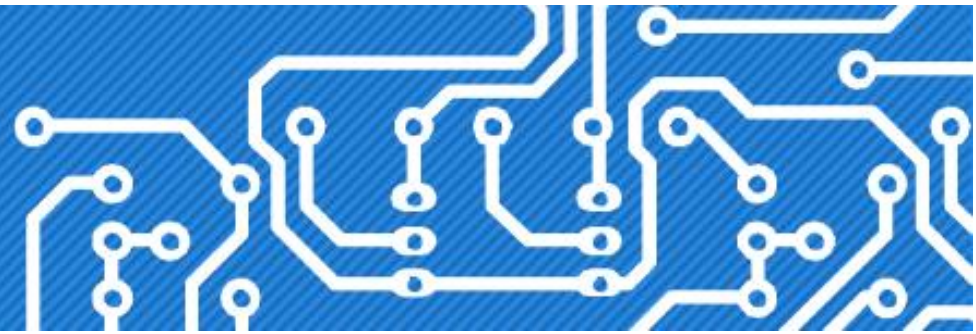




Recent Progress in the Analysis of Electromigration and Stress Migration in Large Multisegment Interconnects

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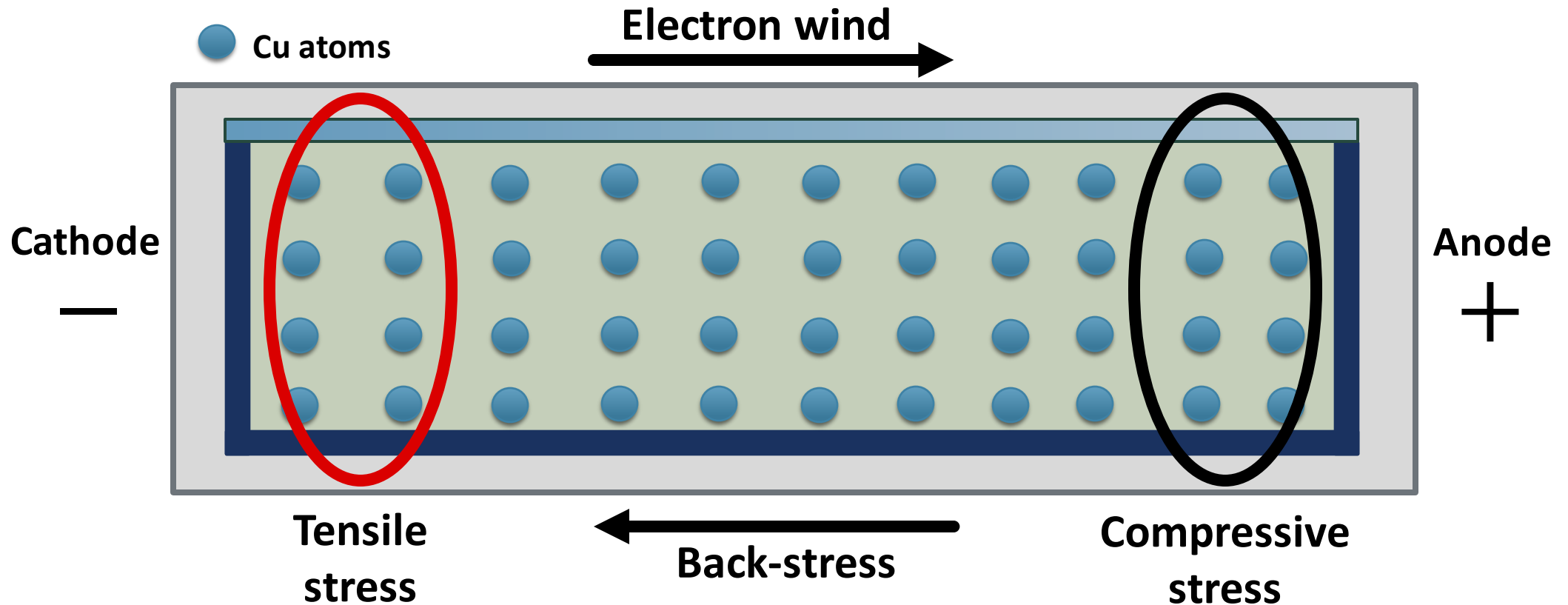


