Towards Designing and Deploying Ising Machines

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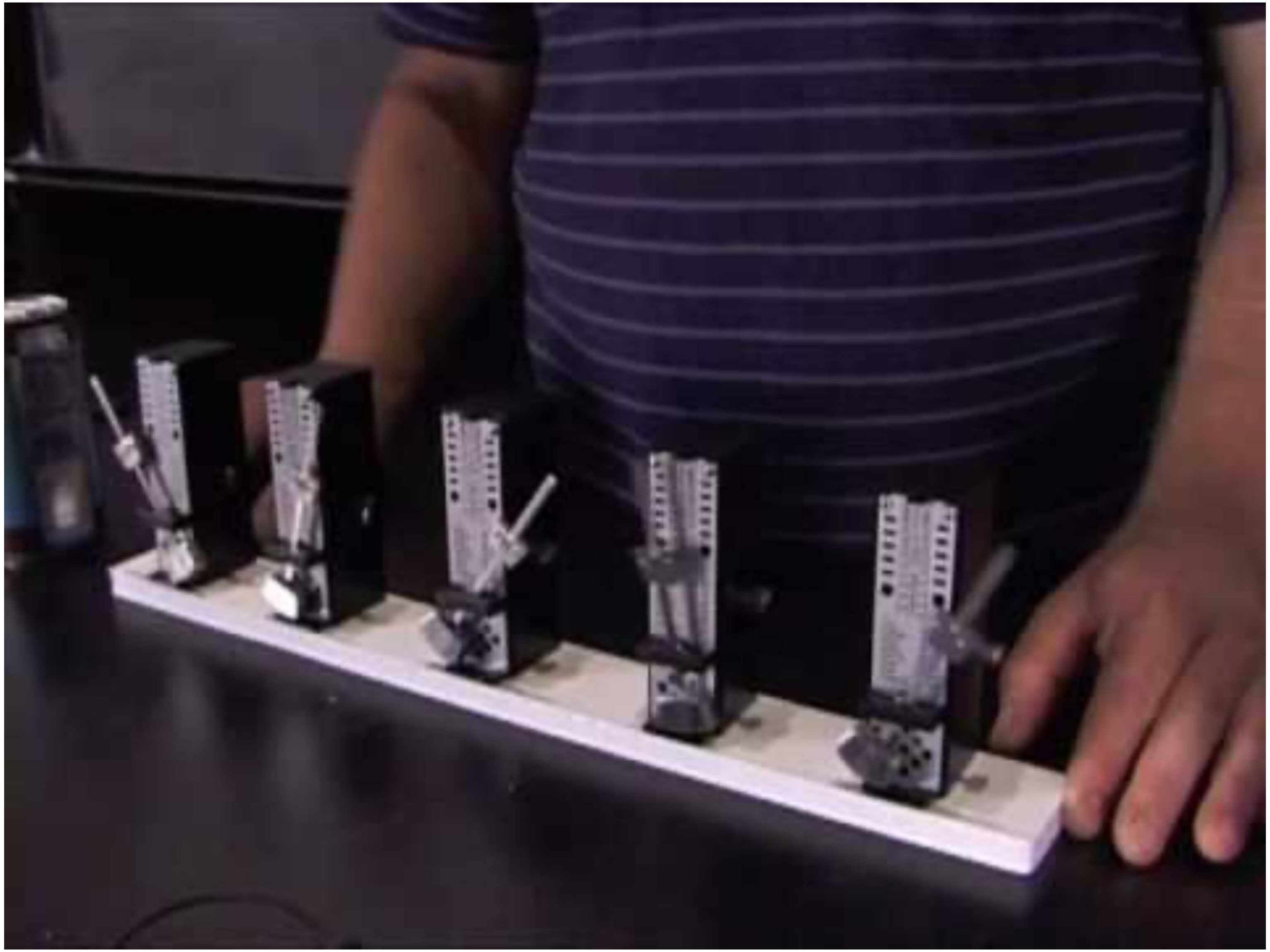
ISPD 2025

The concept

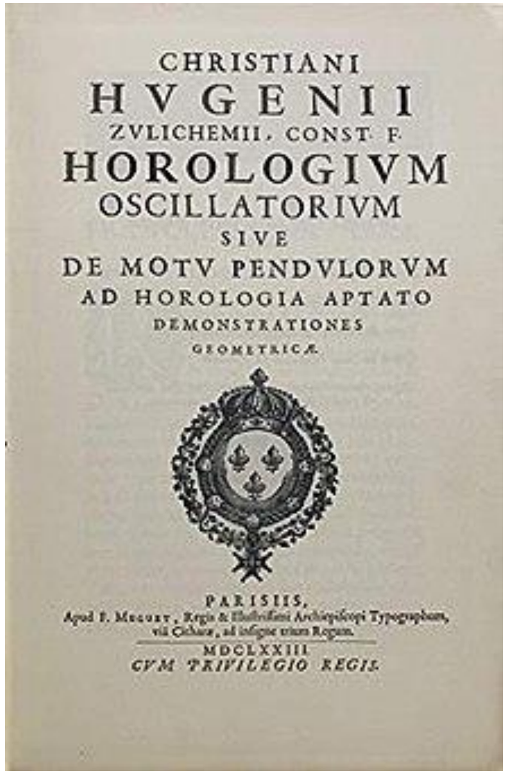
A tale of dancing metronomes



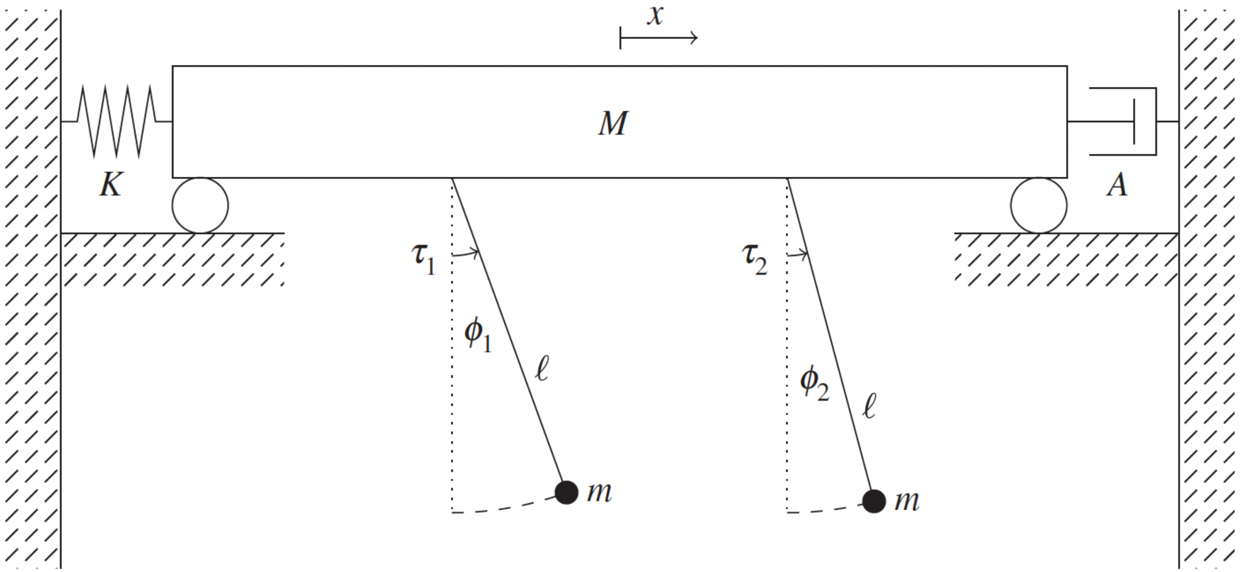
[https://youtu.be/Aaxw4zbULMs]

Huygens’s clocks (1665)

Christiaan Huygens (1629 – 1695)
by Caspar Netscher (1671), Museum Boerhaave, Leiden



The Pendulum Clock: or Geometrical Demonstrations Concerning the Motion of Pendula as Applied to Clocks (1673)



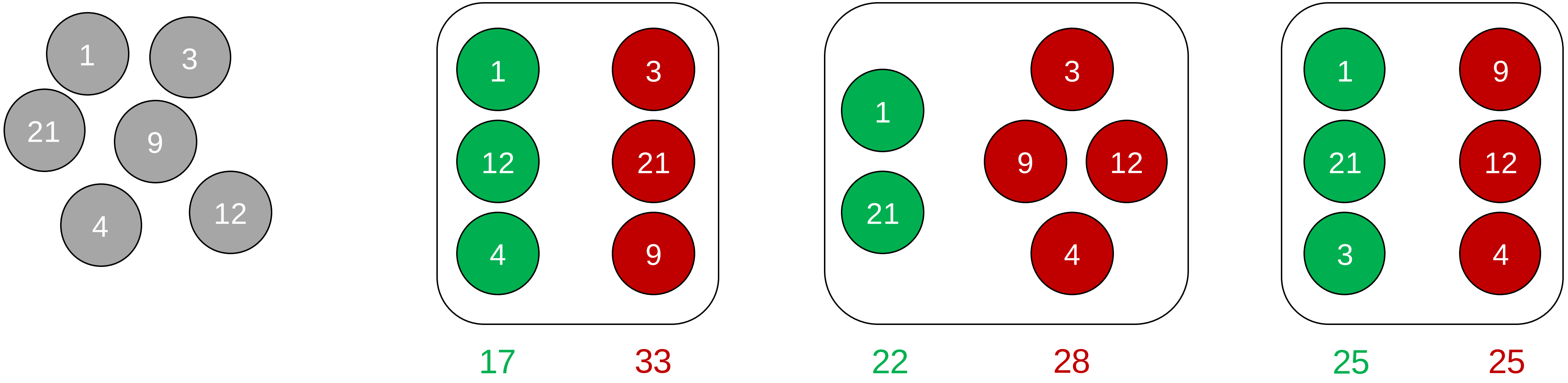
Willms, ”Huygens clocks revisited,” Royal Society Open Science, 4:170777, 2017.

Nice toy, huh?

- But you can also do useful computation with this
  - State = phase of the oscillator
- We can solve optimization problems, e.g., number partitioning (NP-complete)
  - Divide a set of numbers into two subsets whose sum is equal (or as close as possible)

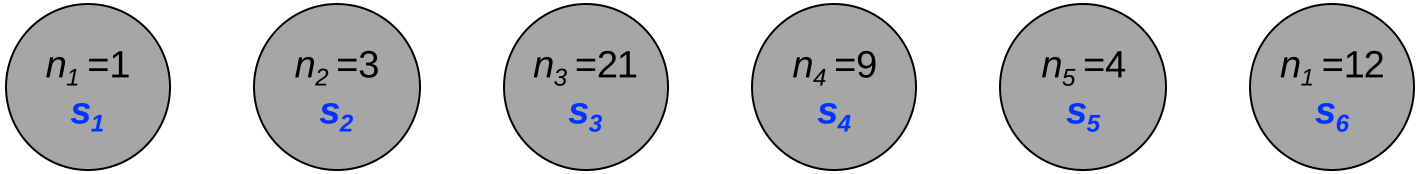
Example

Candidate solutions



A mathematical formulation

- Assign variables $s_i \in \{-1, +1\}$ to each number n_i



- Formulate the problem as minimizing a quadratic

$$\text{minimize } \left(\Sigma\, s_i\, n_i \right)^2$$
$$\text{e.g., minimize } (1\, s_1 + 3\, s_2 + 21\, s_3 + 9\, s_4 + 4\, s_5 + 12\, s_6)^2$$

- This has terms $s_i^2 = 1$ and $s_i\, s_j$, and converts to

$$\text{minimize } \Sigma\, K_{ij}\, s_i\, s_j$$

- Note: Need one reference state! $s_1 = -1 \Rightarrow$ minimize $(-1 + 3\, s_2 + 21\, s_3 + 9\, s_4 + 4\, s_5 + 12\, s_6)^2$

What does this have to do with metronomes?

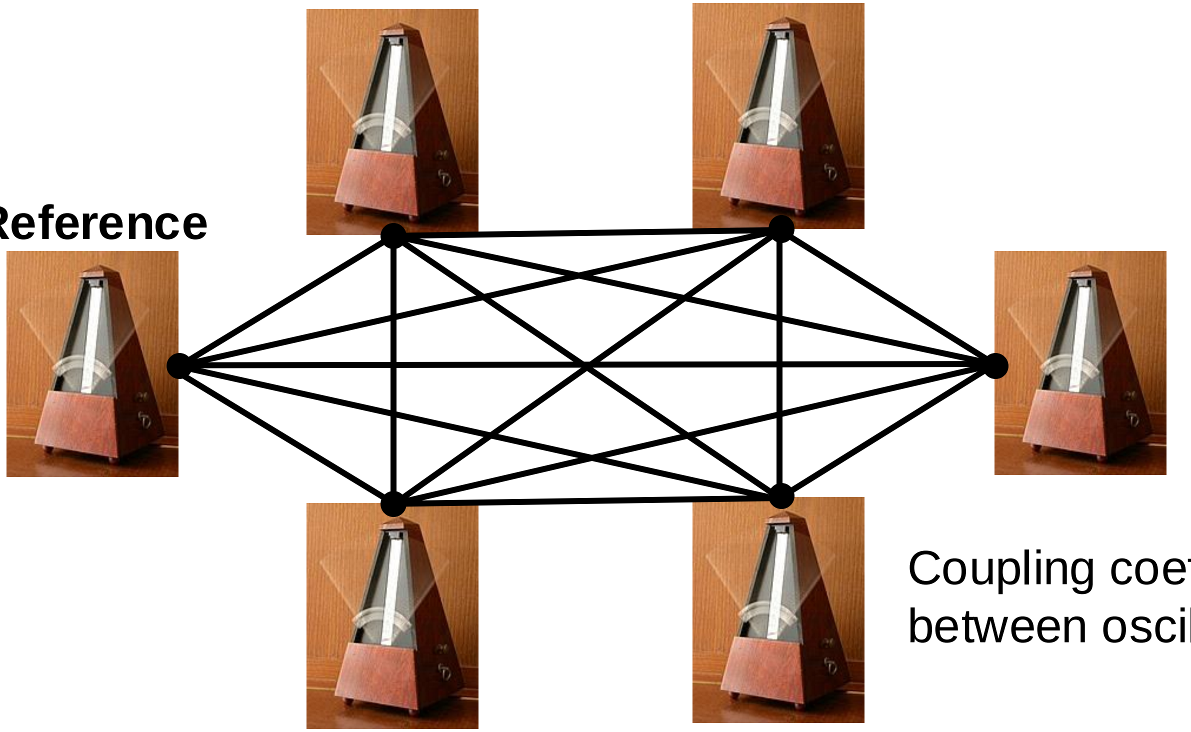
s_i = –1 s_i = +1

- Assign variables $s_i \in \{-1, +1\}$ to each number n_i

$n_1 = 1$, $s_1 = -1$	$n_2 = 3$, $s_2 = +1$	$n_3 = 21$, $s_3 = +1$	$n_4 = 9$, $s_4 = +1$	$n_5 = 4$, $s_5 = -1$	$n_1 = 12$, $s_6 = -1$
Reference					

