigma\, s(v_i)$
- Slack assignment: assign incremental delays
  $\Delta\mathbf{D(V)} = [\Delta d(v_1), \Delta d(v_2), \ldots, \Delta d(v_n)]$ to $\mathbf{V}$

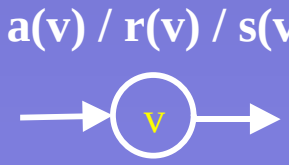
  If $\mathbf{S}_{\Delta}(\mathbf{V}) \geq 0$, the slack assignment is effective and $|\Delta\mathbf{D(V)}|$ is called effective slack.

Slack Assignment Problem

- Potential slack: maximum effective slack $|\Delta \mathbf{D}_m(\mathbf{V})|$
- A slack assignment is (global/) optimal if it leads to potential slack.



Unit delay model



- Slack distribution $\mathbf{S}(\mathbf{V})$ = [5 5 5]
- $\Delta \mathbf{D}(\mathbf{V})$ = [2 1 1] is a slack assignment
- $\Delta \mathbf{D}_m(\mathbf{V})$ = [5 5 0] is a better slack assignment
- Potential slack = $|\Delta \mathbf{D}_m(\mathbf{V})|$ = 10

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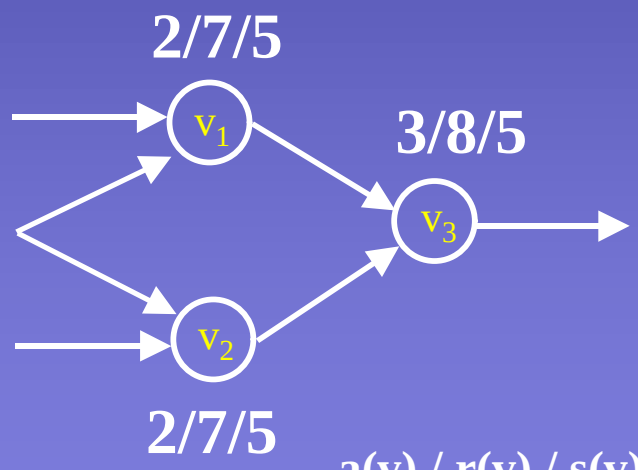
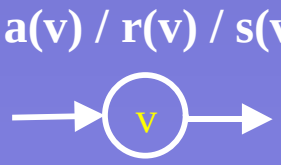
ZSA: Greedy

- Slack represents an upper bound of delay increase while keeping timing performance
- Zero-Slack Algorithm (ZSA) [Nair et al, 1989]:



Unit delay model



- Assign $\Delta d = 2.5$ to v_1 and v_3 each
- Slack distribution becomes [0 2.5 0]
- Assign $\Delta d = 2.5$ to v_2
- We have slack assignment [2.5 2.5 2.5] with effective slack of 7.5

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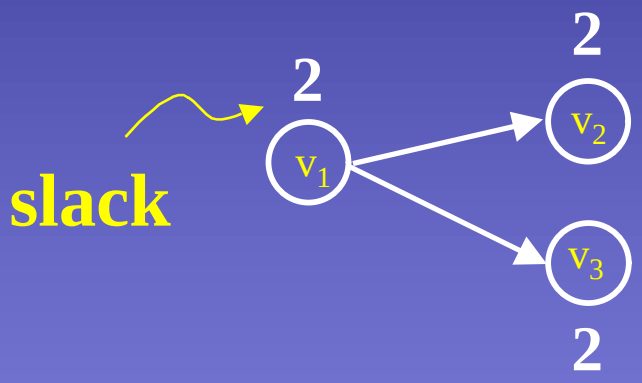
Potential Slack (PS): Basic Idea of Finding PS

- Try to identify a maximal-independent set to increase slack budget (effective slack):



--- Select v_2 and v_3 for slack assignment since v_2 and v_3 are independent nodes

--- Potential slack = 4

- Study slack sensitivity to reduce slack penalty:

Penalty --- When Δd is assigned to node v, the slacks of v's neighbors may be reduced.

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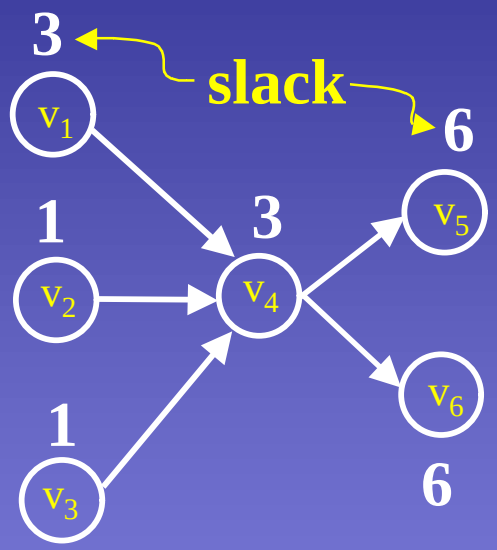
Potential Slack (PS): Basic Idea of Finding PS

An example of slack sensitivity



--- Slacks of nodes v_5 and v_6 are larger than those of v_1 ~ v_4 , the former being sensitive to the latter

--- Order of nodes for slack assignment is important

--- First selecting v_5 and v_6 gives better result

Two different slack assignments:

$\Delta_1\mathbf{D}$ = [3 1 1 0 3 3] , $|\Delta_1\mathbf{D}|$ = 11

$\Delta_2\mathbf{D}$ = [1 1 1 0 5 5] , $|\Delta_2\mathbf{D}|$ = 13

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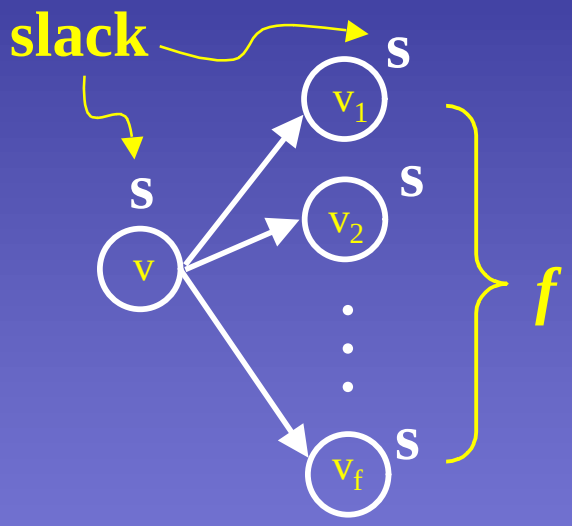
Potential Slack (PS): A typical example



- ZSA: $|\Delta_1 \mathbf{D(V)}| = (f + 1)\, s / 2$
- Optimal: $|\Delta_m \mathbf{D(V)}| = f\, s$
- $|\Delta_m \mathbf{D(V)}| \approx 2\, |\Delta_1 \mathbf{D(V)}|$ for $f >> 1$
- Total slack = $(f + 1)\, s$

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Algorithm: theoretical results

Theorem 1 [Chen-Sarrafzadeh, ICCAD 2000):

Maximal-Independent-Set based Algorithm (MISA) generates an optimal slack assignment.

Theorem 2:

The time complexity of MISA is $O(n^3)$, where n is the number of nodes in a given graph.

(if too complex, build heuristics around it)

Application: gate sizing problem

- Use slack assignment result to select gates to be down-sized.
- High potential slack promises significant area/power reduction.

Application: placement problem

- Potential slack (PS) can be translated into the freedom/flexibility of all signal nets during physical design phase. The more potential slack, the easier to route signal nets without violating the timing constraints.
-
- Potential slack based results are very effective

Conclusions

- Global algorithms are harder to design and analyze, have higher time complexity, however can produce good (optimal?) results.