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Outline

- **Introduction**
- **Electromigration (EM) and Stress Migration Background**
- **EM/Stress Migration Analyses**
- **Multisegment Lines: Analytical Solution by Reflections**
- **Multisegment Lines: Semi-Analytical Solution**
- **Interconnect Trees: Semi-Analytical Solution**
- **Conclusion and Future Challenges**

EM/Stress Migration Background



EM/Stress Migration Background

Physics-based modeling

(Korhonen et al., JAP 1993)

The diagram illustrates the physics-based modeling equation for EM/Stress Migration. The equation is
$$\frac{\partial \sigma}{\partial t} = \frac{\partial}{\partial x} \left[\kappa \left(\frac{\partial \sigma}{\partial x} + \beta j \right) \right]$$
. Blue arrows point from various parts of the equation to descriptive labels: an arrow from σ points to 'Stress'; an arrow from κ points to 'Atomic flux due to Back-stress'; an arrow from $\frac{\partial \sigma}{\partial x}$ points to '[given params]'; an arrow from βj points to 'Atomic flux due to Electron wind'; and an arrow from j points to 'Current density'.

Stress

[given params]

Current density

Atomic flux due to Back-stress

Atomic flux due to Electron wind

- Diffusion-type Partial Differential Equation
- Must hold in every segment of the interconnect

EM/Stress Migration Background

Boundary Conditions

Blocking terminal



$$\frac{\partial \sigma}{\partial x} + \beta j = 0$$

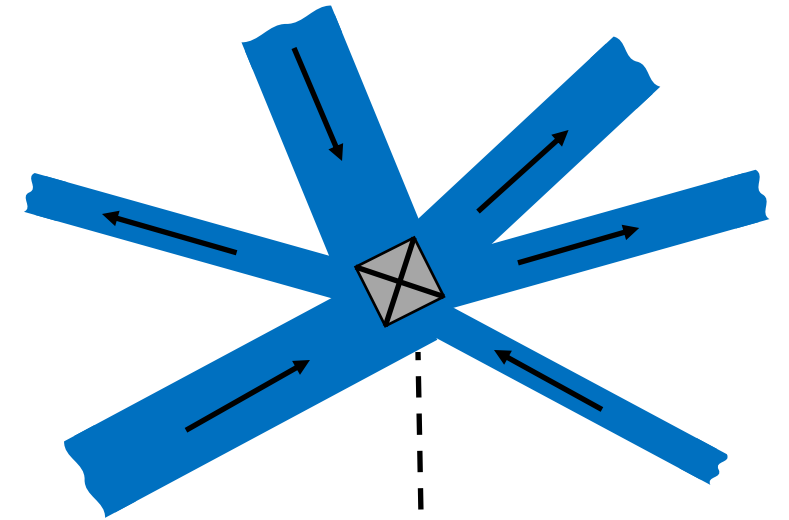
Intermediate point of
multisegment line



$$\sigma_1 = \sigma_2$$

$$\frac{\partial \sigma_1}{\partial x} + \beta j_1 = \frac{\partial \sigma_2}{\partial x} + \beta j_2$$

Junction of N segments with
varying cross-sections a_i



$$\sigma_1 = \dots = \sigma_N$$

$$\sum_{i=1}^{N_{out}} a_i \left(\frac{\partial \sigma_i}{\partial x} + \beta j_i \right) = \sum_{i=1}^{N_{in}} a_i \left(\frac{\partial \sigma_i}{\partial x} + \beta j_i \right)$$