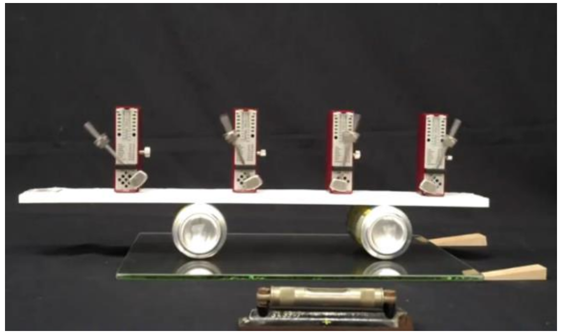
minimize $\Sigma\, K_{ij}\, s_i\, s_j$

Reference

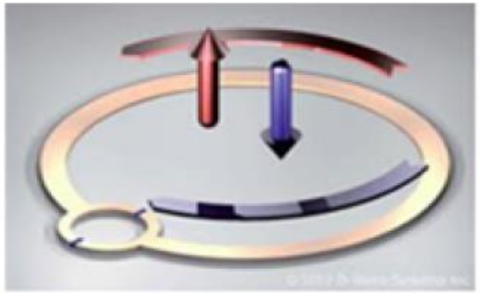


Coupling coefficients K_{ij} between oscillators *i and j*

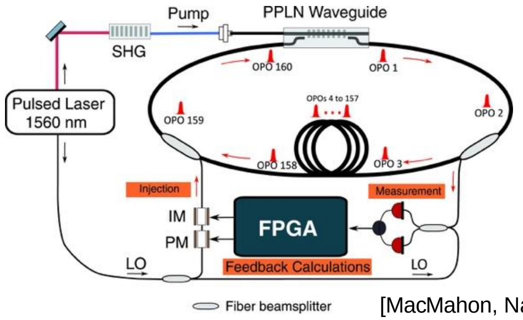
Coupled oscillator technologies



Metronomes

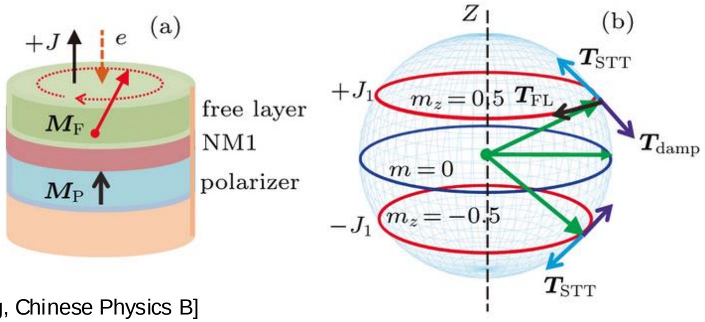


Superconducting



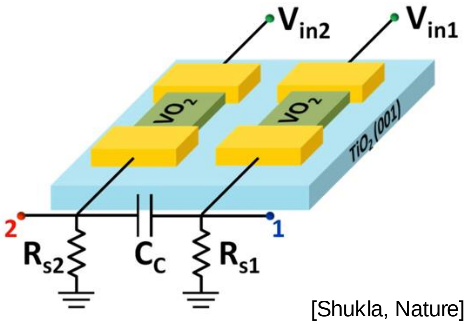
[MacMahon, Nature]

Optical



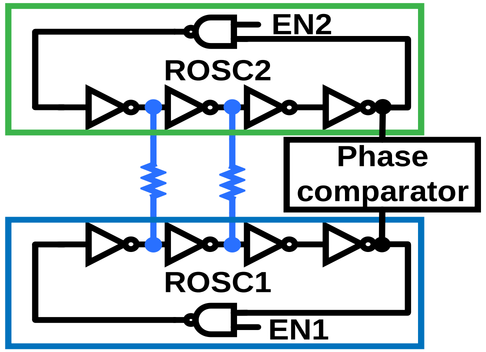
[Zheng, Chinese Physics B]

Spintronic



[Shukla, Nature]

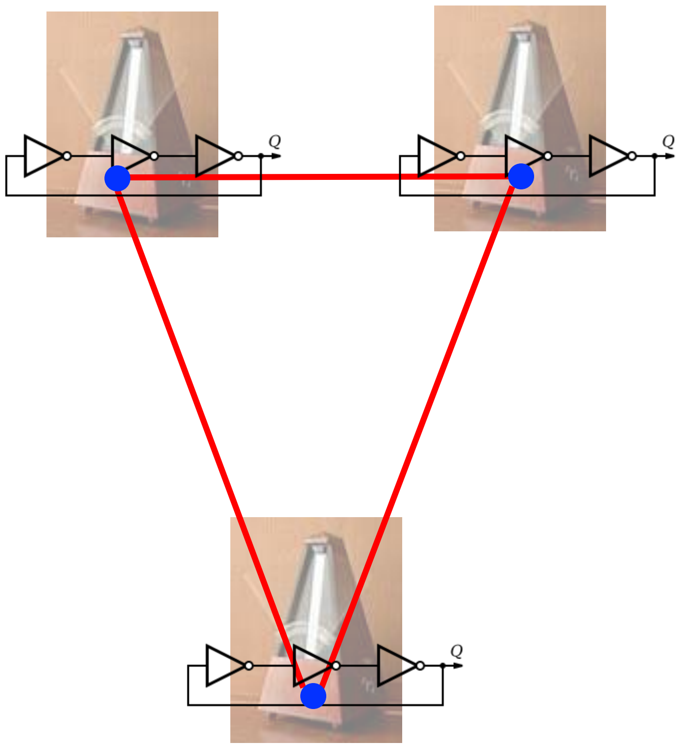
Phase change



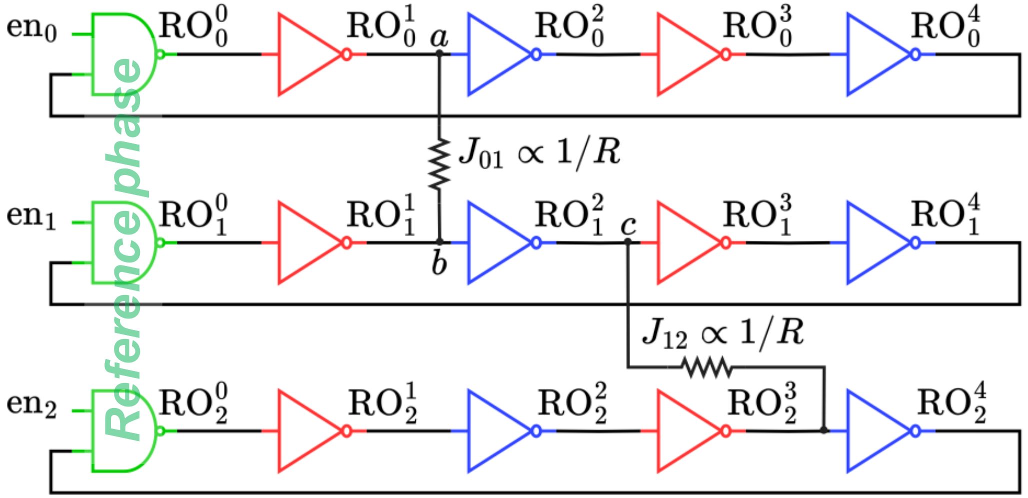
CMOS

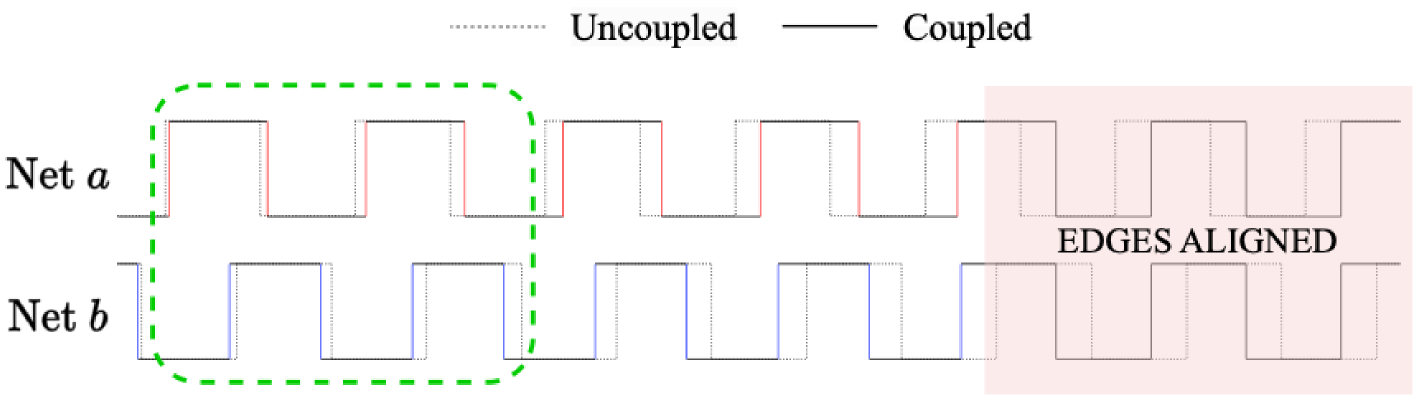
Analog computing with coupled oscillators

- Using ring oscillators

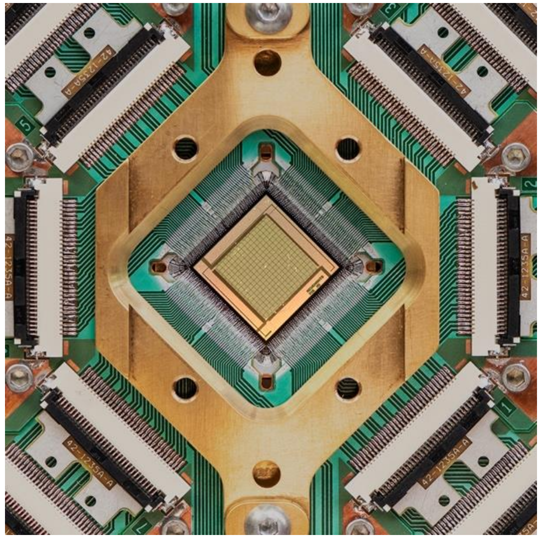


- Could have positive or negative coupling

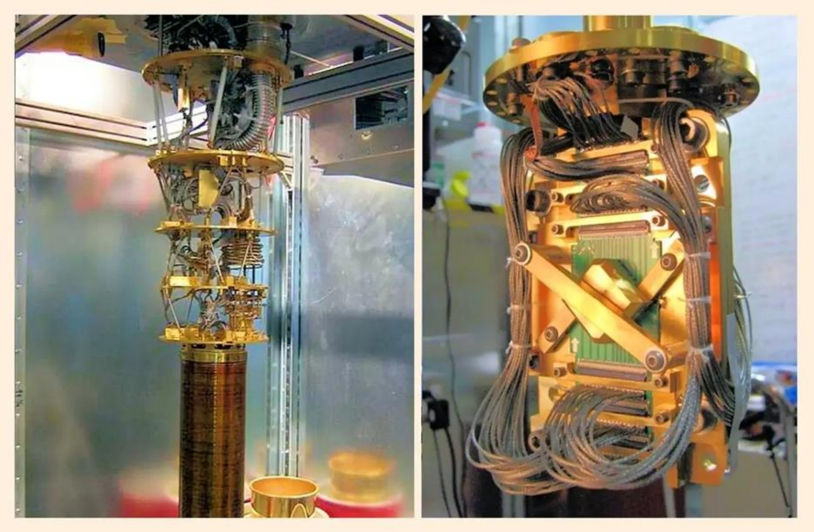




Analog computing with coupled oscillators



[dwavesys.com]

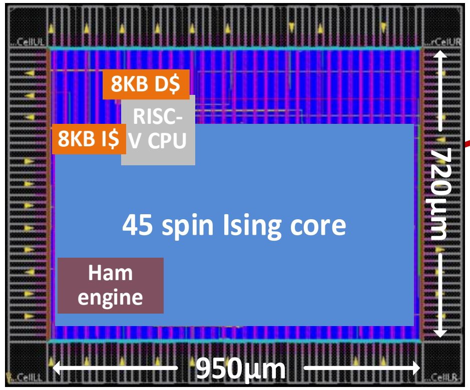


https://www.thebrighterside.news/post/in-a-global-first-quantum-computers-crack-rsa-and-aes-data-encryption/

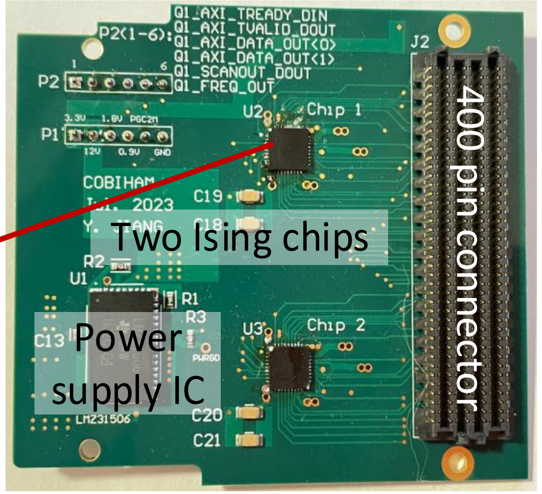
20mK

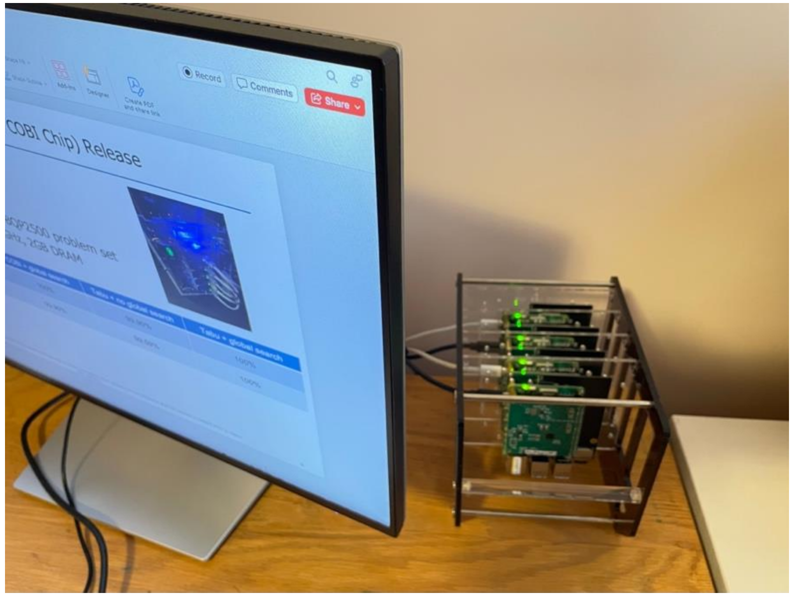
Classical quantum annealer

COBI
(Coupled oscillator based Ising)