- Is it worth it? Deep understanding on the structures of the solution
  - What if too complex to implement?
  - What if takes too much time to run?

Approximation Algorithms

Approximation Algorithm

- Problem : *P*
- Objective : minimize *f* (*f* : the optimal value*)
- Instance *I* ¸ *P*
- α-Approximation Algorithm *A* (Heuristic) for *P*

$$f(I,A) \leq \alpha . f^{*}(I)$$

Approximation Algorithm

- How to Evaluate Quality of Heuristics?
  1. Analyzing the data obtained by running on set of benchmark
     → Only comparison among set of samples!
  2. Deriving Lower Bounds or limits of approximability using approximation analysis methods
     → Not Easy to Analyze Lower Bounds!

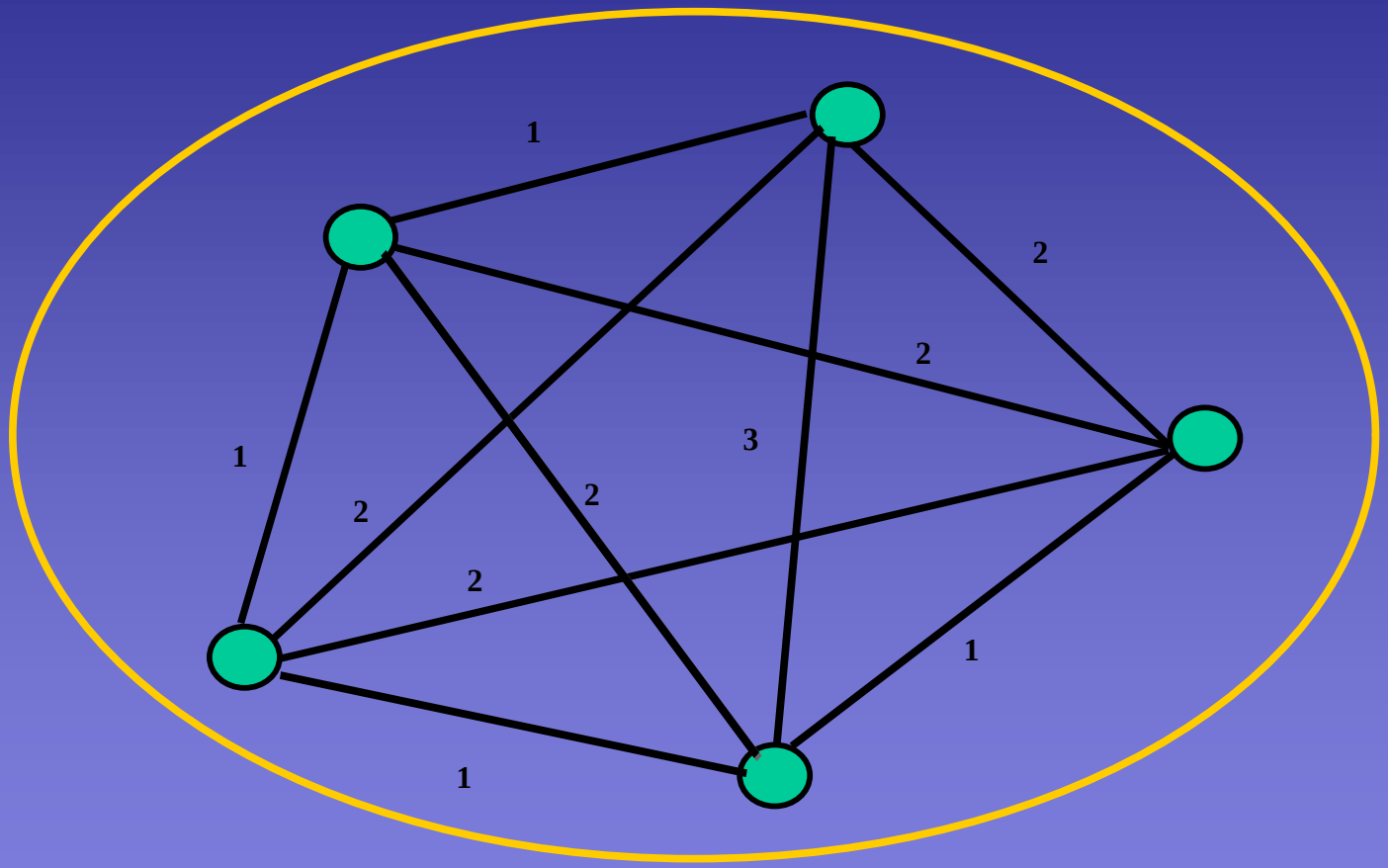
- What can analyzing performance of an algorithm tell us?
  - If the approximation is good enough
  - Approximation algorithms bound the search space

Example: Clustering Algorithm

- Problem:
  - Given an undirected weighted graph ***G***, objective function ***f***, and integer ***k***,
  - Partition the graph into ***k*** clusters such that value of ***f*** is minimized.
- α-Approximation Algorithm:
  - Proposed by Gonzalez (1985)
  - Assumptions:
    - ***G*** is Complete Graph
    - ***f*** is to minimize the maximum weight of an edge inside a cluster
    - Weights of edges satisfy triangle inequality
  - Complexity: $O(k \cdot n)$
    - n is number of nodes in graph G

Example: Clustering Algorithm

- Generate two clusters on complete graph G (k=2).



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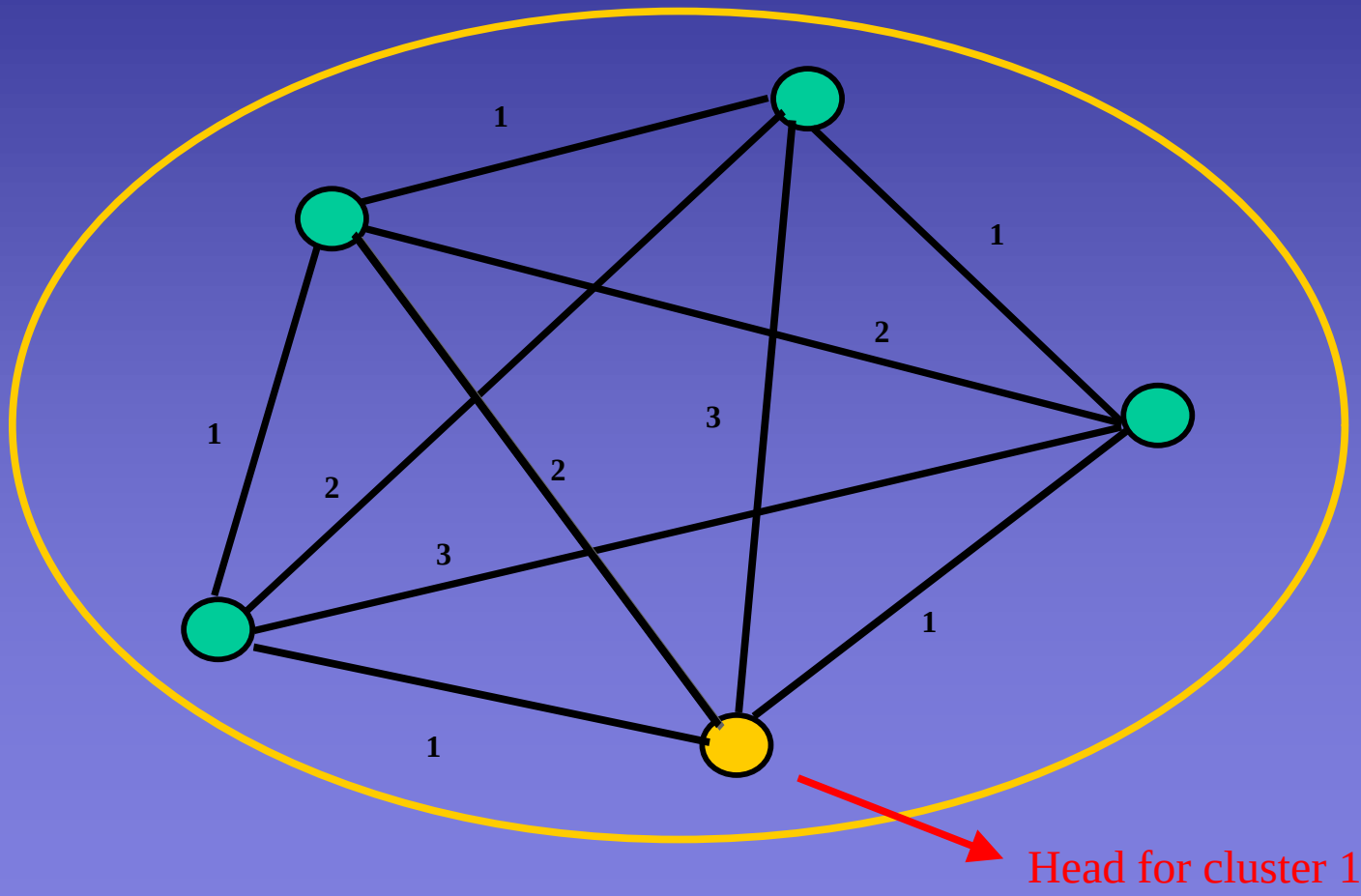
Example: Clustering Algorithm