Stress continuity

Conservation of
atomic flux

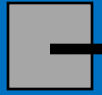
EM/Stress Migration Analyses

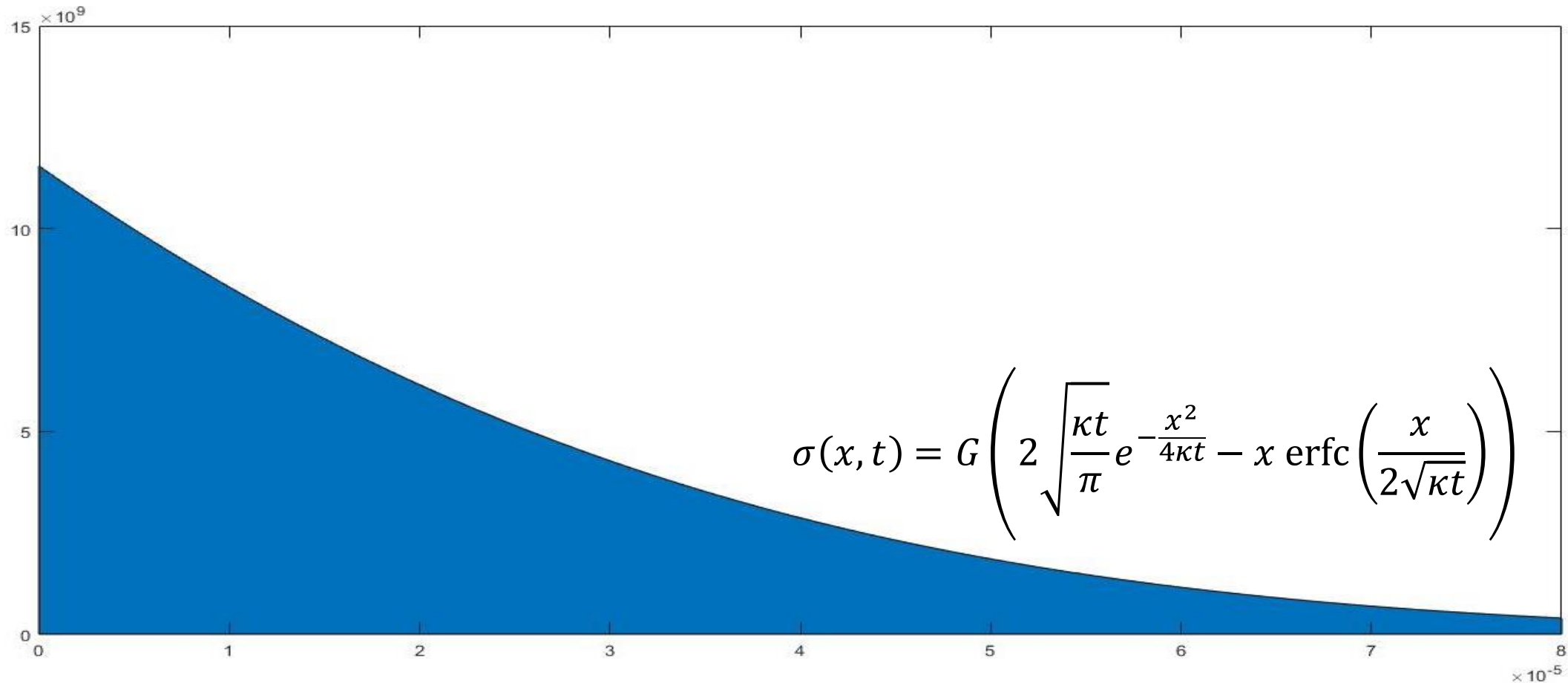
- **If tensile stress exceeds critical value $\Rightarrow$ void is nucleated**
(further electron wind leads to void growth, and eventual open circuit)
- **Steady-state analysis**
 - Solution for $t \rightarrow \infty$ (when transients have settled and driving forces reach equilibrium)
 - Can take very long time to reach (much longer than the expected chip lifetime)
 - Used to identify wires where critical stress is never attained ($\Rightarrow$ immortal wires)
 - Quite mature
- **Transient analysis**
 - Stress evolution over time
 - Identifies if potentially mortal wires from steady-state analysis are susceptible to EM ($\Rightarrow$ can attain critical stress) during chip lifetime