FPGA Mezzanine Card





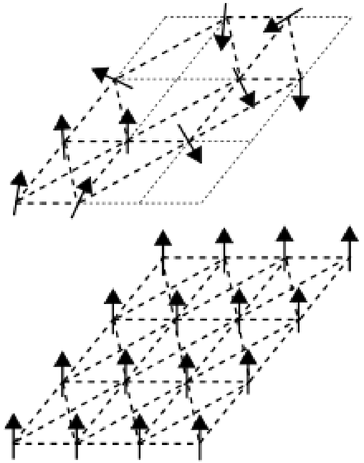
Room Temperature

~10mW

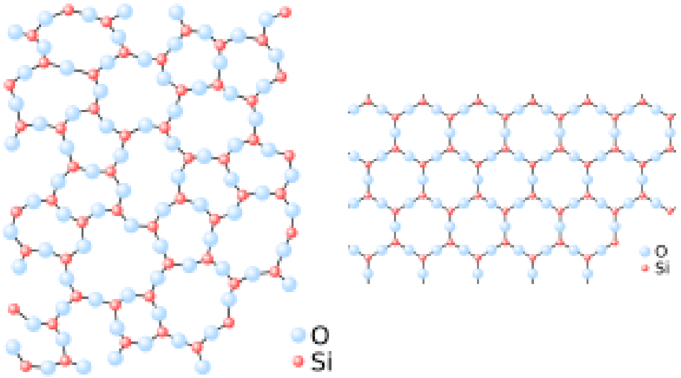
More concretely…

“Ising spin glasses”

- Magnetic systems with potentially disordered magnetization



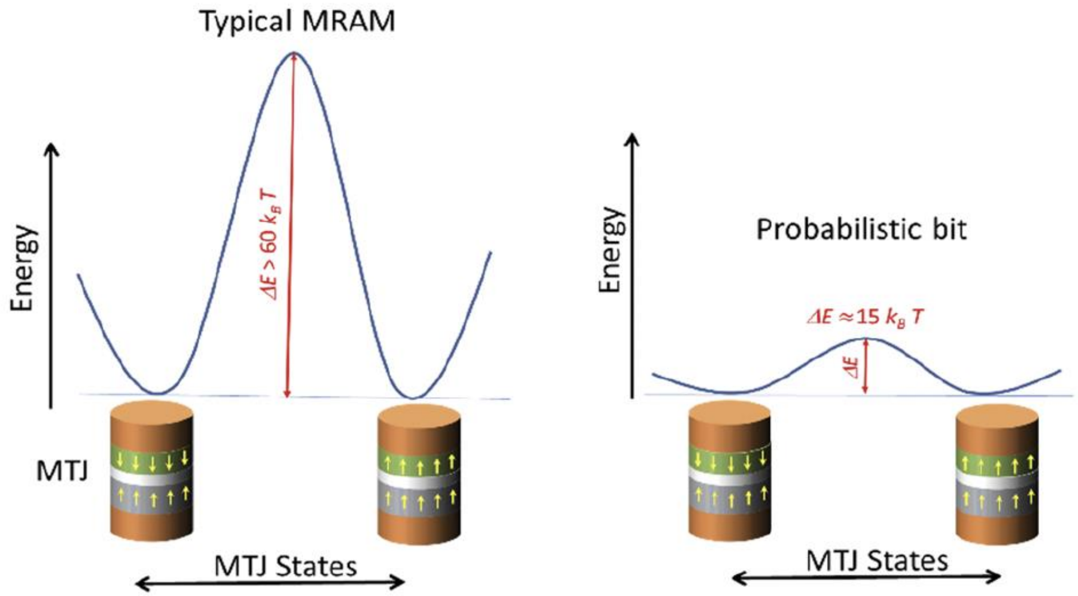
- Analogy with “regular” glass



Glass Quartz

[Wikipedia, Wikimedia Commons]

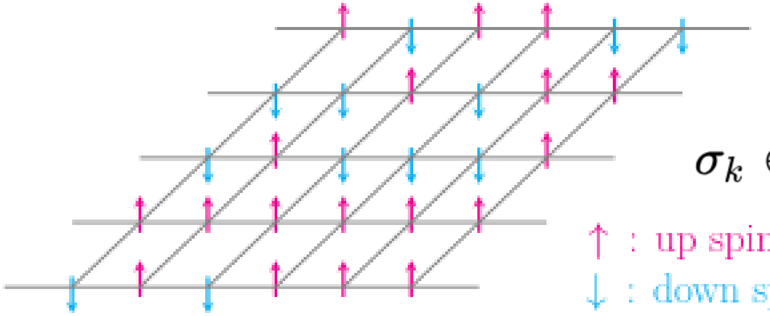
- Has been explored with magnetic tunnel junctions (MTJs)



[Navau and Sort, APL Materials 2021]

Ising computation for dummies (the Wikipedia version)

- Misnomer: formulated by Wilhelm Lenz
- 1D model solved by Ising (Lenz's student)
- 2D model: magnetic dipole moments in a lattice



- Configuration probability

$$P_\beta(\sigma) = \frac{e^{-\beta H(\sigma)}}{Z_\beta},$$

where $\beta = 1/(k_B T)$, and the normalization constant

$$Z_\beta = \sum_\sigma e^{-\beta H(\sigma)}$$

[Wikipedia, Wikimedia Commons]

- Energy given by a *Hamiltonian* function

$$H(\sigma) = -\sum_{\langle ij\rangle} J_{ij}\sigma_i\sigma_j - \mu\sum_j h_j\sigma_j$$

$J_{ij} > 0$, the interaction is called ferromagnetic,
$J_{ij} < 0$, the interaction is called antiferromagnetic,
$J_{ij} = 0$, the spins are *noninteracting*.

$h_j > 0$, the spin site *j* desires to line up in the positive direction,
$h_j < 0$, the spin site *j* desires to line up in the negative direction,
$h_j = 0$, there is no external influence on the spin site.

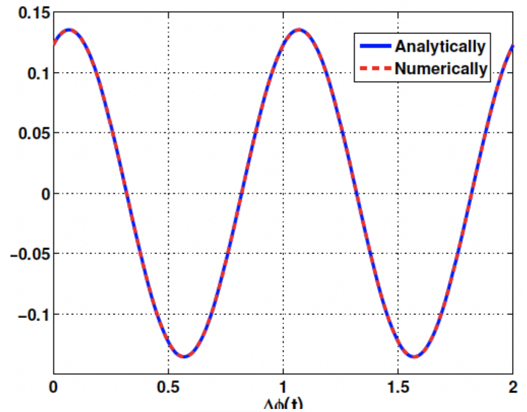
- One way to use this: [Lucas, Frontiers in Physics 2024]
  - Map a problem to this coupled system
  - Start from known solution H_0 to the Hamiltonian ; evolve from H_0 to H with temperature T

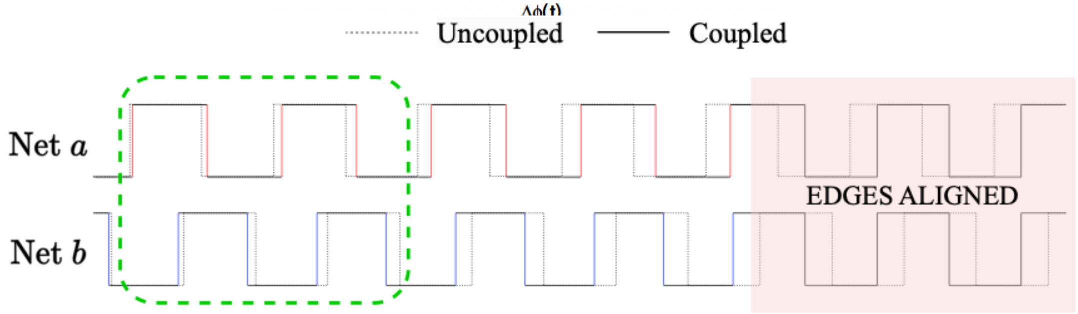
$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_P$$

Timing analysis of coupled oscillators

- Generalized Adler equation for N coupled oscillators [Bhansali and Roychowdhury, ASPDAC09]

$$\frac{d\phi_i(t)}{dt} = (\omega_i - \omega^*) + \omega_i \sum_{j=1, j\neq i}^{N} c_{ij}(\phi_{ij}(t))$$

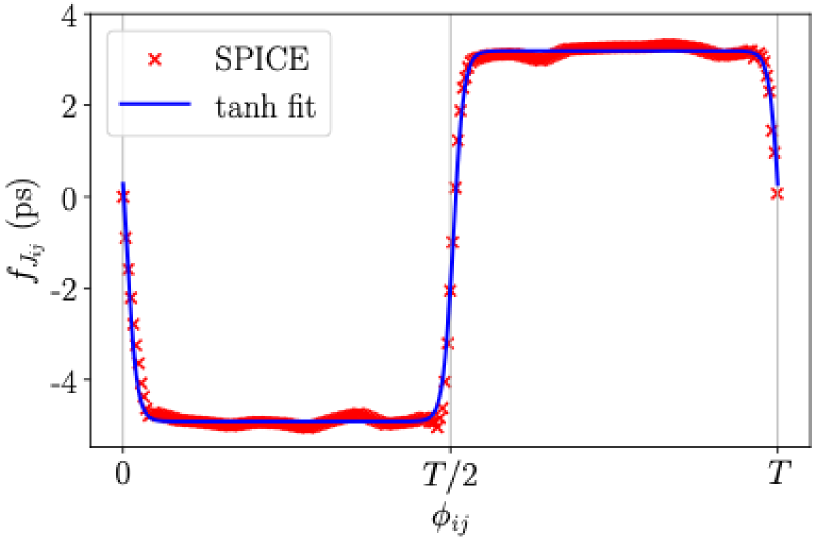




- We have shown that this is a continuous-time approximation of a series of discrete events

$$c_{ij} = f_{J_{ij}}(\phi_{ij})/(2\pi)$$

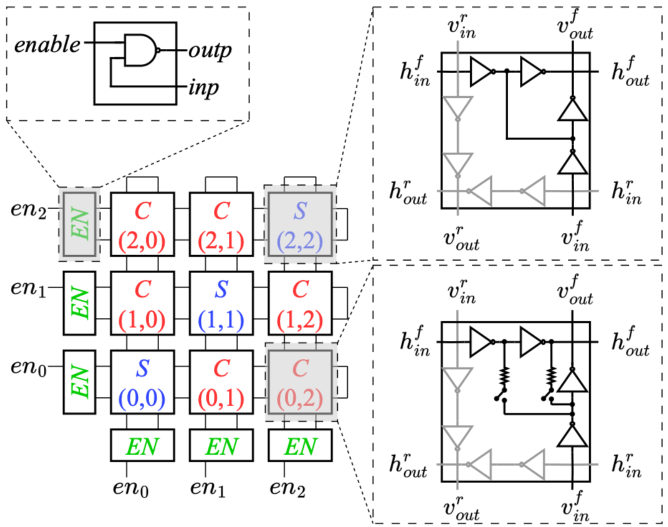
where f is the delay shift per cycle

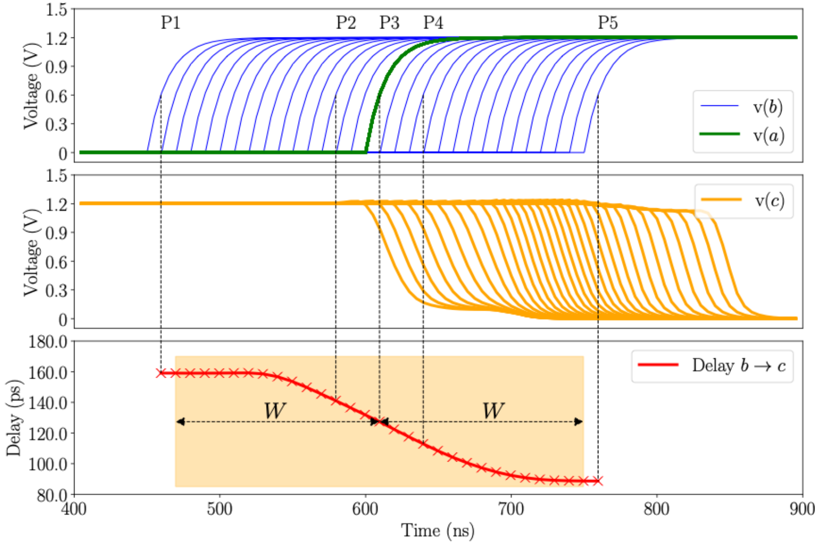


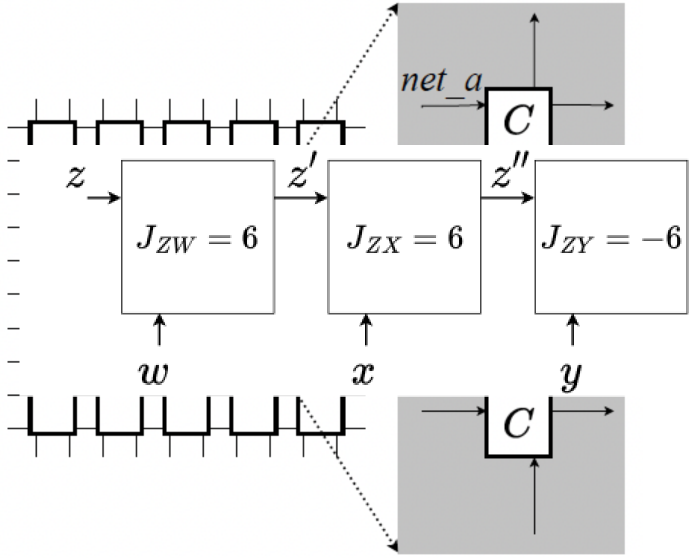
[Kumar *et al.*, arXiv:2502.19399, 2025]

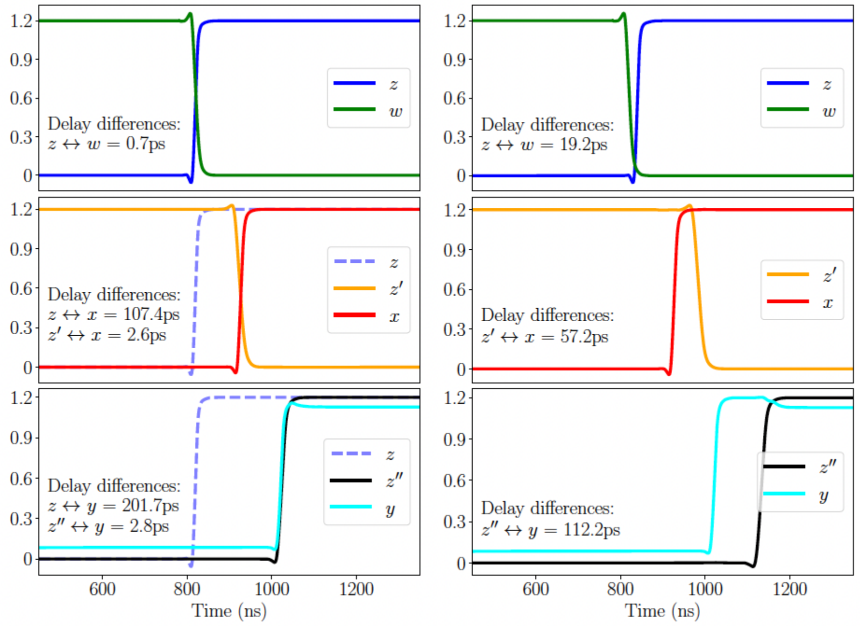
Timing analysis of coupled oscillators (contd.)