- Generate a cluster including all the nodes.
- Pick an arbitrary node as head of current cluster.



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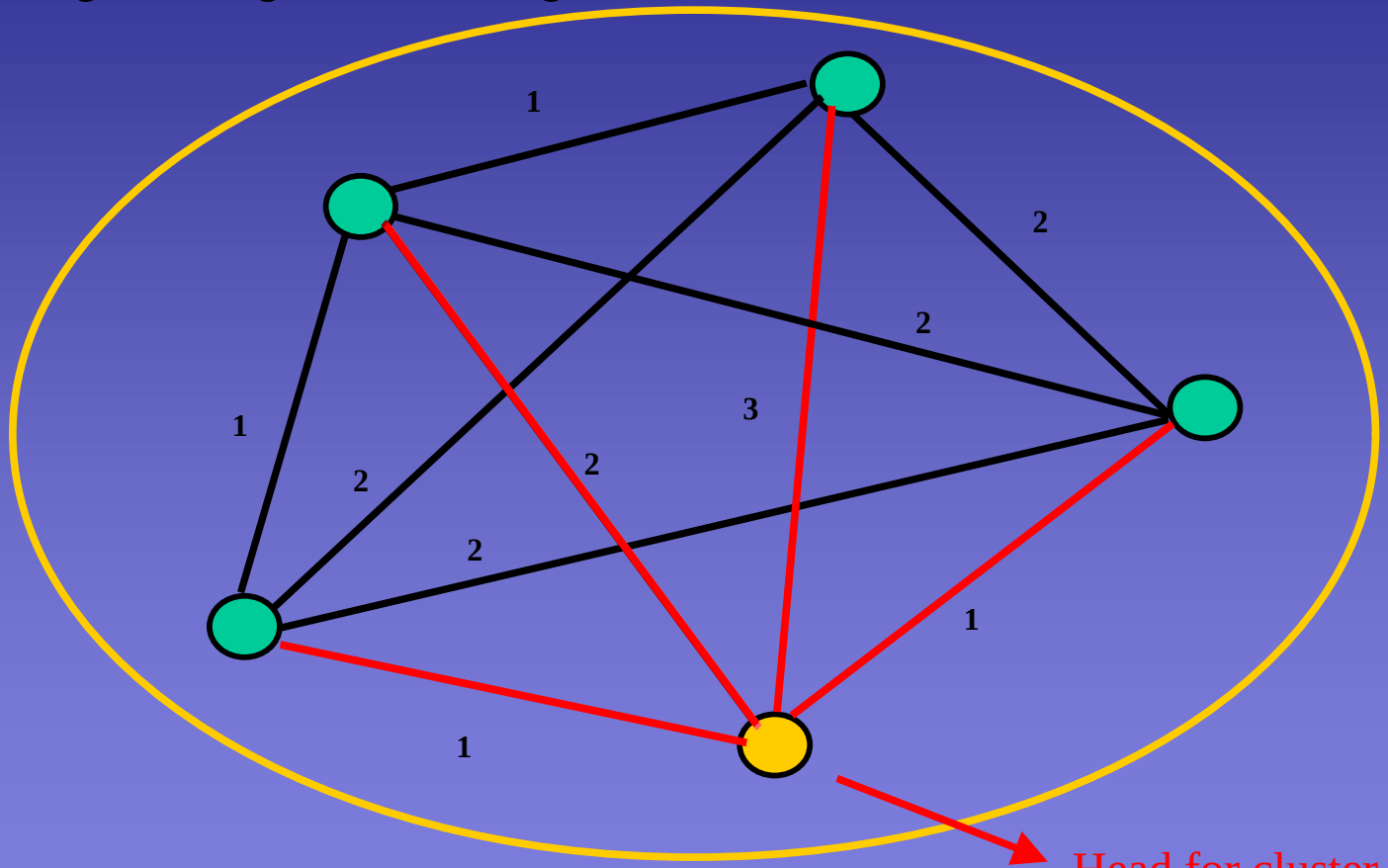
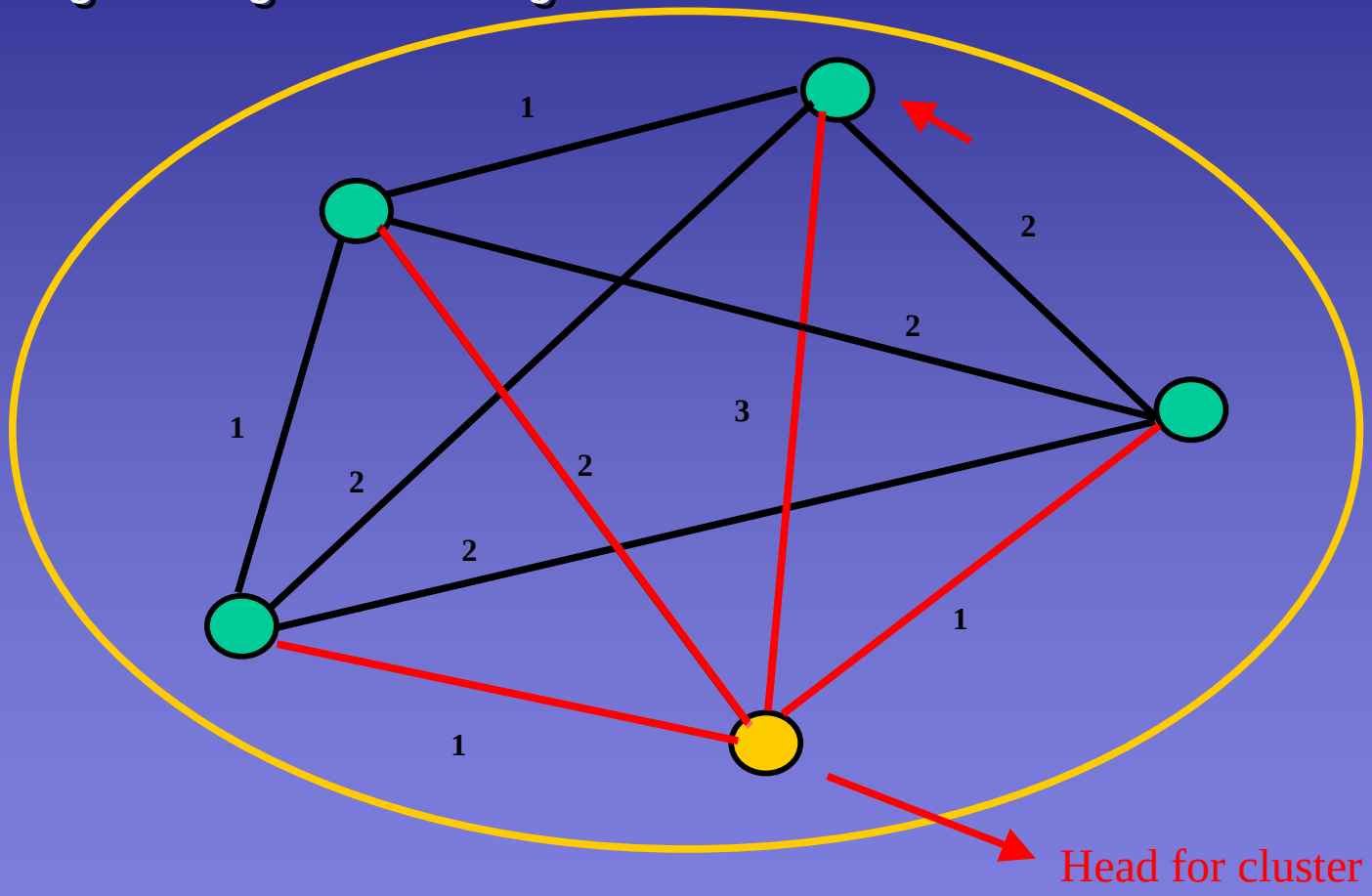
Example: Clustering Algorithm