Single-segment line: solution by reflections



Single-segment line: solution by reflections


$$\frac{\partial \sigma}{\partial x} = -\beta j \equiv -G$$

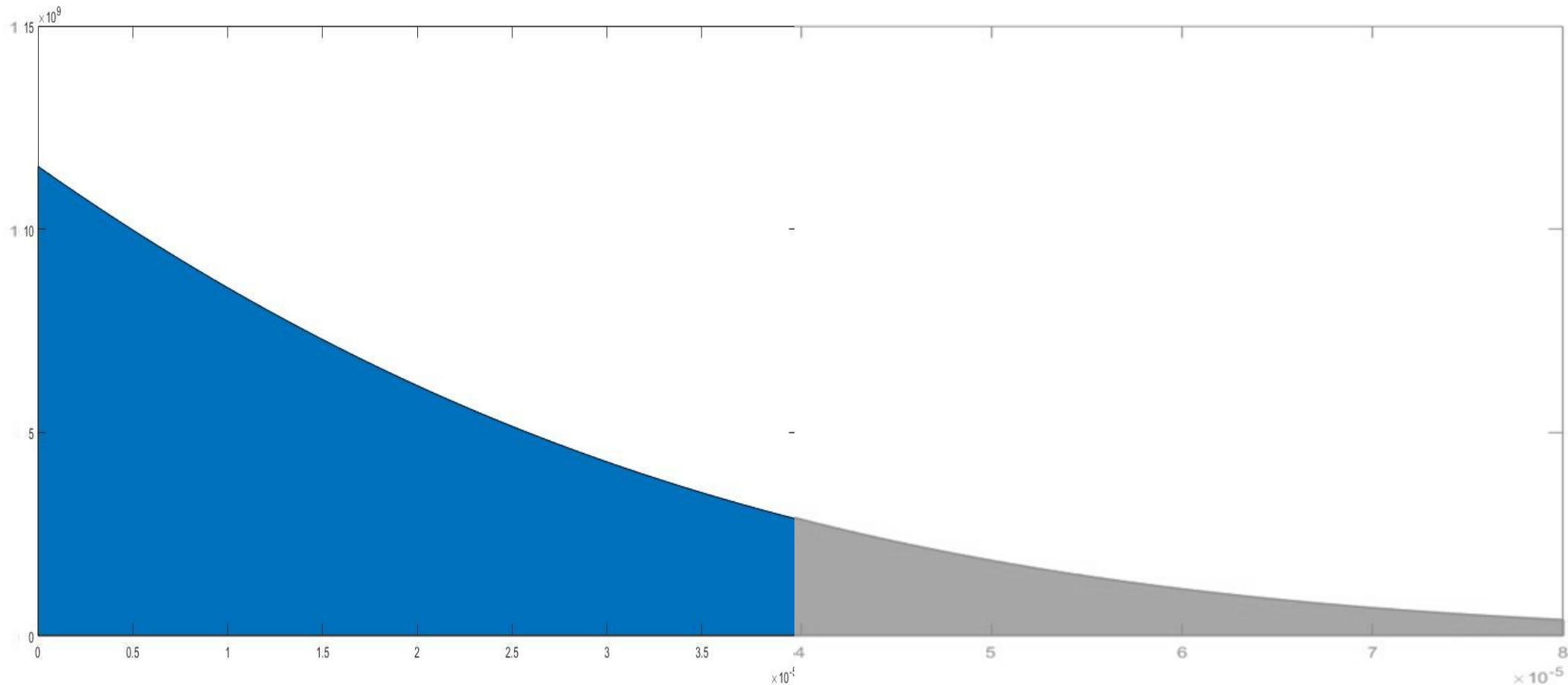


Single-segment line: solution by reflections



A small gray square represents a material element. A black arrow points to the right from its center, indicating the direction of the stress gradient.