- For RO-based array: can use principles of timing analysis
  - Delay is a function of phase difference
- Event-driven timing
  - Seek events within the window that influences a stage delay





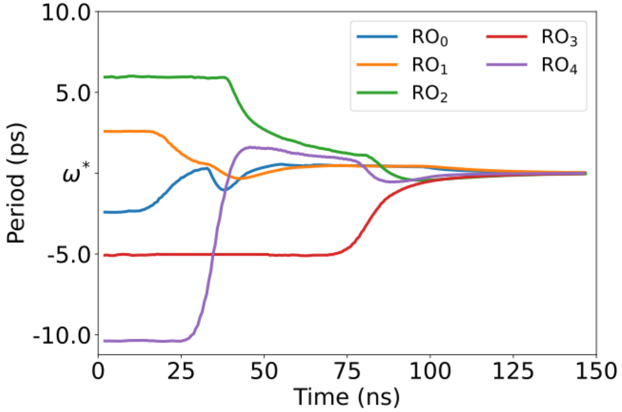




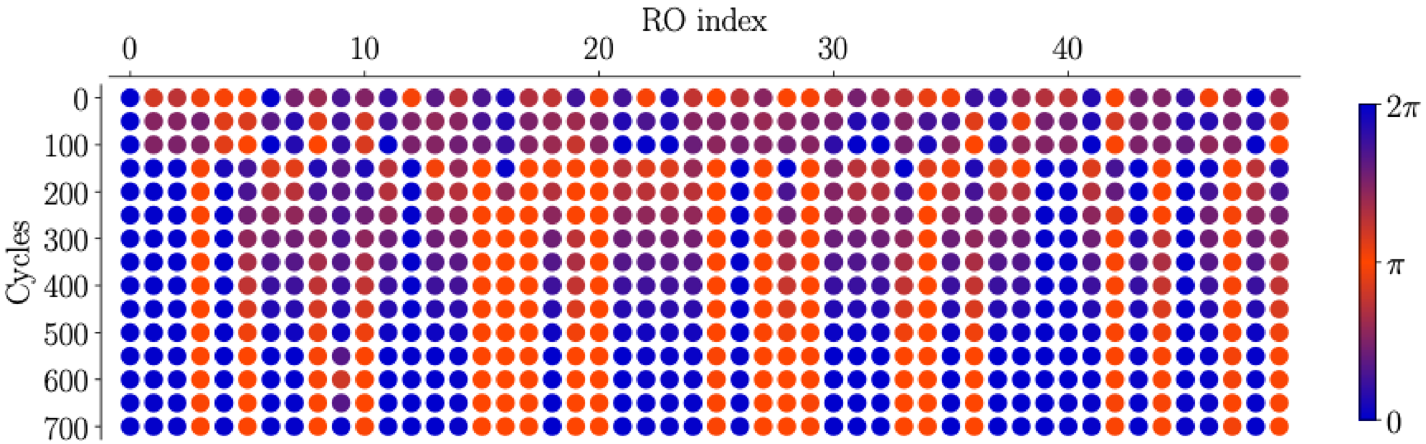
[Kumar *et al.*, arXiv:2502.19399, 2025]

Simulation of all-to-all coupled arrays using DROID

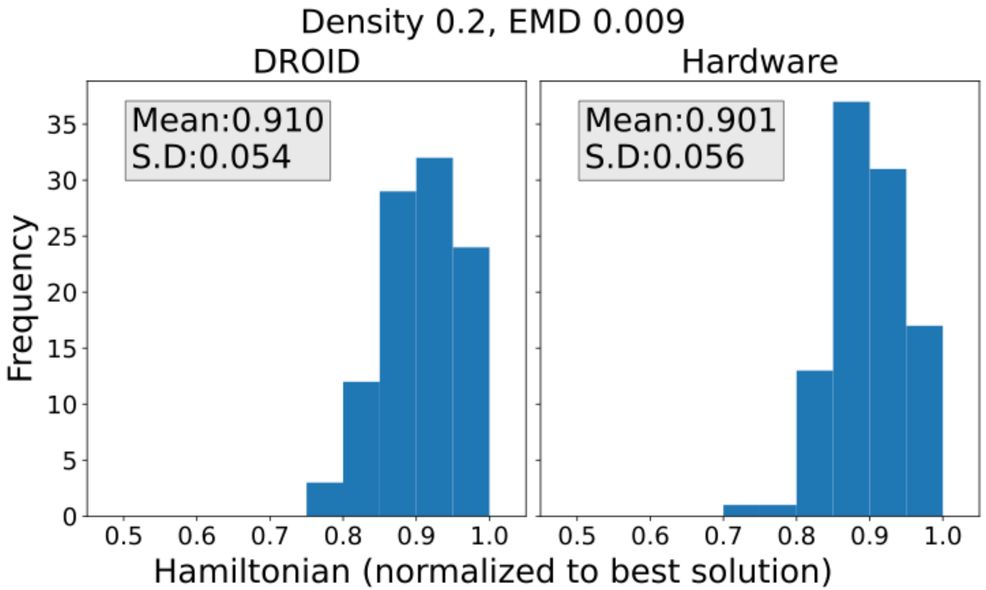
- Evolution of RO periods using DROID



- Evolution of the solution for 50 coupled ROs



- Comparison of the solution

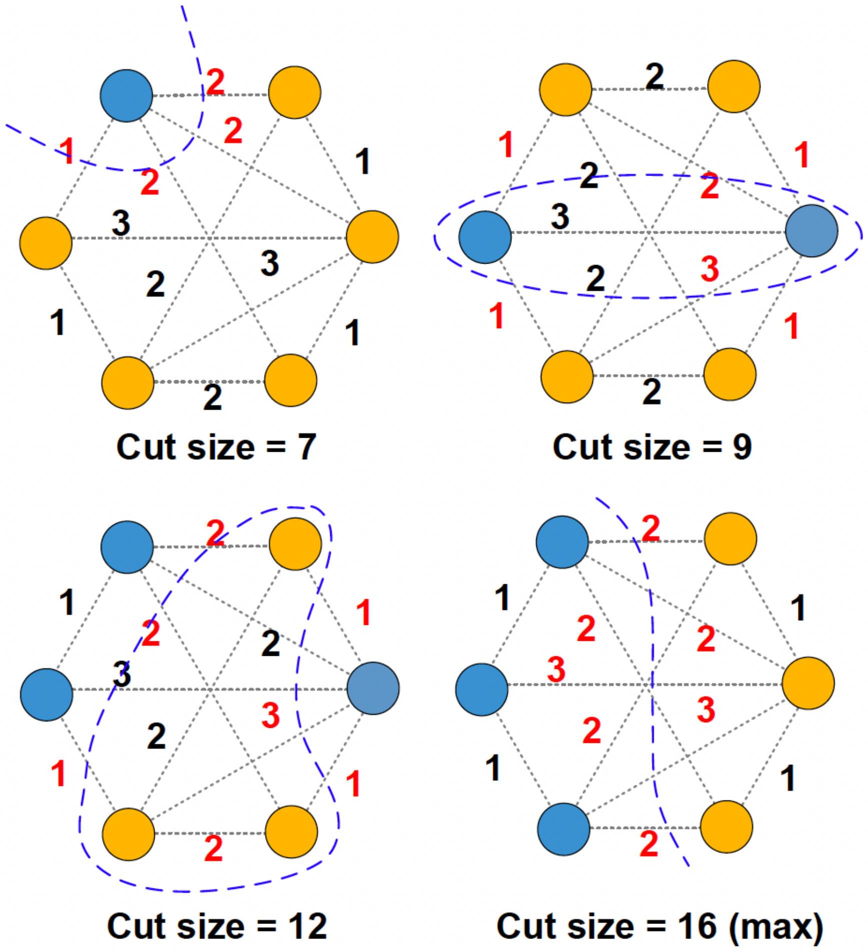


[Kumar *et al.*, arXiv:2502.19399, 2025]

Applications

Max-cut on an Ising solver

- NP-hard problem, considered to be the “easy” problem for Ising



$$H(\sigma) = \sum_{i,j} w_{ij}\,\sigma_i \sigma_j$$
$$= \sum_{diff\ group} (-w_{ij}) + \sum_{same\ group} w_{ij}$$
$$= \sum_{diff\ group} (-w_{ij}) + \left[\sum_{i,j} w_{ij} - \sum_{diff\ group} w_{ij}\right]$$
$$= \sum_{all} w_{ij} - 2 \times \underbrace{\sum_{diff\ group} w_{ij}}_{\text{Cut size}}$$

H = Hamiltonian of the system

σ_i = Spin status of magnet i {+1 or -1}

w_{ij} = weight between magnets i and j

Other problems can be modeled in Ising form

0–1 ILP

min $\mathbf{c}^\mathrm{T}\,\mathbf{x}$

subject to $S\,\mathbf{x} = \mathbf{b}$, $x_i \in \{0,1\}$

- Minimize unconstrained Hamiltonian

$$H = H_A + H_B$$

$$H_A = A\sum_{j=1}^{m}\left[b_j - \sum_{i=1}^{N} S_{ji}x_i\right]^2 \qquad H_B = -B\sum_{i=1}^{N} c_i x_i$$

- Factorizing 15

$$\mathrm{p} = (x_1\ 1)_2\,, \mathrm{q} = (x_2\ x_3\ 1)_2$$
$$H = (15 - pq)^2$$

$$H = 128x_1x_2x_3 - 56x_1x_2 - 48x_1x_3 + 16x_2x_3 - 52x_1 - 52x_2 - 96x_3 + 196$$

$$H_{mod} = 200x_1x_2 - 48x_1x_3 - 512x_1x_4 + 16x_2x_3 - 512x_2x_4 + 128x_3x_4 - 52x_1 - 52x_2 - 96x_3 + 768x_4 + 196$$

[Lucas, Frontiers in Physics, 2014] [Jiang *et al.*, Scientific Reports 2018]

Integer knapsack problem

Maximize knapsack cost $\mathcal{C} = \sum_{\alpha=1}^{N} c_\alpha x_\alpha$

subject to knapsack weight

$$\mathcal{W} = \sum_{\alpha=1}^{N} w_\alpha x_\alpha \leq W$$

- Introduce variable y_n ≡ knapsack has weight n
- Maximize unconstrained Hamiltonian

$$H = H_A + H_B$$

$$H_A = A\left(1 - \sum_{n=1}^{W} y_n\right)^2 + A\left(\sum_{n=1}^{W} n y_n - \sum_{\alpha} w_\alpha x_\alpha\right)^2$$

$$H_B = -B\sum_{\alpha} c_\alpha x_\alpha$$

All-to-all interactions between variables!

QUBO formulations

- QUBO = quadratic unconstrained Boolean optimization
  - Variables $x_i \in \{0,1\}$

$$\min_{\mathbf{x}} F(\mathbf{x}) = \mathbf{x}^T Q \mathbf{x} = \sum_{i=1}^{n} Q_{ii} x_i + \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} Q_{ij} x_i x_j$$

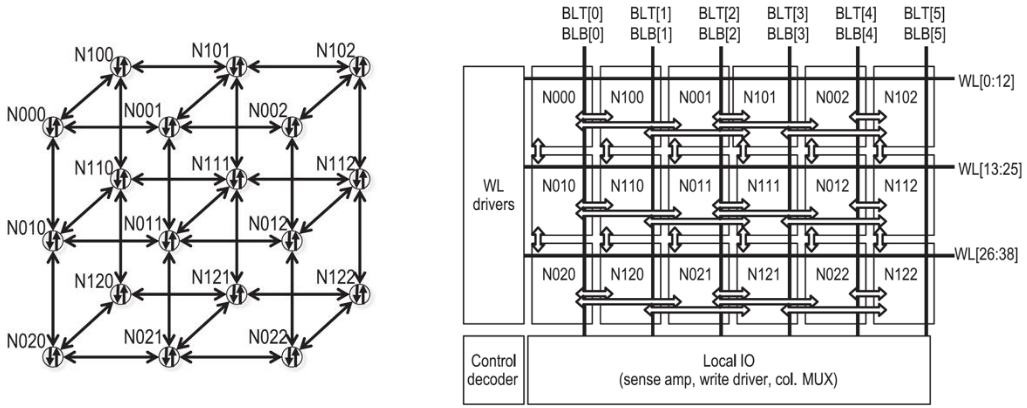
- Equivalent Ising formulation
  - Variables $s_i \in \{-1,1\}$

$$x_i = (s_i + 1)/2$$

$$\min_{\mathbf{s}} F(\mathbf{s}) = \underbrace{\sum_{i} h_i s_i}_{\text{field}} + \underbrace{\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} J_{ij} s_i s_j}_{\text{coupling}}$$

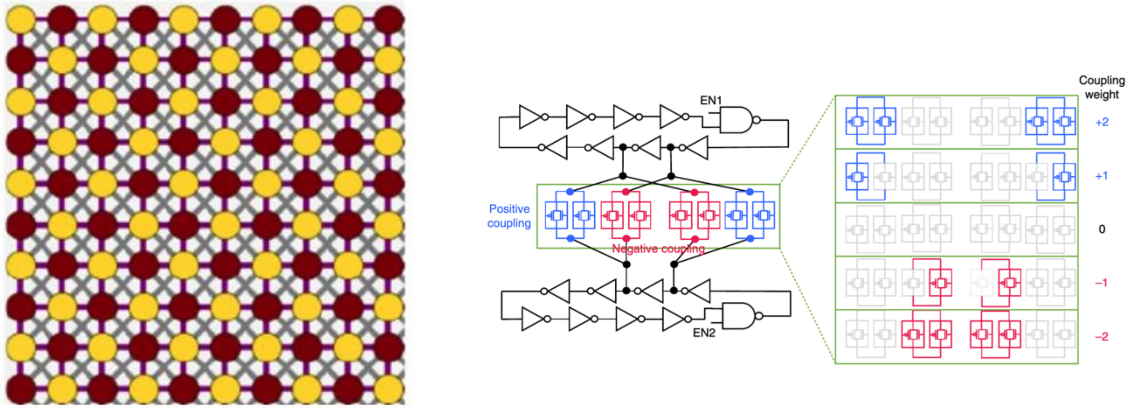
Implementations of Ising machines

- Lattice graph (SRAM-based)



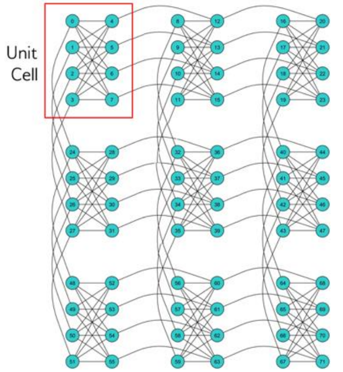
[Yamaoka, JSSC 2016]