- Select the node that is connected to the head of cluster through the edge with greatest weight



Example: Clustering Algorithm

- Select the node that is connected to head of its cluster through the edge with greatest weight.



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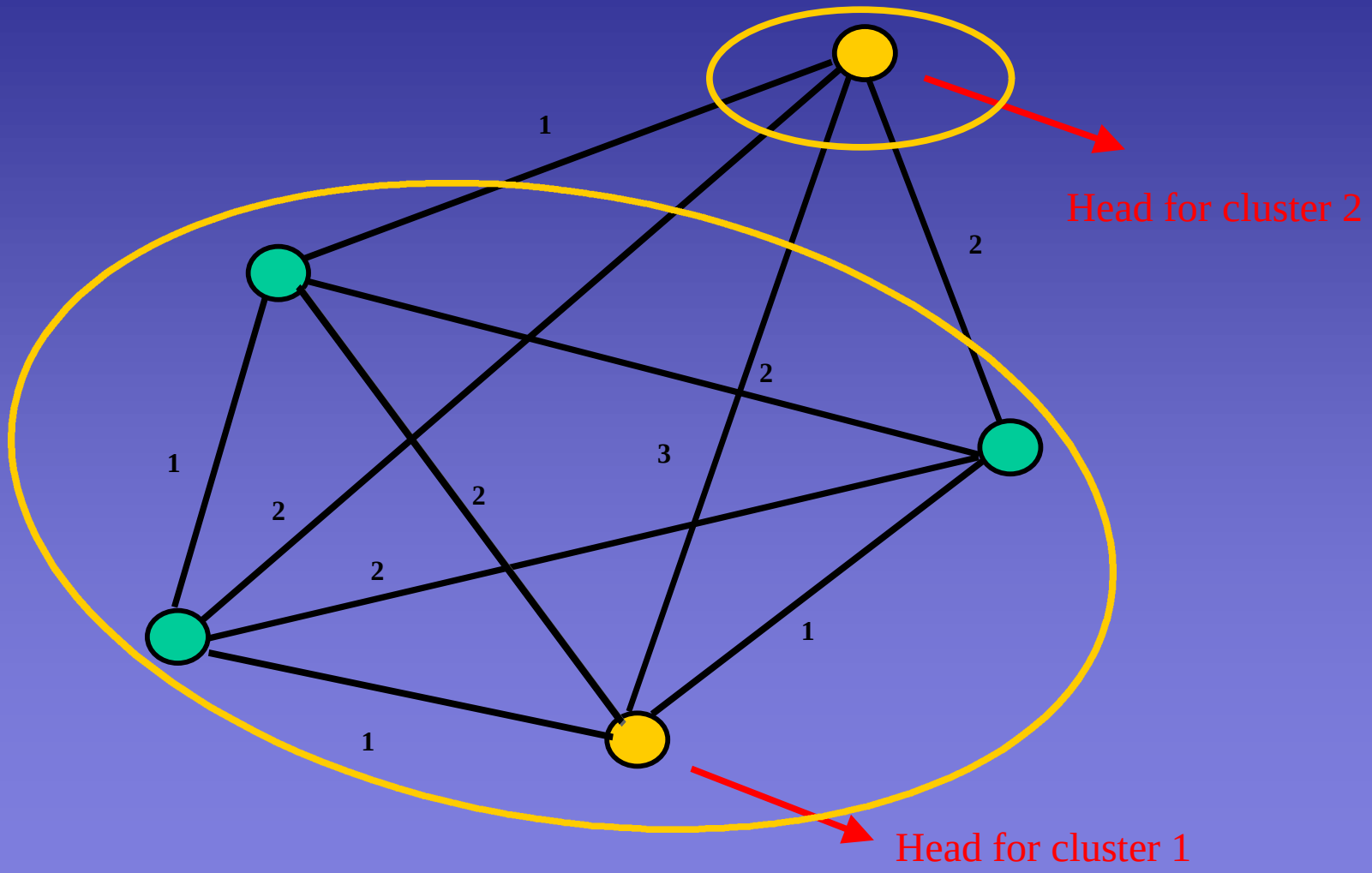
Example: Clustering Algorithm

- Separate the node, being the head of new cluster.