$$\frac{\partial \sigma}{\partial x} = -G$$

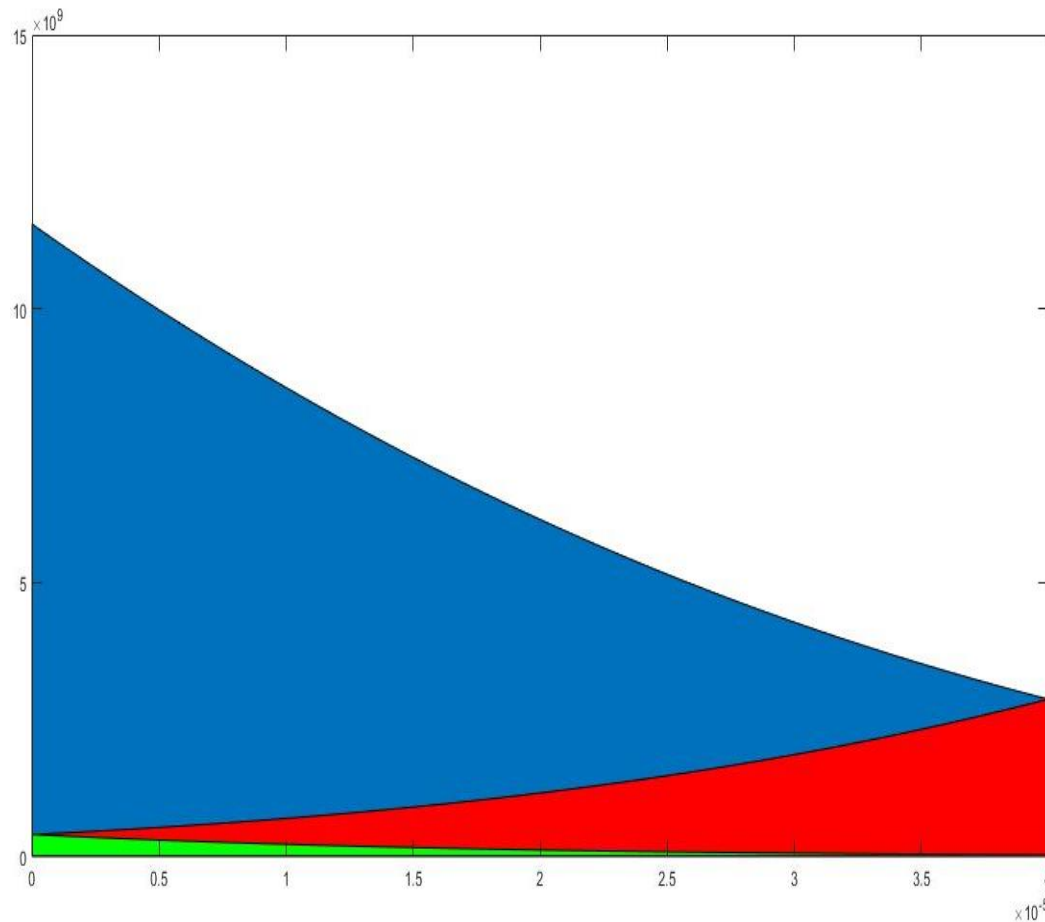


Single-segment line: solution by reflections



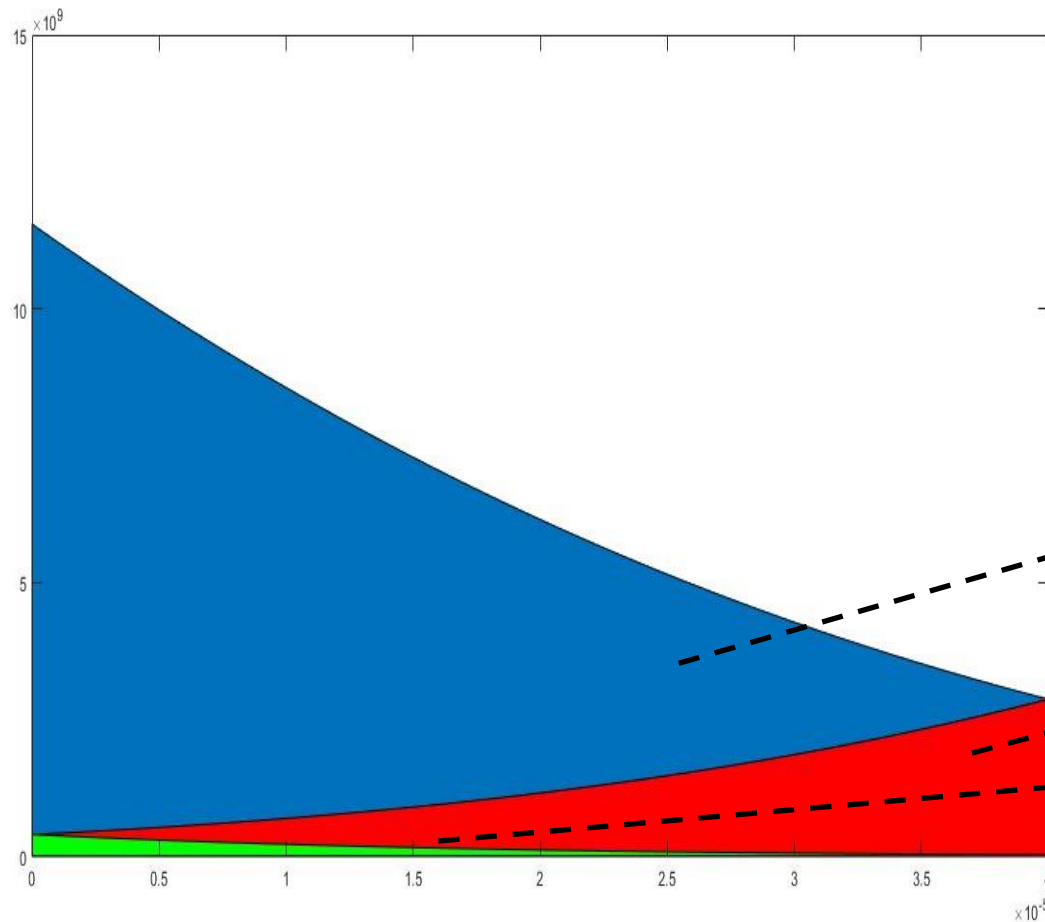
A small gray square represents a material element. A black arrow points to the right from its center, indicating the direction of the stress gradient.