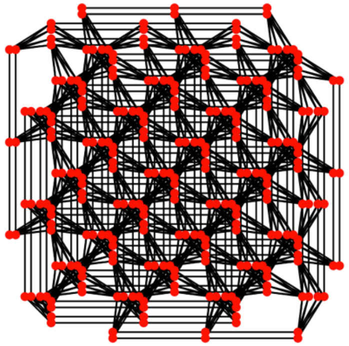
- King's graph CMOS ASIC



[Moy, Nature Electronics 2022]

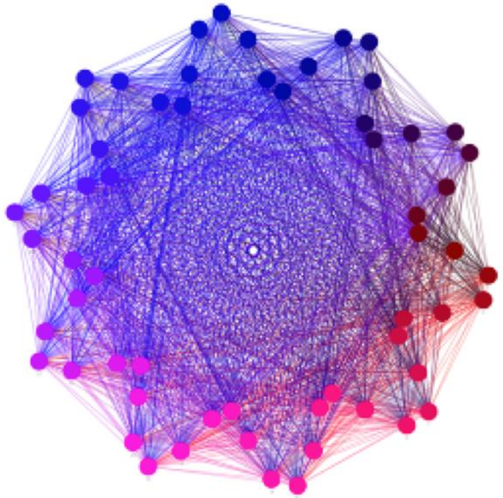
- D-Wave





[dwave.com]

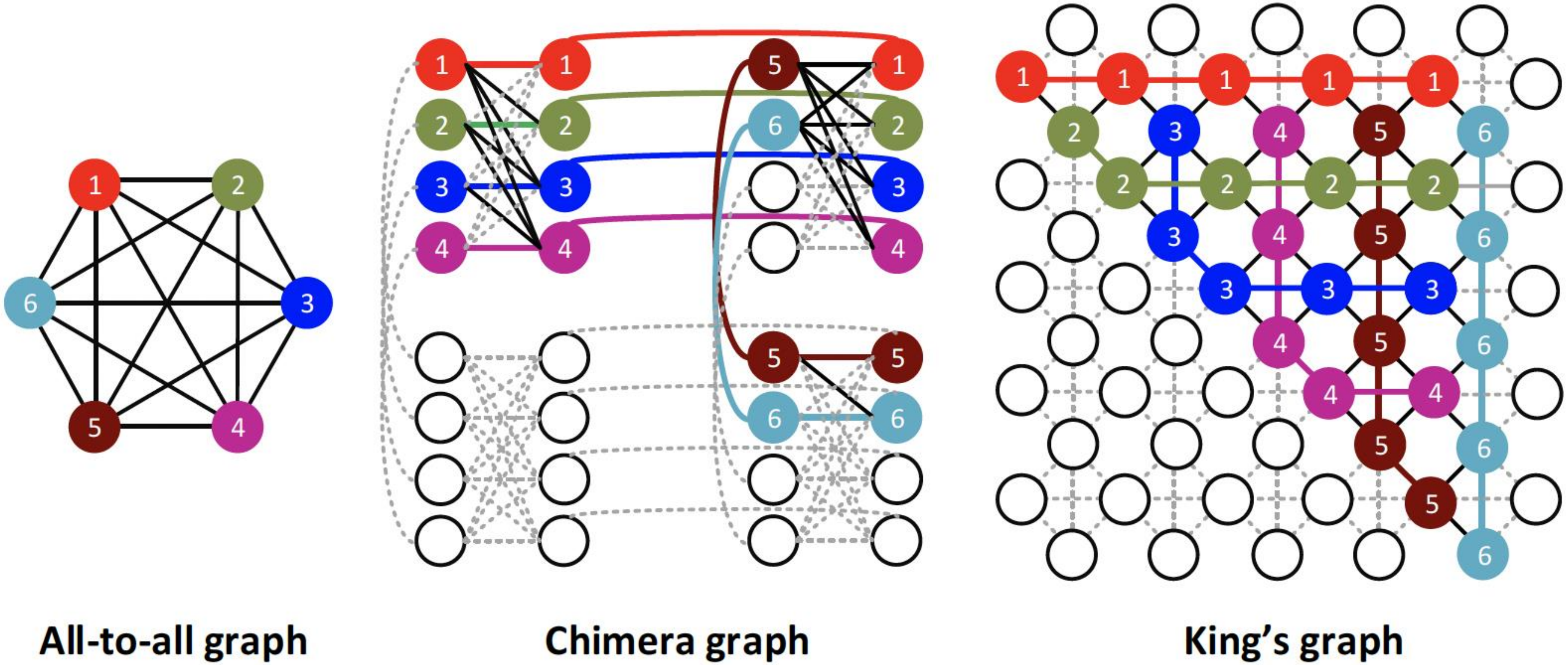
- Ideal structure: "all-to-all connected"



Planar vs. A2A and minor embedding

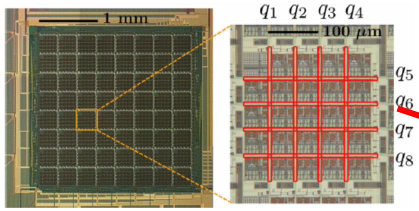
- Number of connections in lattice-based/King's graph structures = O(n) for n coupled oscillators
- Can program O(n) out of O(n^2) possible connections
- Minor embedding: mapping logical spins to physical spins
- More replication = weaker spin strength

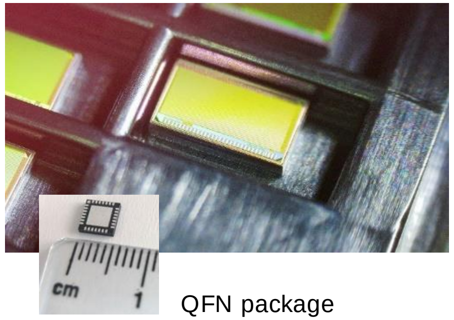
Topology	Connectivity
King's graph (Hitach)	8
Chimera graph (D-Wave)	6
Pegasus graph (D-Wave)	15
Zephyr (D-Wave)	20
All-to-all (This work)	48-1 = 47



[Lo *et al.*, Nature Electronics 23]

Quantum vs. Quantum-Inspired



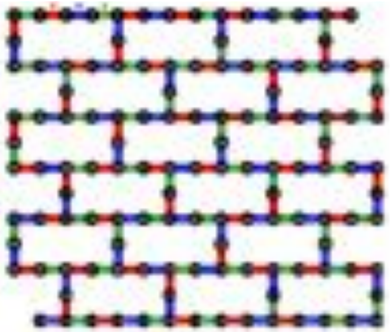
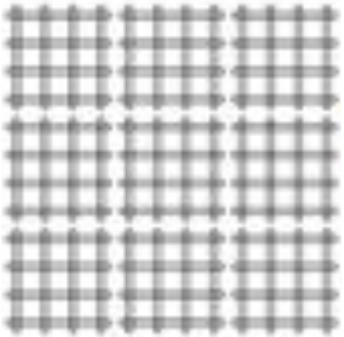
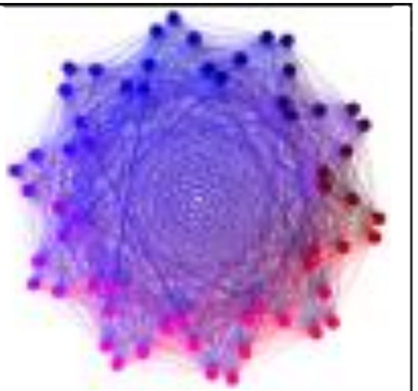


QFN package

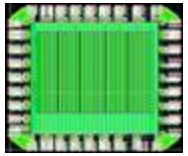
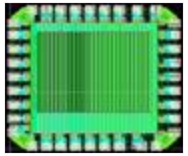
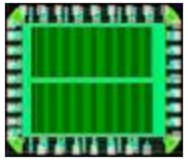
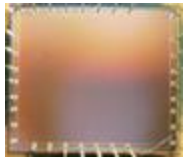
	Quantum Annealer	CMOS Ising Chip
Qubits	5,000+ physical qubits = 47 (all-to-all)	59 physical qubits
Connectivity	15	58 (native all-to-all)
Couplers	35,000+	1,711
Weight resolution	~5 bit	~5 bit
Power consumption	25,000W	0.010W
Technology	Superconducting	Foundry (TSMC)

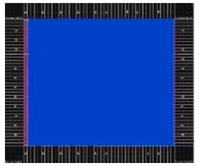
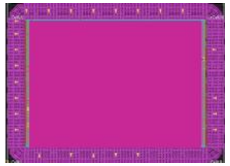
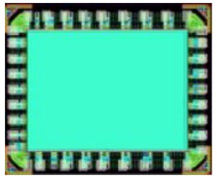
[D-Wave, IOT World Today]

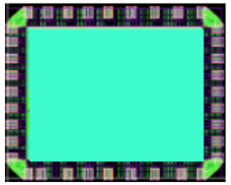
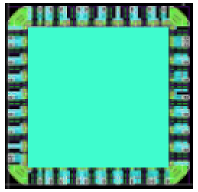
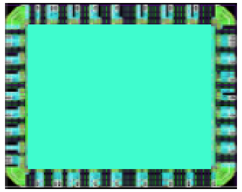
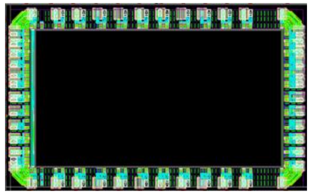
Key challenge: Connectivity

Architecture	 King's graph	 Eagle	 Chimera	 All-to-all
# of qubits	289 (equivalent to 17 all-to-all)	127	2,000+ (equivalent to ~50 all-to-all)	59
Connectivity	8 (max)	3 (max)	6	58
Weight	0, -1	N/A	N/A	5 bit
Device type	Rydberg atoms	Superconducting	Superconducting	CMOS (room temp.)
Minor embedding	Required	Required	Required	Not required
Power consumption	N/A	N/A	25,000W	0.010W
Reference	Science 2022 [Ebadi]	Nature 2023 [Kim]	D-Wave website	This work

CMOS Ising chip prototypes

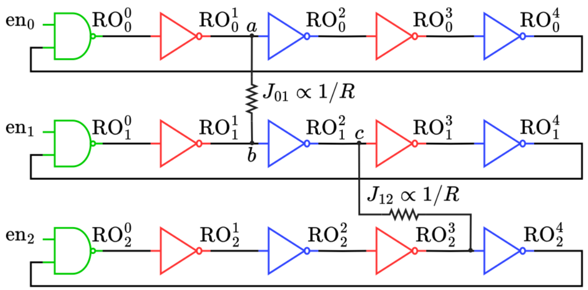
Chip name	COBISHIL (28nm, Jan. 2023)	COBITX (28nm, Jan. 2023)	COBIFIXED (28nm, Mar 2023)	COBIFIXED65 (65nm, May 2023)
Chip Layout				

Chip name	COBIHAM (28nm, May 2023)	COBIRISCV2 (28nm, Jun 2023)	COBIGRADIENT (28nm, Sep 2023)	COBITRANSPOSE (28nm, Dec 2023)
Chip Layout				

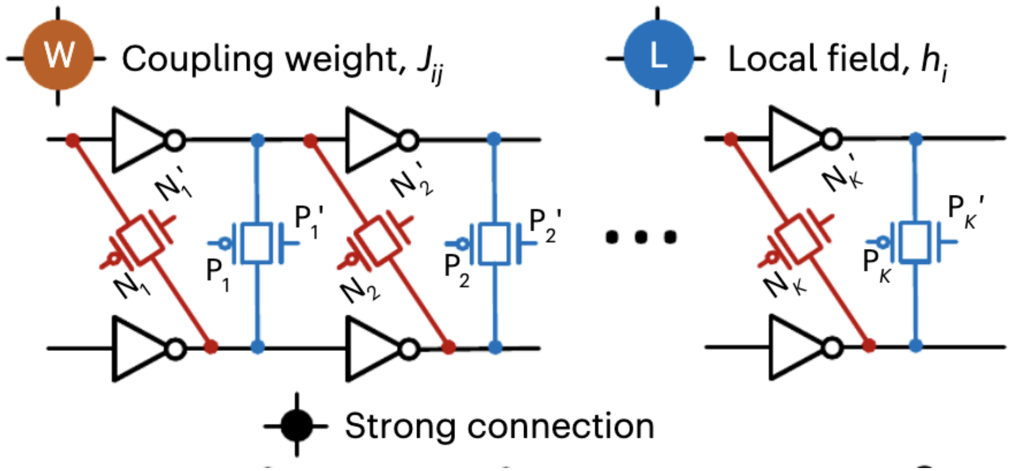
Chip name	COBIFIXED28 (28nm, Jan. 2024)	COBIDUALRES (28nm, Feb. 2024)	COBIFIXED28P (28nm, Mar. 2024)	COBIFIVE (28nm, May 2024)
Chip Layout				