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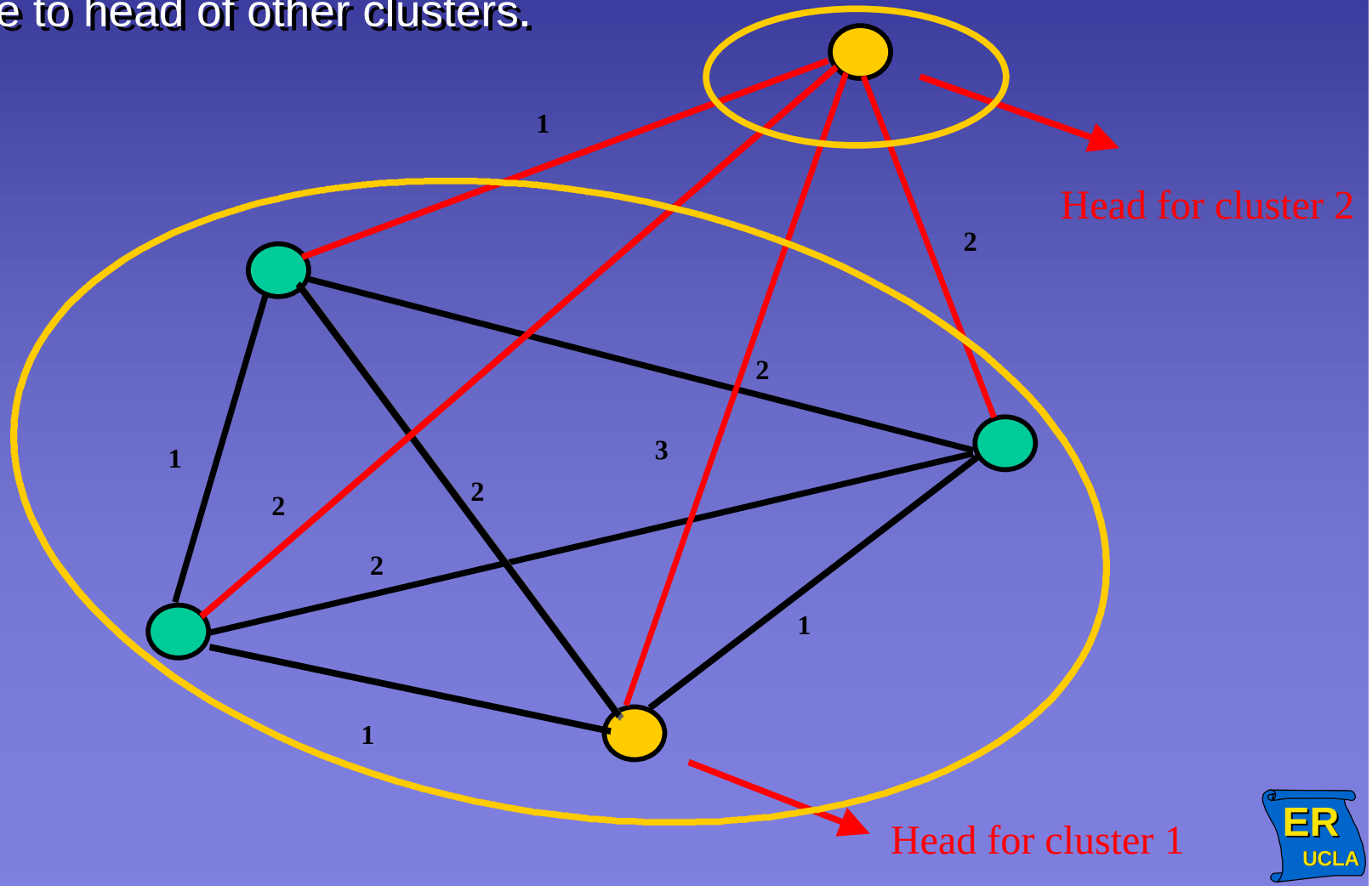
Example: Clustering Algorithm

- Move the nodes inside the clusters connected to head of new cluster with lower weight compared to weight of edges connecting those to head of other clusters.



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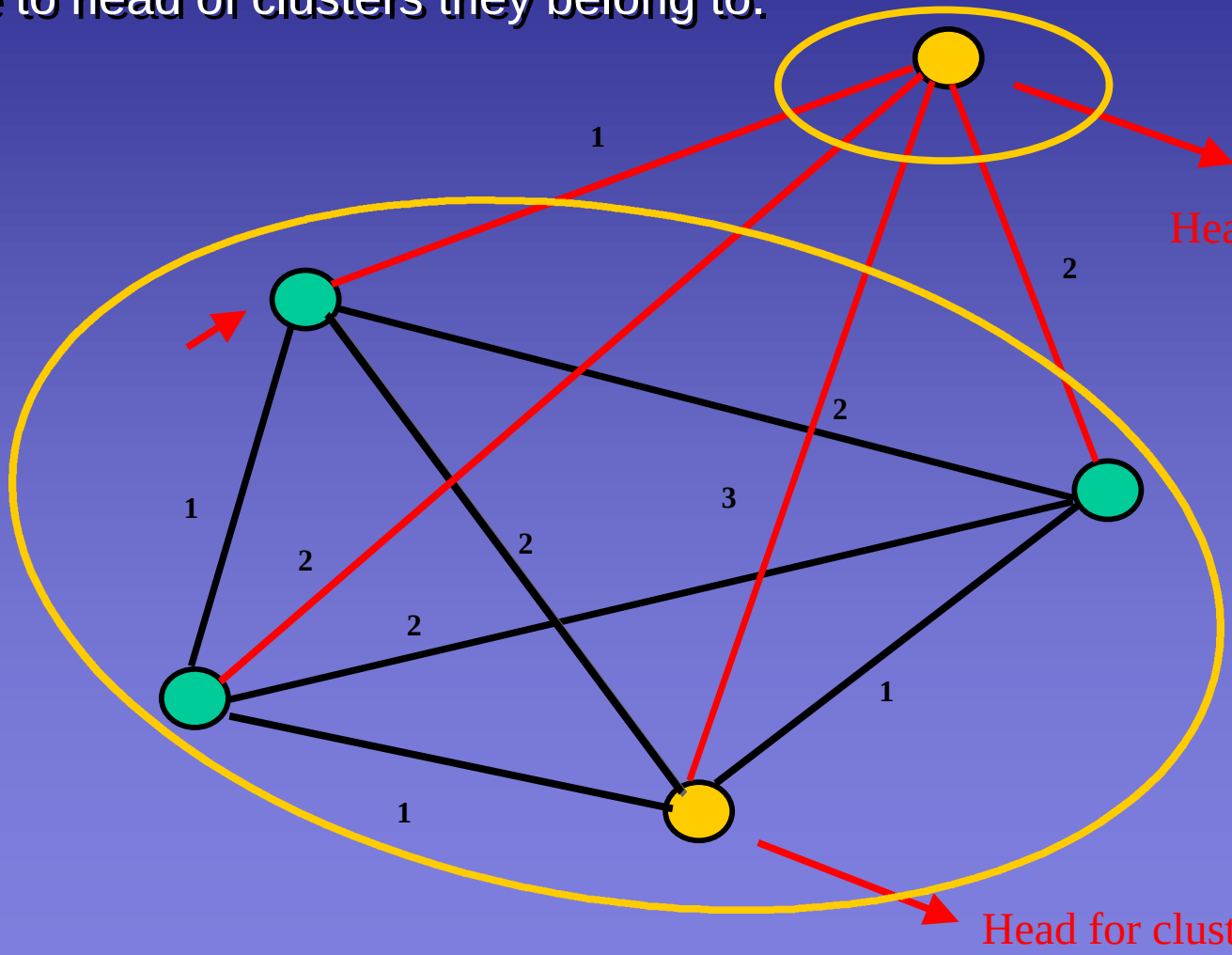
Example: Clustering Algorithm

- Move the nodes inside the clusters connected to head of new cluster with lower weight compared to weight of edges connecting those to head of clusters they belong to.