$$\frac{\partial \sigma}{\partial x} = -G$$



Single-segment line: solution by reflections


$$\frac{\partial \sigma}{\partial x} = -G$$



Stress-Wave Analogy

Fundamental component

Reflection (1st order)

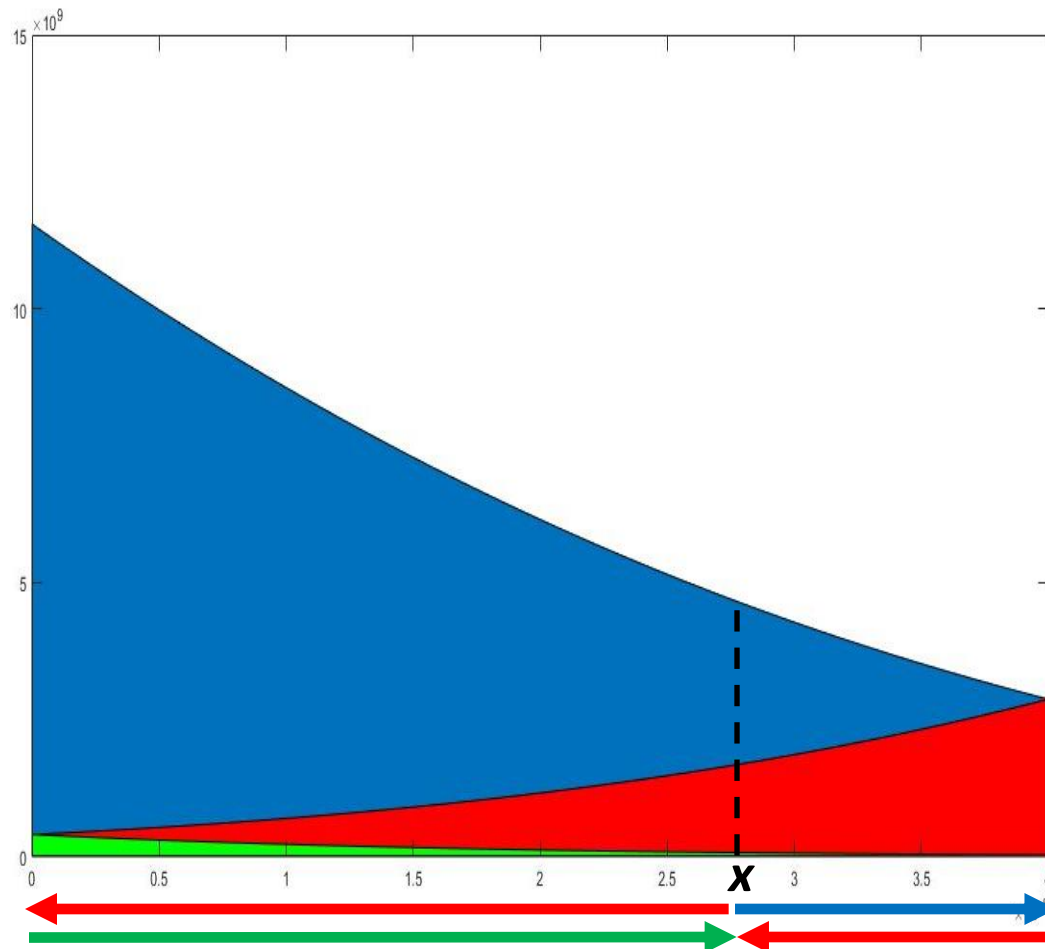
Reflection (2nd order)