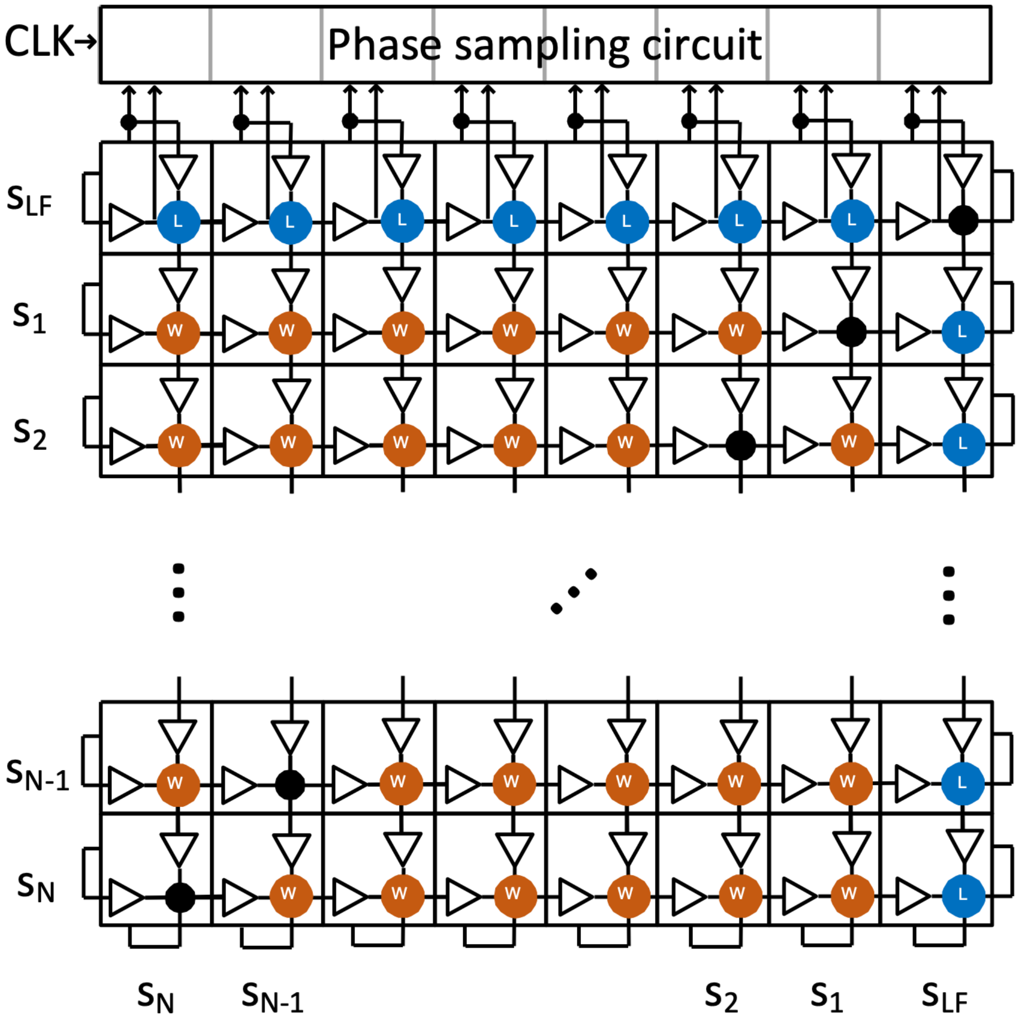
COBI oscillator array

- Coupling strengths depend on wire parasitics



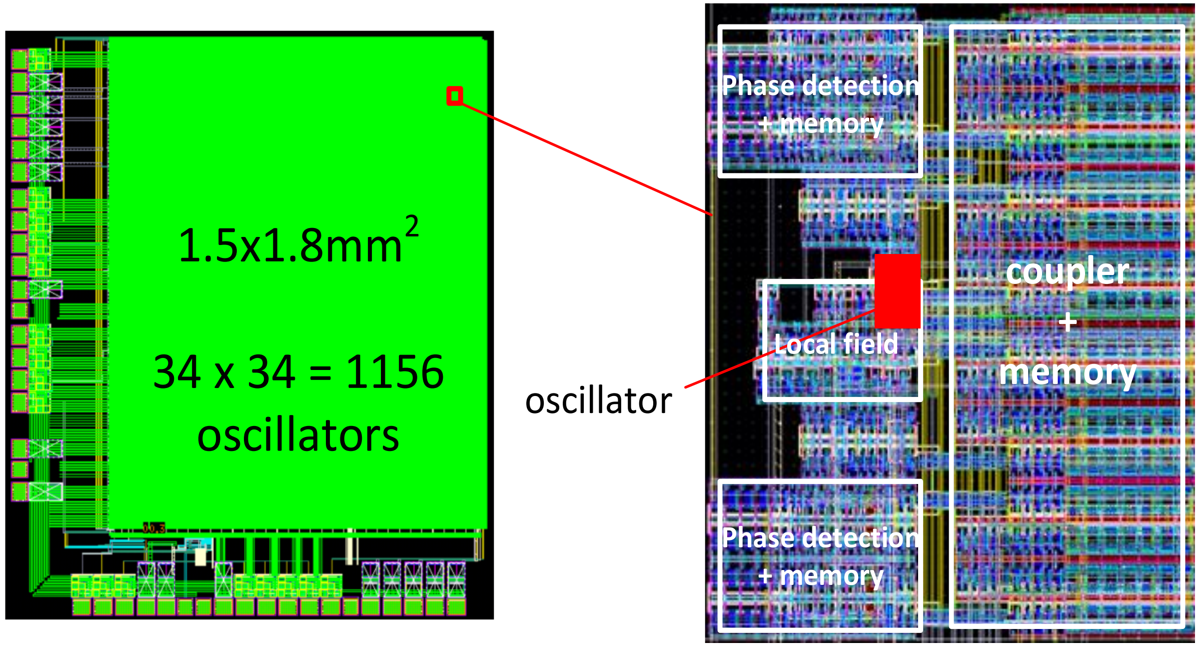
- Need complete symmetry in the structure





COBI oscillator array (contd.)

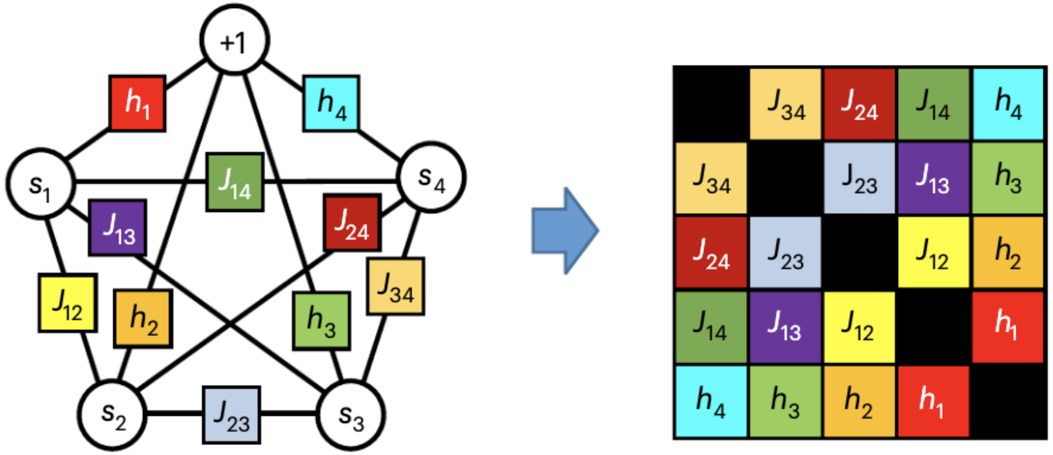
Chip layout



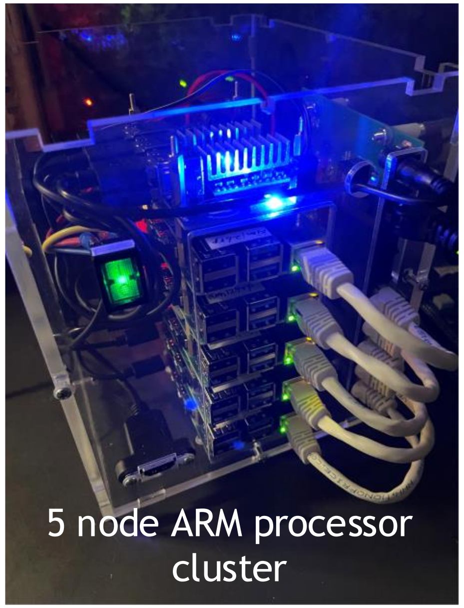
Oscillator area < 3% of COBI chip.
Rest is memory, coupler, logic, control, and IO.

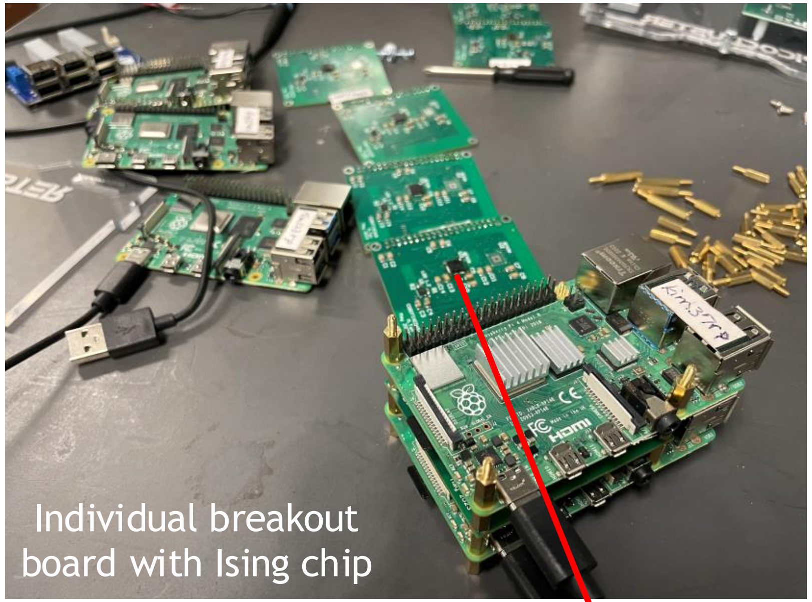
Problem mapping

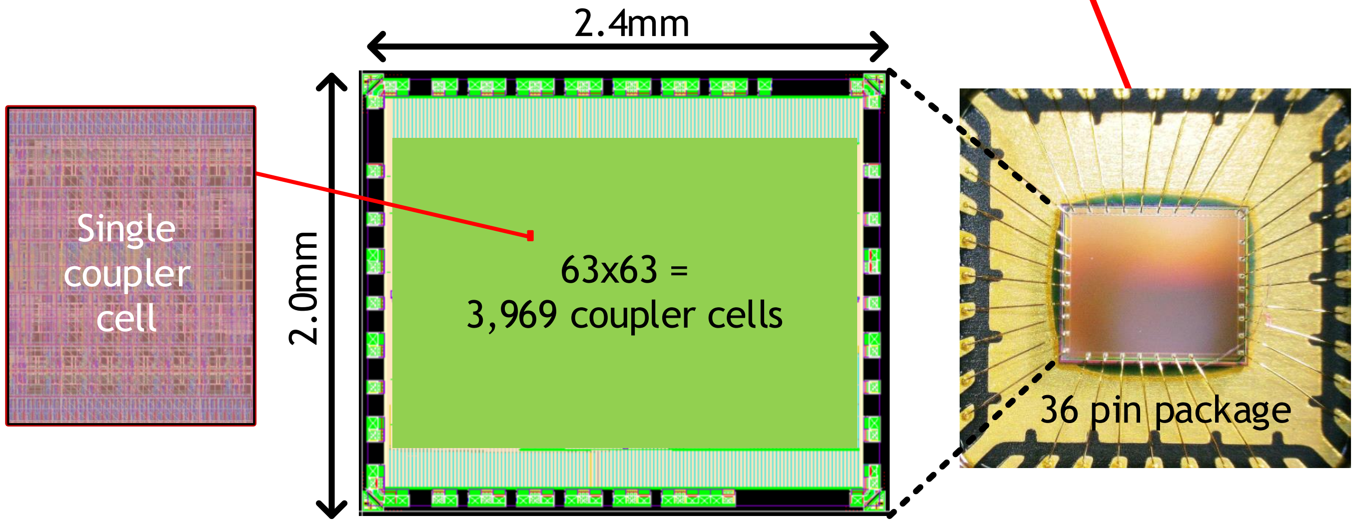
$$H = h_1 s_1 + h_2 s_2 + h_3 s_3 + h_4 s_4 + J_{12} s_1 s_2 + J_{13} s_1 s_3 + J_{14} s_1 s_4 + J_{23} s_2 s_3 + J_{24} s_2 s_4$$



Improved 59-spin A2A Ising solver chip



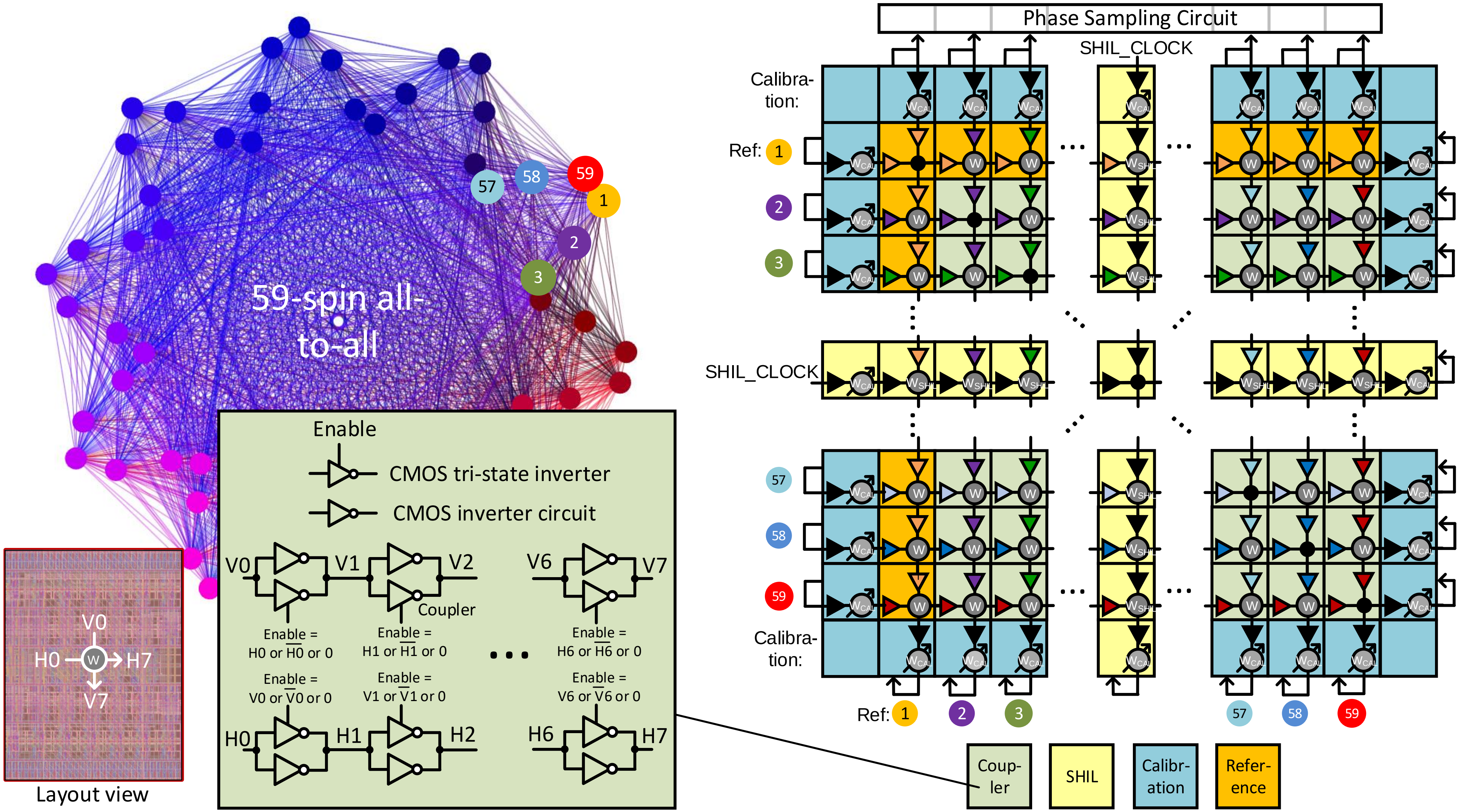




Technology	TSMC 65 nanometer
Supply voltage	1.2V
Die size	2.4 mm x 2.0 mm
Array size	63 x 63
# of spins	59 spins + 2 SHIL + 2 calibration = 63 cells
Connectivity	58 (all-to-all)
Coupler resolution	-14, -13, …, 13, 14 (29 levels)
# of couplers	59 x 58 / 2 = 1,711
Power consumption	8.2mW (graph density = 0.0) 10.0mW (graph density = 1.0)
Oscillation frequency	5.19 MHz (graph density = 0.0) 5.36 MHz (graph density = 1.0)
Operating temperature	Room temperature
Key features	Zero static power coupler, sub harmonic injection locking (SHIL), individual frequency calibration

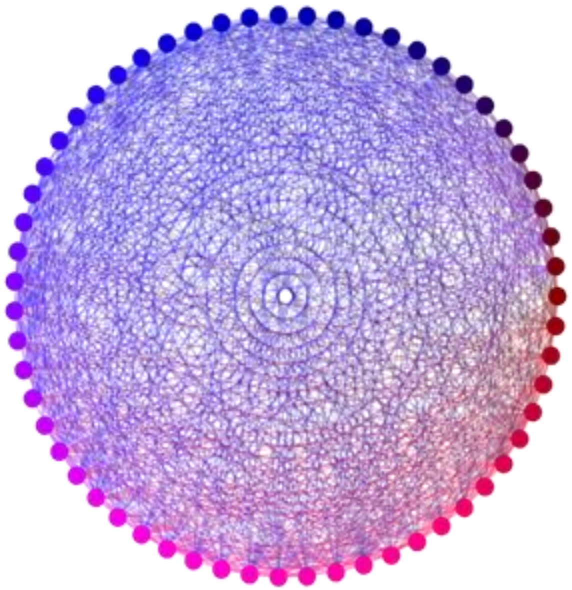
[Cılasun *et al.*, https://doi.org/10.21203/rs.3.rs-4208492/v1]

Array architecture and coupler design



[Cılasun *et al*., https://doi.org/10.21203/rs.3.rs-4208492/v1]

Chip Programming



59 spin all-to-all

Spin ID			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30			31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	1	0	4	4	4	2	4	7	4	7	7	6	3	7	7	4	4	7	4	4	1	7	7	7	7	4	2	7	3	5	4	3	1	1	2	4	7	7	7	3	7	5	4	5	3	7	7	2	5	5	1	3	3	3	3	4	7	5	3	4	3	4	7	0
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Enable bits | Calibration bits | SHIL bits | Jij weights | hi weights

[Cılasun *et al.*, https://doi.org/10.21203/rs.3.rs-4208492/v1]

Solving real problems in the real world

The solution is inherently probabilistic!