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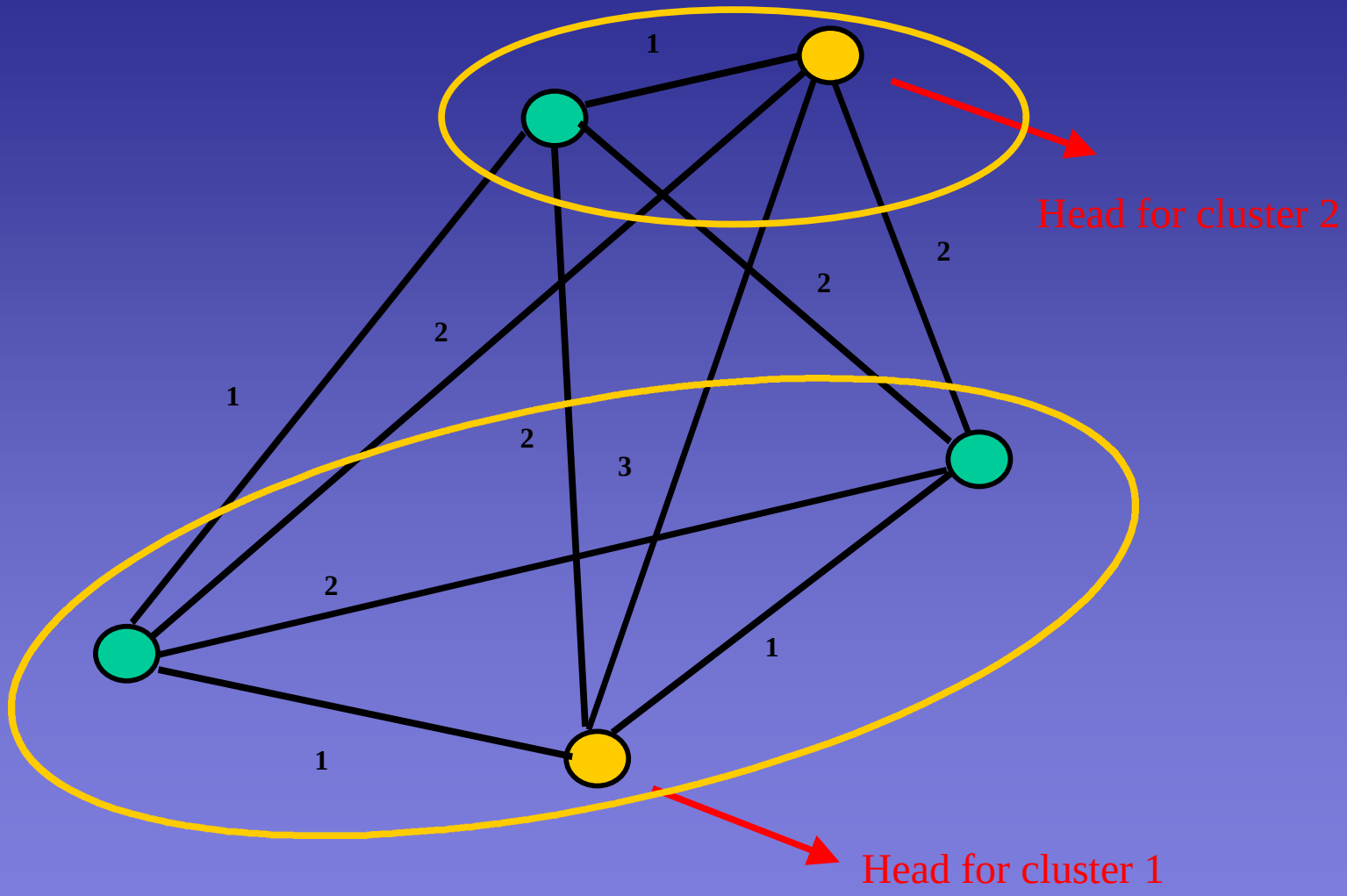
Example: Clustering Algorithm



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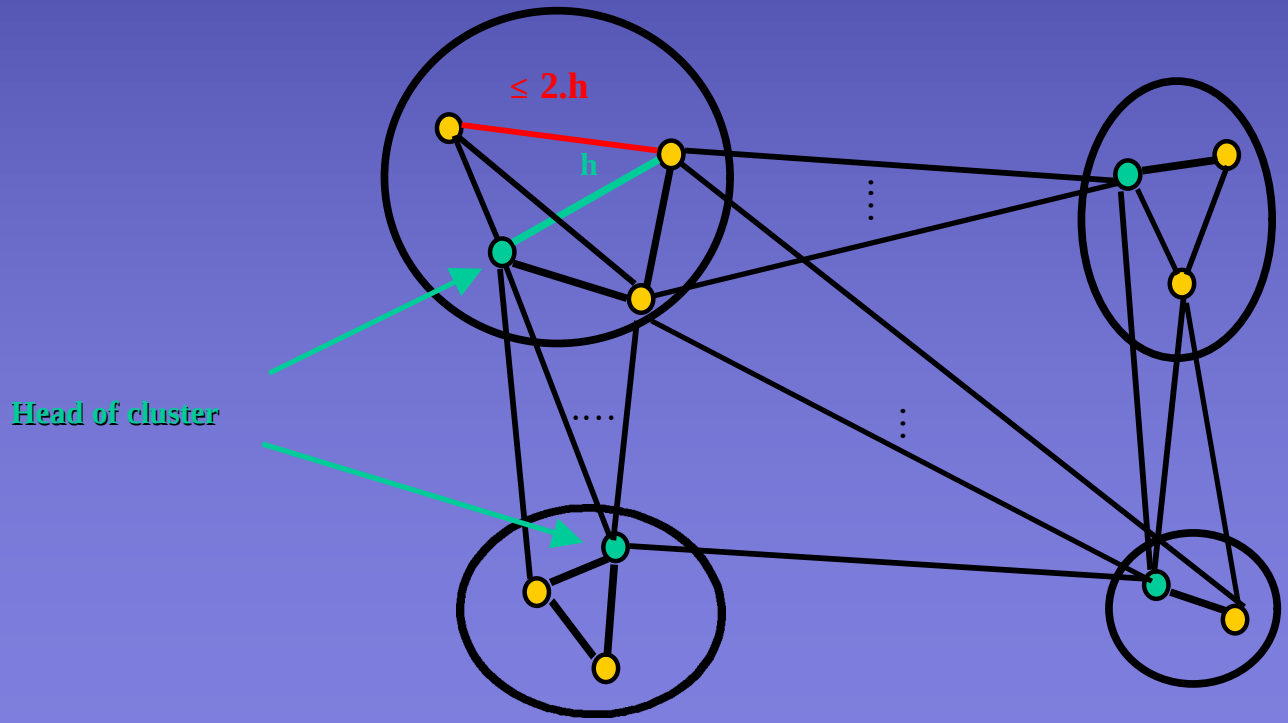
Example: Clustering Algorithm

- It has been proven:
  - Proposed algorithm gives a solution within 2 times the optimal solution ($\alpha = 2$).
  - Computed approximation bound is the best possible for such problem (unless P=NP).
  - This implies that problem of approximation within $2-\varepsilon$ is NP-Complete.

Analysis of 2-approximation clustering algorithm

- h is defined as the maximum weight of edge inside clusters connecting to head of cluster.
- According to triangle inequality no edge in clusters has greater weight than 2 x h (G is a complete graph).



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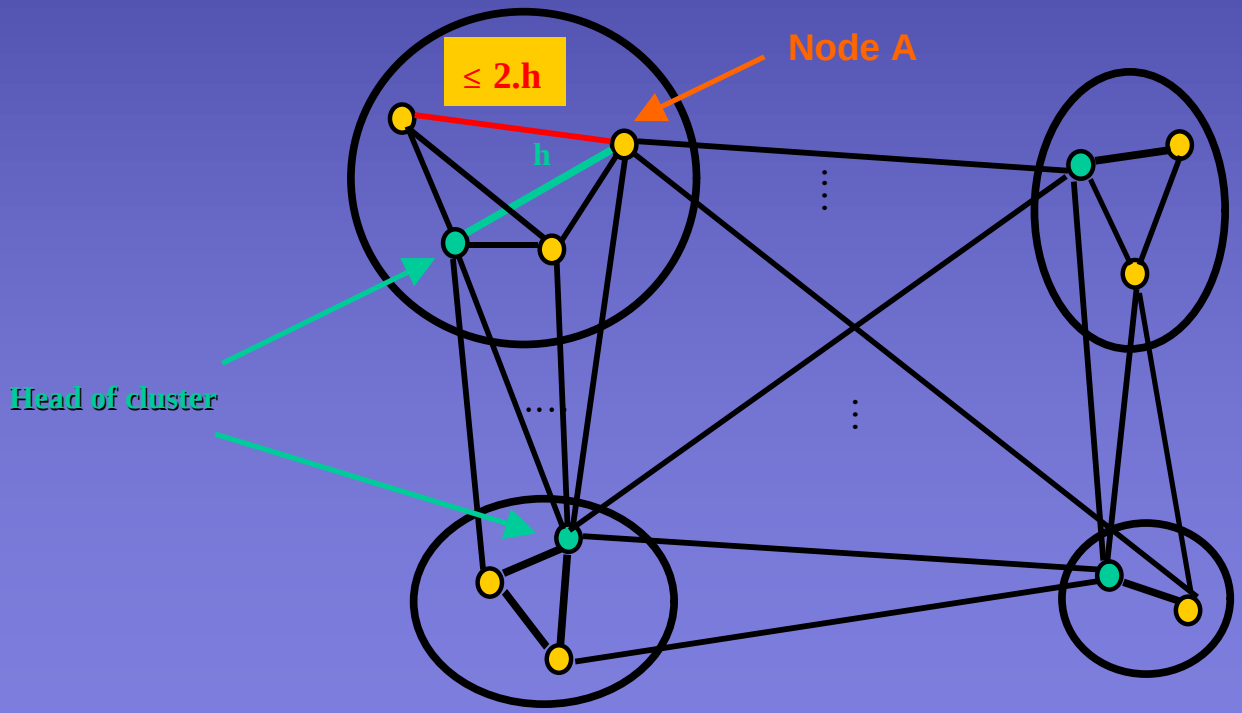
Analysis of 2-approximation clustering algorithm