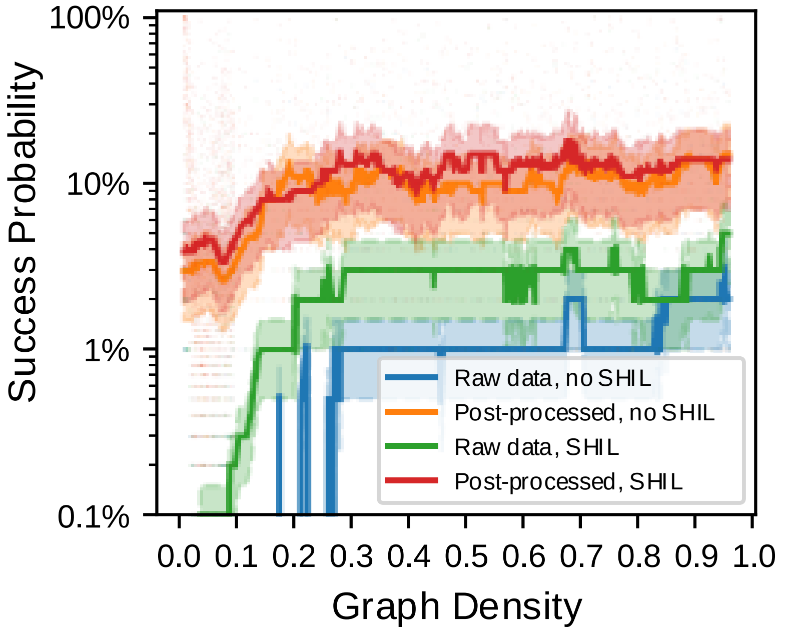
- Requires repeated solutions – not a problem if each solution is fast

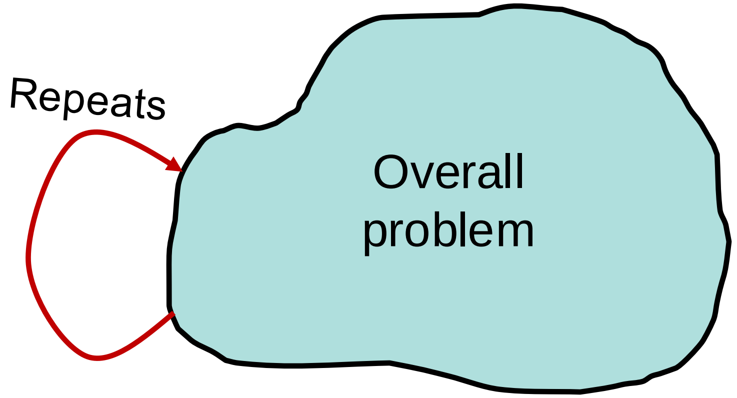
- Let p = probability of achieving a solution
- Over n attempts, probability of success = $1 - (1-p)^n$

- Time-to-solution (TTS) metrics
  - To achieve p_{target} (e.g., 99%) accuracy,

$$p_{target} = 1 - (1-p)^n$$

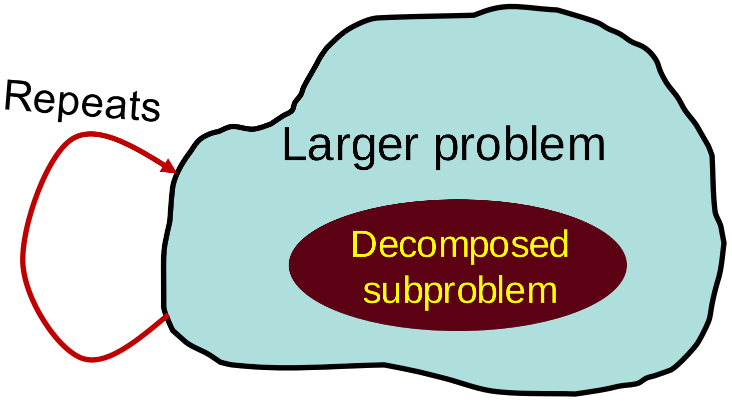
$$n = \log(1 - p_{target})/\log(1 - p)$$

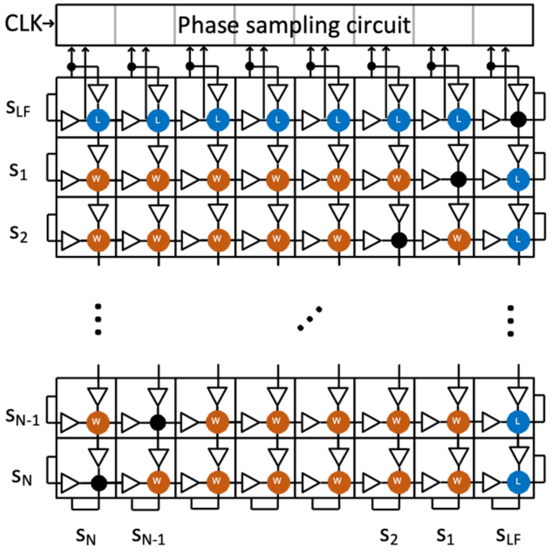


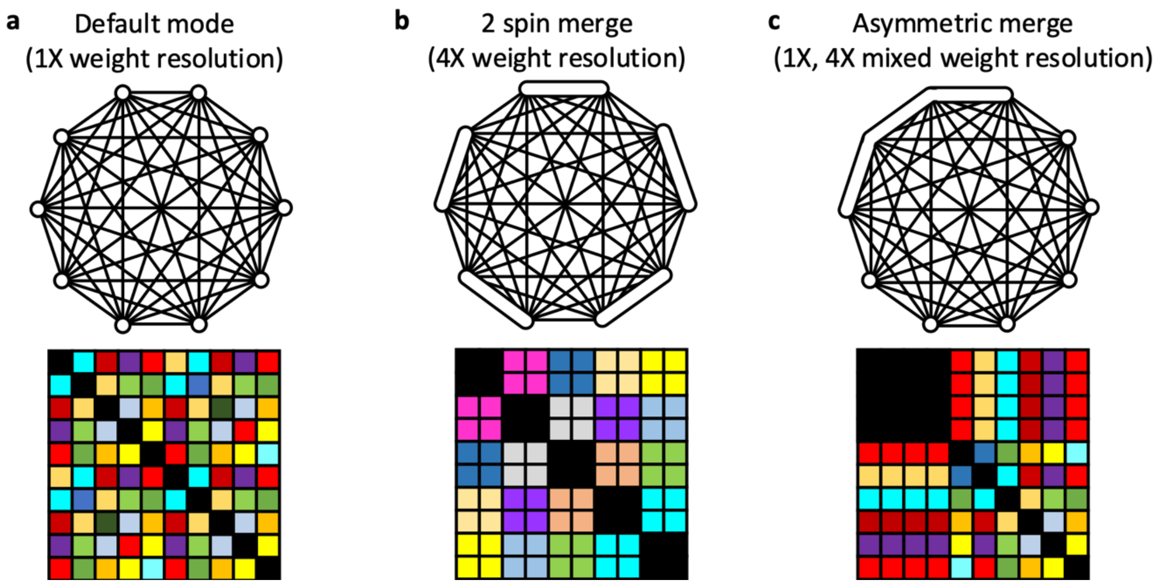


Living with the limitations of the hardware

Limitation	Solution
Limited hardware size	• Problem decomposition
Limited coupling weights	• Map to multiple spins • Truncation
Precision improvement	• Scaling • Truncation

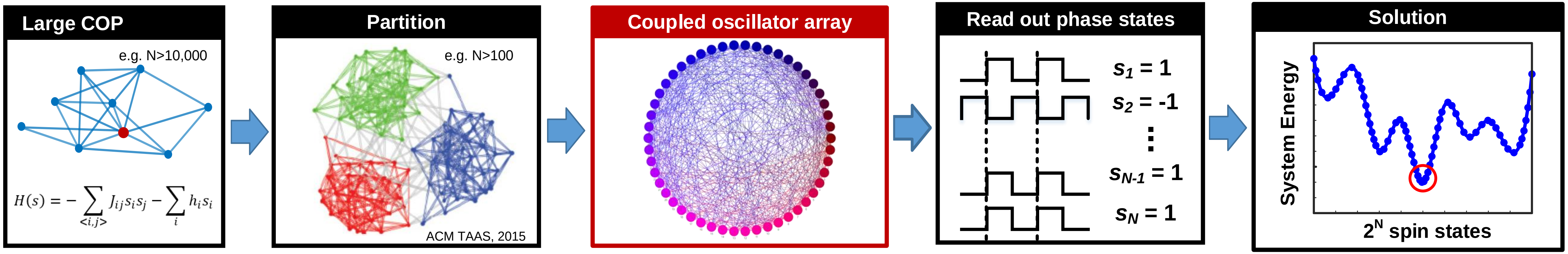






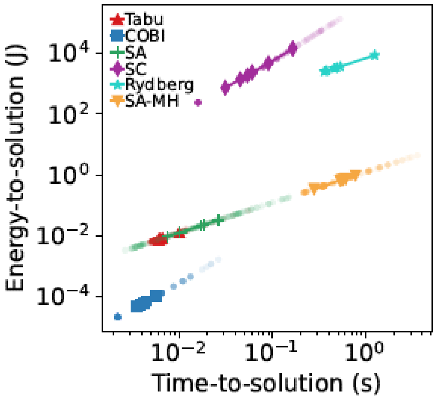
[Cılasun *et al.*, Scientific Reports 2024]

Solving large problems with a decomposer

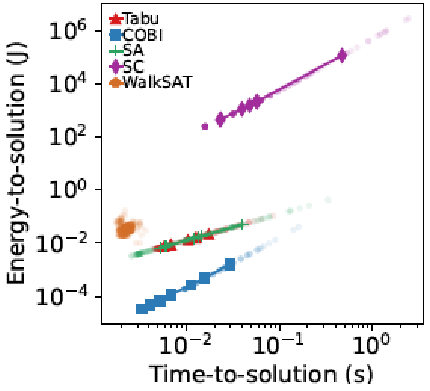


$$H(s) = -\sum_{<i,j>} J_{ij} s_i s_j - \sum_i h_i s_i$$: Ising Hamitonian

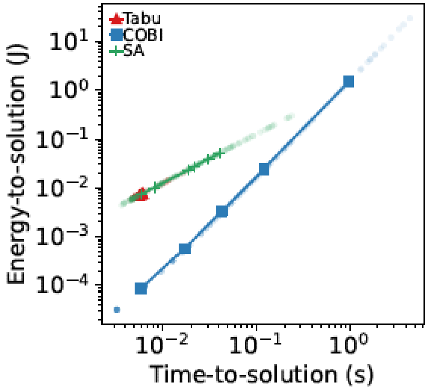
s_i, s_j : Spin state {+1 or -1} J_{ij} : Coupling strength h_i : local field strength



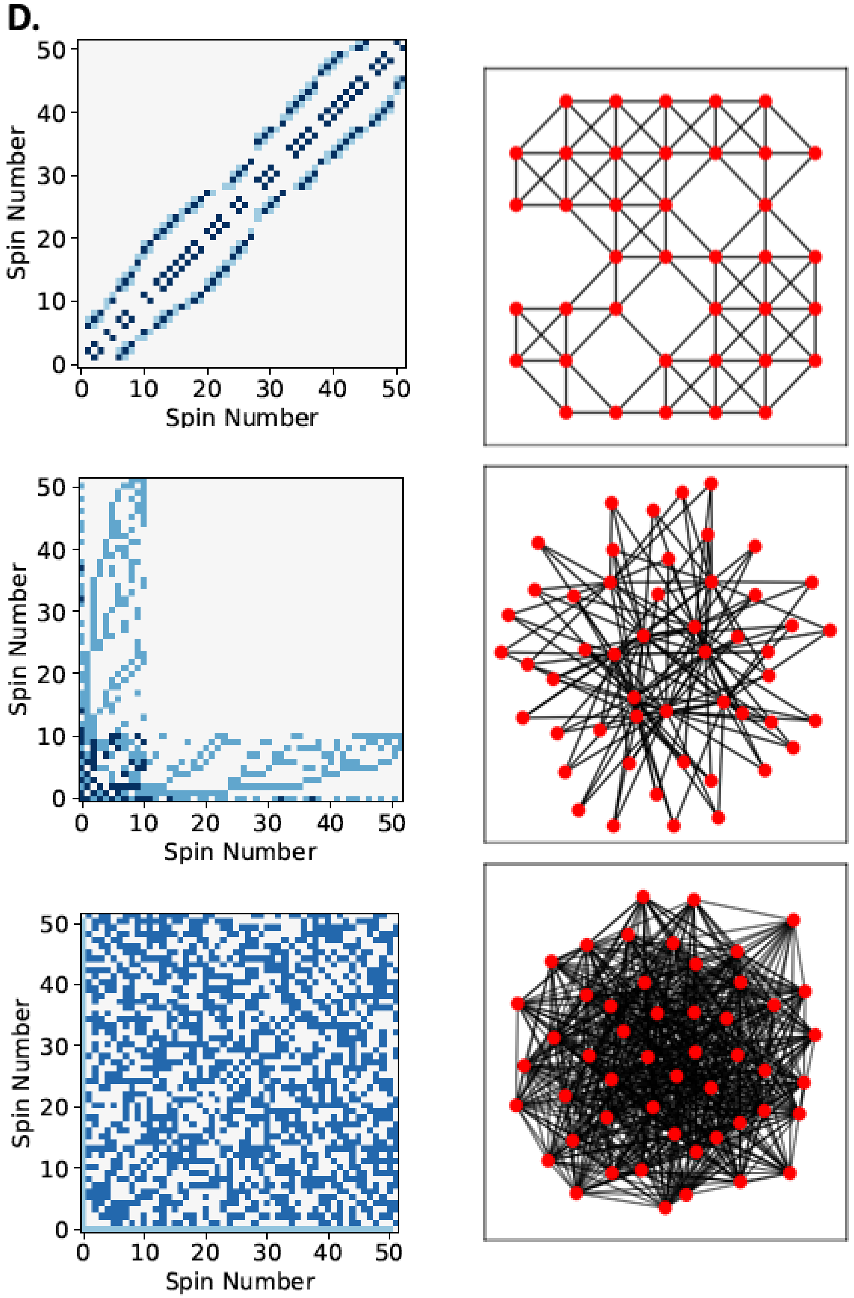
Planar MIS
(n=39, 51)