- Node A is connected to head of cluster with weight h.
- Since Node A has not been chosen to be head of other clusters (before or after cluster 1 was constructed), it implies that the weight of edges connecting Node A to heads of other clusters are at least h.



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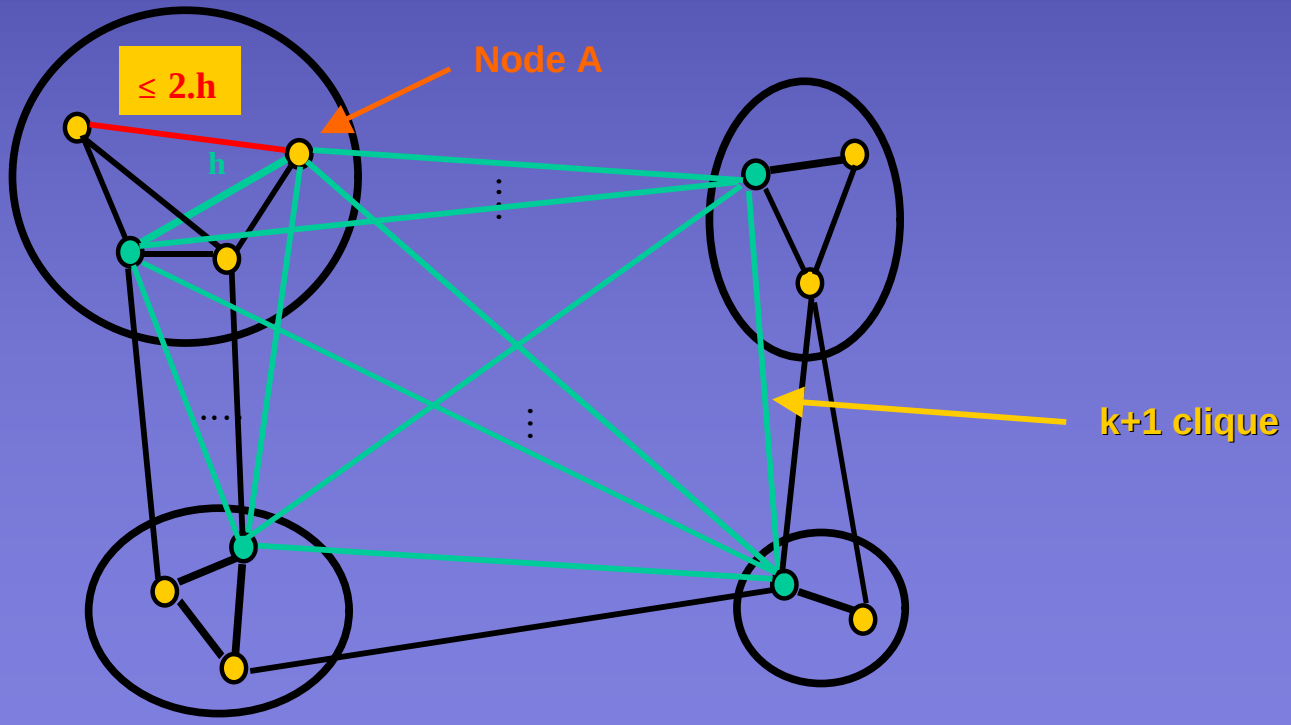
Analysis of 2-approximation clustering algorithm

- Node A and heads of clusters (k nodes) generate a k+1 clique.
- Weights of edges in clique is at least h.
- If there is a (k+1)-clique with edges having greater weight than h, at least one edge of this clique will be inside one of the k clusters.

(k+1)-clique of weight h observed in G

→ $f^*(G) \geq h$



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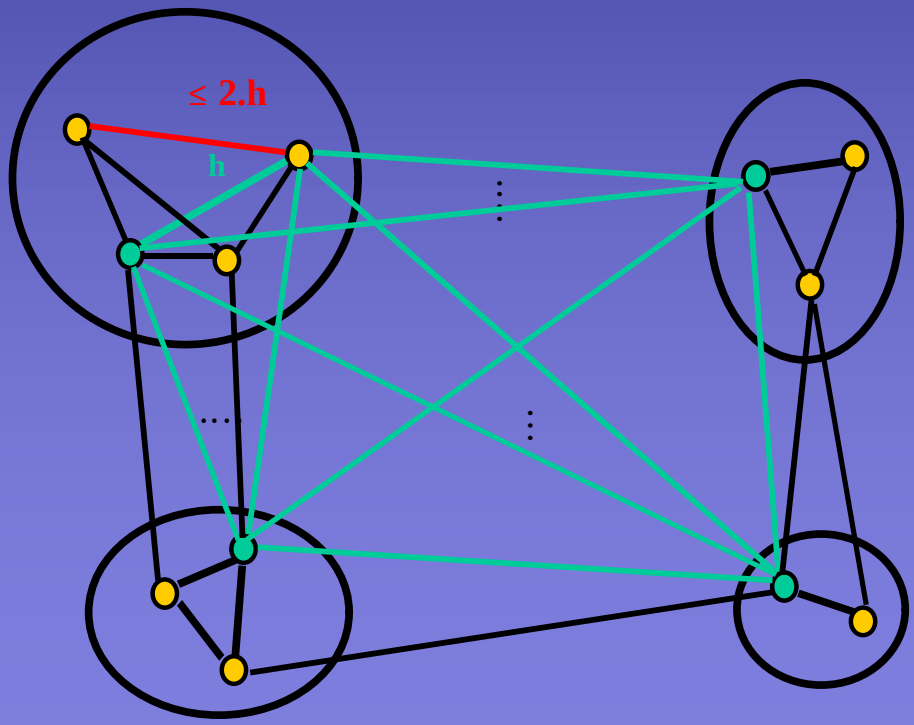
Analysis of 2-approximation clustering algorithm

- Maximum weight of an edge inside any cluster is great than h.
- In solution obtain by approximation algorithm, no edge with weight greater than 2 x h is inside any cluster.

$$f(A,G) \leq 2 \cdot f^{*}(G)$$



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Conclusions

- Important to analyze algorithms
- Better to start with a simple algorithm, analyze it, add more local optimization methods, and improve the performance
- Let computational effort guide us towards the correct road to develop better algorithms for hard problems.

Probabilistic Algorithms

Probabilistic Algorithms

- Definition – an algorithm that makes random choices during execution
- Examples
  - Simulated Annealing
  - Genetic Algorithms
  - Karger's Contraction Algorithm for Clustering
- Use probablistic analysis to give:
  - Expected runtime
  - Bound on solution quality

Probabilistic algorithms often run faster, are easier to implement and describe than comparable deterministic algorithms

Contraction Algorithm for Clustering/partitioning

Input: directed acyclic graph $G = (V, E)$, number of partitions k

Output: k sets of vertices

begin

while |V| > k

do choose and edge $e(u,v)$ at random

contract both u and v

end

Simple to describe with good runtime and bound on solution quality

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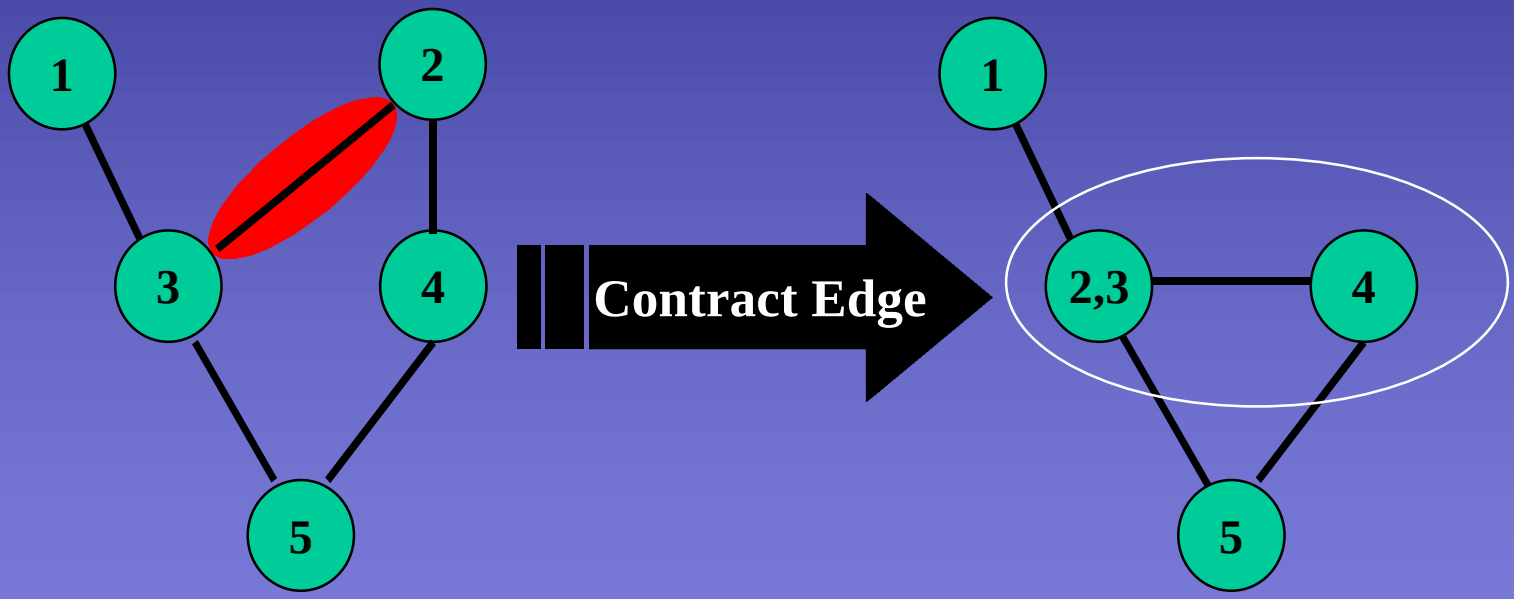
Example

- Edge contraction
  - Remove vertices connected to edge
  - Replace with one vertex and maintain connectivity



"Simple" Algorithm with Analysis yields powerful results

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Solution Quality Properties

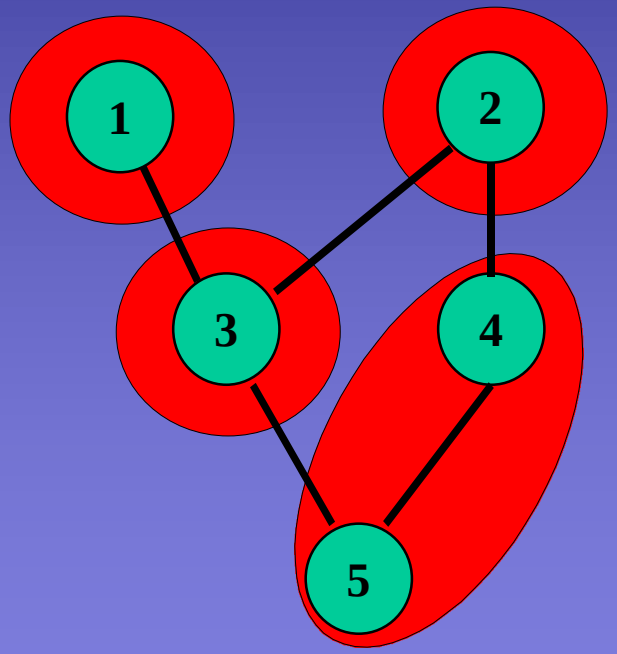
- *Using probabilistic analysis we can get bounds on solution quality*
- ***Theorem:*** *Stopping the Contraction Algorithm when r vertices remain yields a minimum r-way cut with probability at least*

$$r\binom{n}{r-1}^{-1}\binom{n-1}{r-1}^{-1}$$

- ***Corollary:*** *If we perform $O(n^2 \log n)$ independent contractions to two vertices, we find a min-cut with high probability.*
- *You want it faster: combine this with FM*

Proof of Theorem

- Choose *r - 1* vertices at random, each of these vertices is in its own cluster (*r* = 4)
- The remaining vertices are in the final cluster



- *f* is the number of edges cut (*f = 4*)
- *m* is the number of graph edges (*m = 5*)

$$E[f] = \left[1 - \left(1 - \frac{r-1}{n}\right)\left(1 - \frac{r-1}{n-1}\right)\right] m$$

Probability that a single edge is cut

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Proof of Theorem (cont)

$$E[f] = \left[1 - \left(1 - \frac{r-1}{n}\right)\left(1 - \frac{r-1}{n-1}\right)\right] m$$

- *f* is no less than the value of the minimum *r*-cut, E[*f*] is the value of the minimal cut
- The probability that a particular minimum r-cut survives the reductions process until there are r vertices remaining is at least

$$\prod_{u=r+1}^{n}\left(1 - \frac{r-1}{u}\right)\left(1 - \frac{r-1}{u-1}\right) = \prod_{u=r+1}^{n}\left(1 - \frac{r-1}{u}\right)\prod_{u=r+1}^{n}\left(1 - \frac{r-1}{u-1}\right)$$

$$= r\binom{n}{r-1}^{-1}\binom{n-1}{r-1}^{-1}$$

Insights into Clustering Problem

- Probability theory also gives tremendous insight into the clustering problem

- **Corollary:** *The number of minimum r-cuts of a graph is no more than* $O(n^{2(r-1)})$.

- **Corollary:** *The number of r-cuts within a factor of k of the optimum is* $O(n^{2k(r-1)})$.

"Simple" probabilistic algorithms give quality and runtime bounds and insight into the problem

Don’t be afraid to try wild ideas

- Crossing-based placement
- Embedding into intermediate graphs (netlist to mesh)

Final Comments

- Algorithm innovation is needed in all aspect of PD
- More so, as problems are getting more complex
- We need to look at
  - new paradigms,
  - novel algorithms,
  - concepts that may not go into a CAD tool (right away)
- Who is responsible for doing all these?
  - Industry
  - Academia