SAT
(n=10,11)



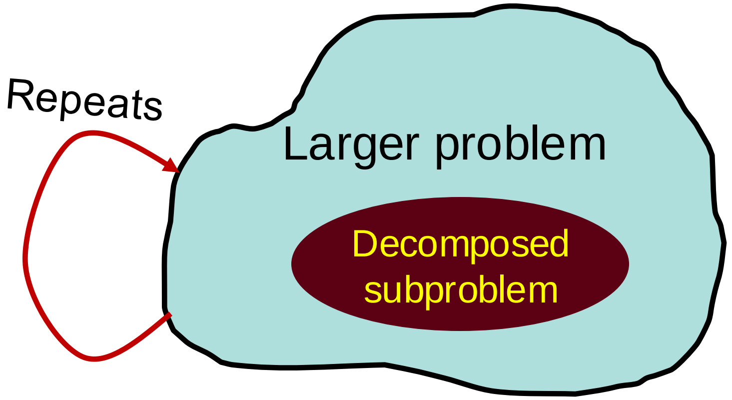
Nonplanar MIS
(n=51)



[Cılasun *et al.*, https://doi.org/10.21203/rs.3.rs-4208492/v1]

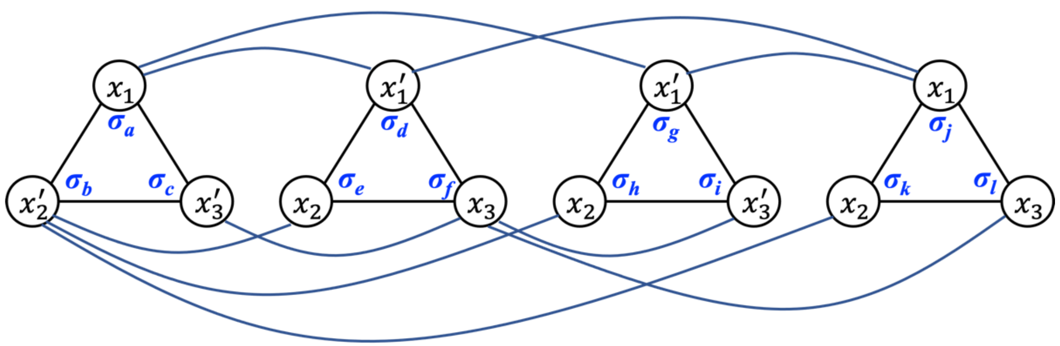
Exploiting parallelism

- Multicore COBI chips can accelerate solution



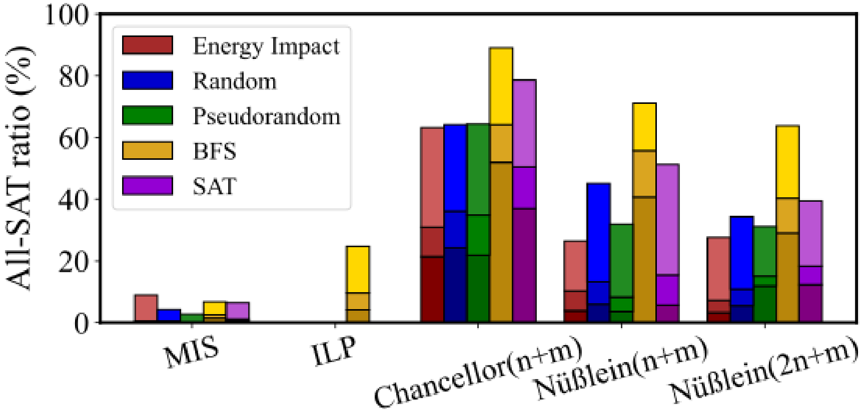
Problem mapping: 3SAT on the COBI Ising chip

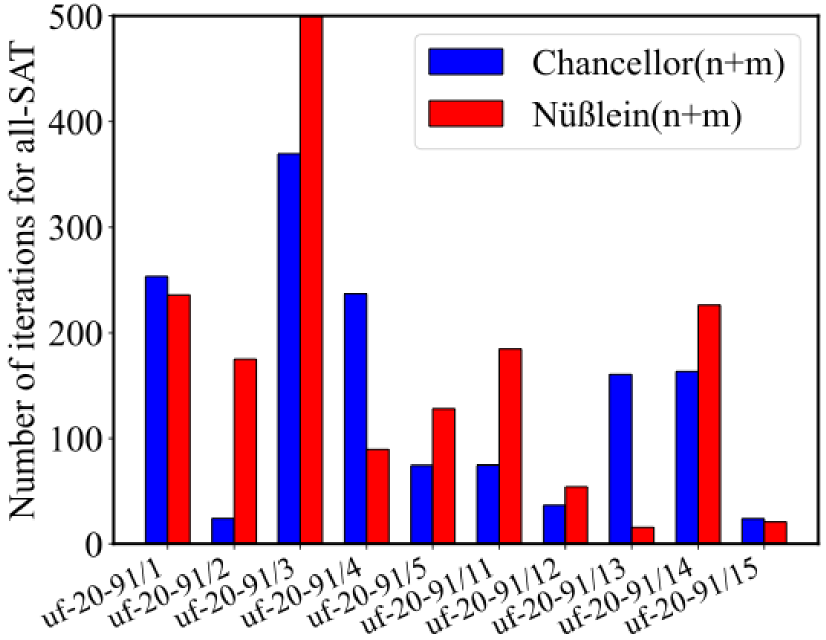
- 3SAT parameters: *n* variables, *m* clauses
- Various possible mappings
  - Maximal independent set (3*m* spins)



$$f = (x_1 \vee x_2' \vee x_3')(x_1' \vee x_2 \vee x_3)(x_1' \vee x_2 \vee x_3')(x_1 \vee x_2 \vee x_4)(x_4 \vee x_5' \vee x_6)(x_4' \vee x_5 \vee x_6)$$

  - Chancellor, Nüßlein formulations (*n* + *m* spins)
  - ILP formulation (*n* + 2*m* spins)



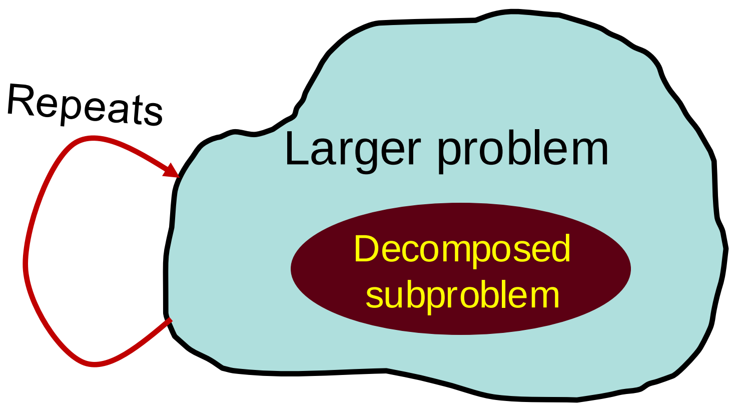


[Cılasun *et al.*, Scientific Reports 2024]

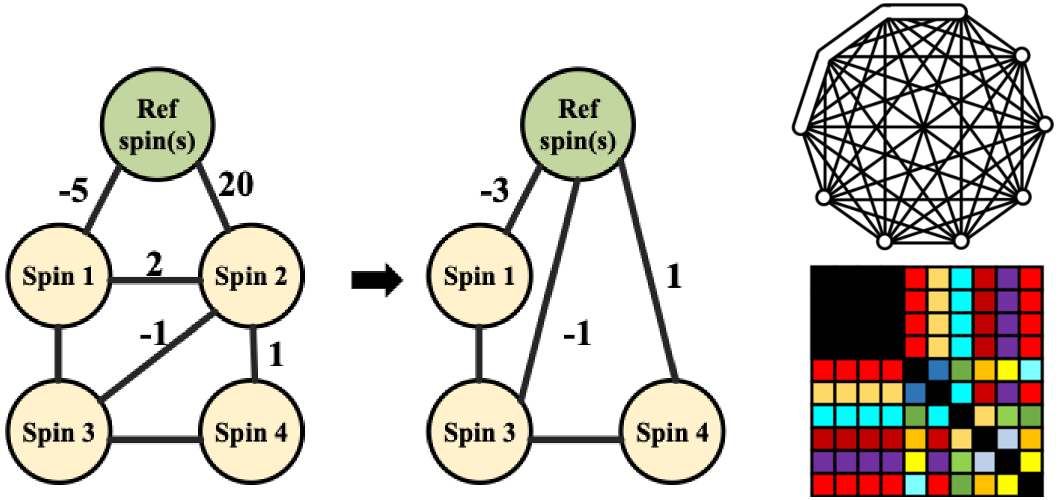
Problem decomposition and frozen spins

- All spins outside the decomposed segment are “frozen” to values in the previous iteration
- Impact of freezing a spin in this expression

$$\min_{\mathbf{s}} F(\mathbf{s}) = \underbrace{\sum_{i} h_i s_i}_{\text{field}} + \underbrace{\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} J_{ij} s_i s_j}_{\text{coupling}}$$



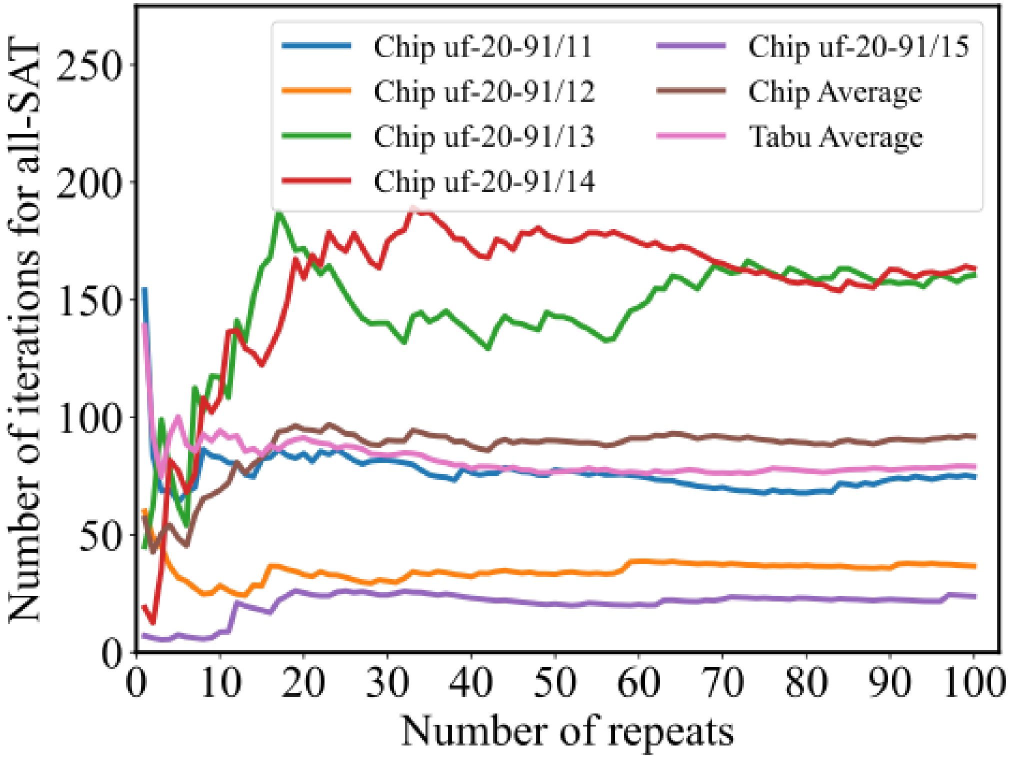
- Frozen $s_i \Rightarrow$ field term constant $\Rightarrow$ removed
- Frozen s_i and $s_j \Rightarrow$ coupling constant $\Rightarrow$ removed
- Unfrozen s_i and $s_j \Rightarrow$ coupling term kept as is
- Frozen s_i but not $s_j \Rightarrow$ coupling $\rightarrow$ field term
  - Field terms can become very large!
  - Theoretical result: if field term $h_i s_i$ satisfies an inequality, it forces the spin s_i value to ± 1



[Cılasun *et al.*, Scientific Reports 2024]

Some early runtime comparisons

Benchmark	Runtime (ms)			Energy (mJ)		
	Tabu	WalkSAT	Chip	Tabu	WalkSAT	Chip
uf20-91-11	4322	2.351	12.83	367,370	199.8	0.128
uf20-91-12	1596	2.337	6.30	135,660	198.6	0.063
uf20-91-13	5095	2.506	27.58	50,950	213.0	0.276
uf20-91-14	7148	2.470	28.07	433,075	210.0	0.281
uf20-91-15	1591	2.377	4.09	607,580	202.0	0.041



Very recent results on ~200-variable SAT problems show great promise

[Cılasun *et al.*, Scientific Reports 2024]

Conclusion

- Quantum-inspired Ising solvers can be manufactured in today's foundry processes
- QUBO challenges and future directions
  - Problem formulation for larger constrained optimization problems
  - Higher-order interactions beyond QUBO
    - Ongoing work on *k*-SAT shows promise
  - Mundane issues:
    - Huge QUBO file size, memory bottleneck
    - ASIC designs with integrated pre- and post-processing, PCIe, memory interface
- In future: Massively parallel solutions (1000 cores ≡ 10W)

Thanks to

- You, the audience
- Hüsrev Cılasun, Ahmet Efe, Ulya R. Karpuzcu, Chris H. Kim, Abhimanyu Kumar, Hao Lo, Will Moy, Nafisa Sadaf Prova, Ziqing Zeng
- DARPA QuICC, NSF ASCENT