Single-segment line: solution by reflections



A small gray square represents a material element. A black arrow points to the right from its center, indicating the direction of the stress gradient.

$$\frac{\partial \sigma}{\partial x} = -G$$



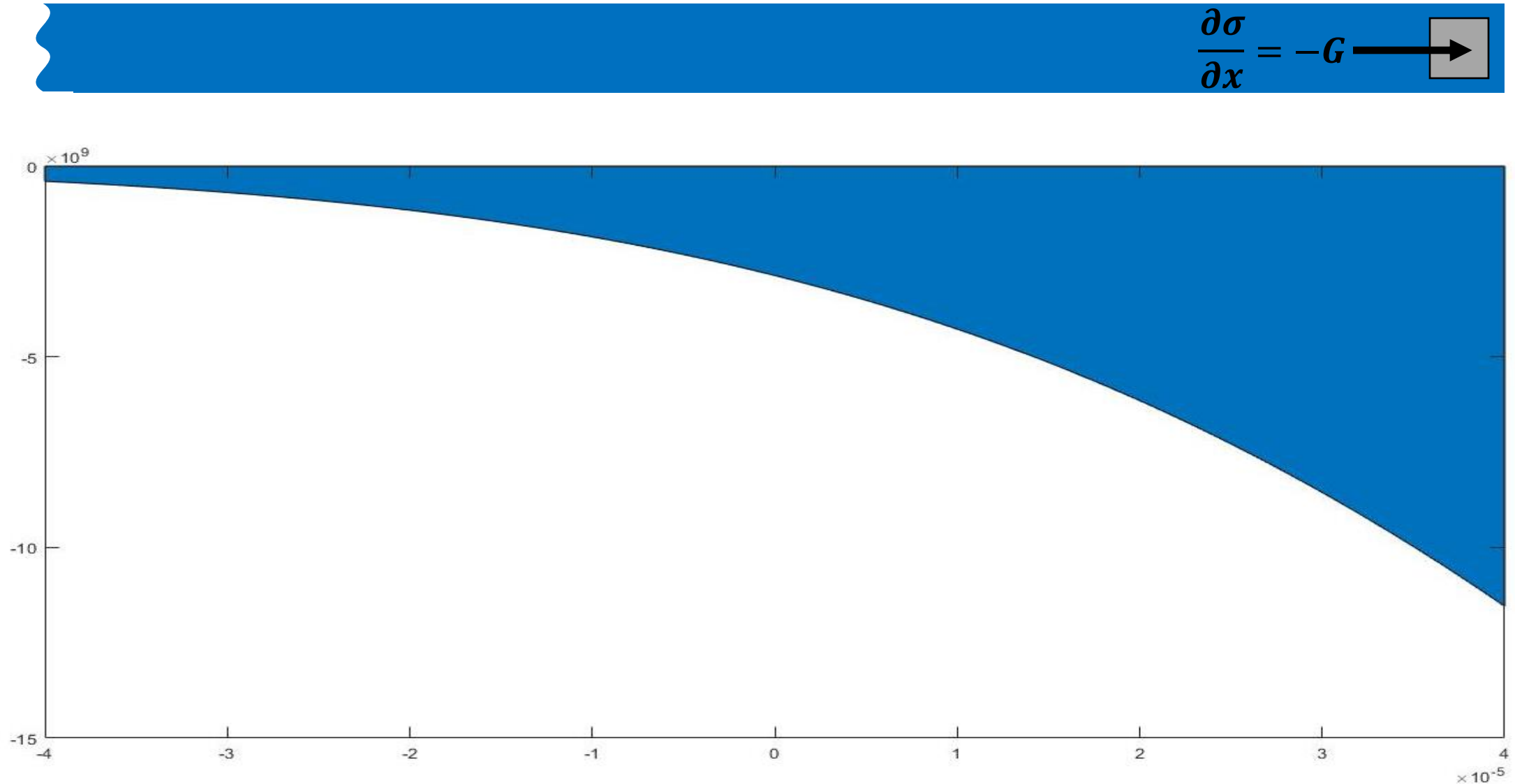
Single-segment line: solution by reflections



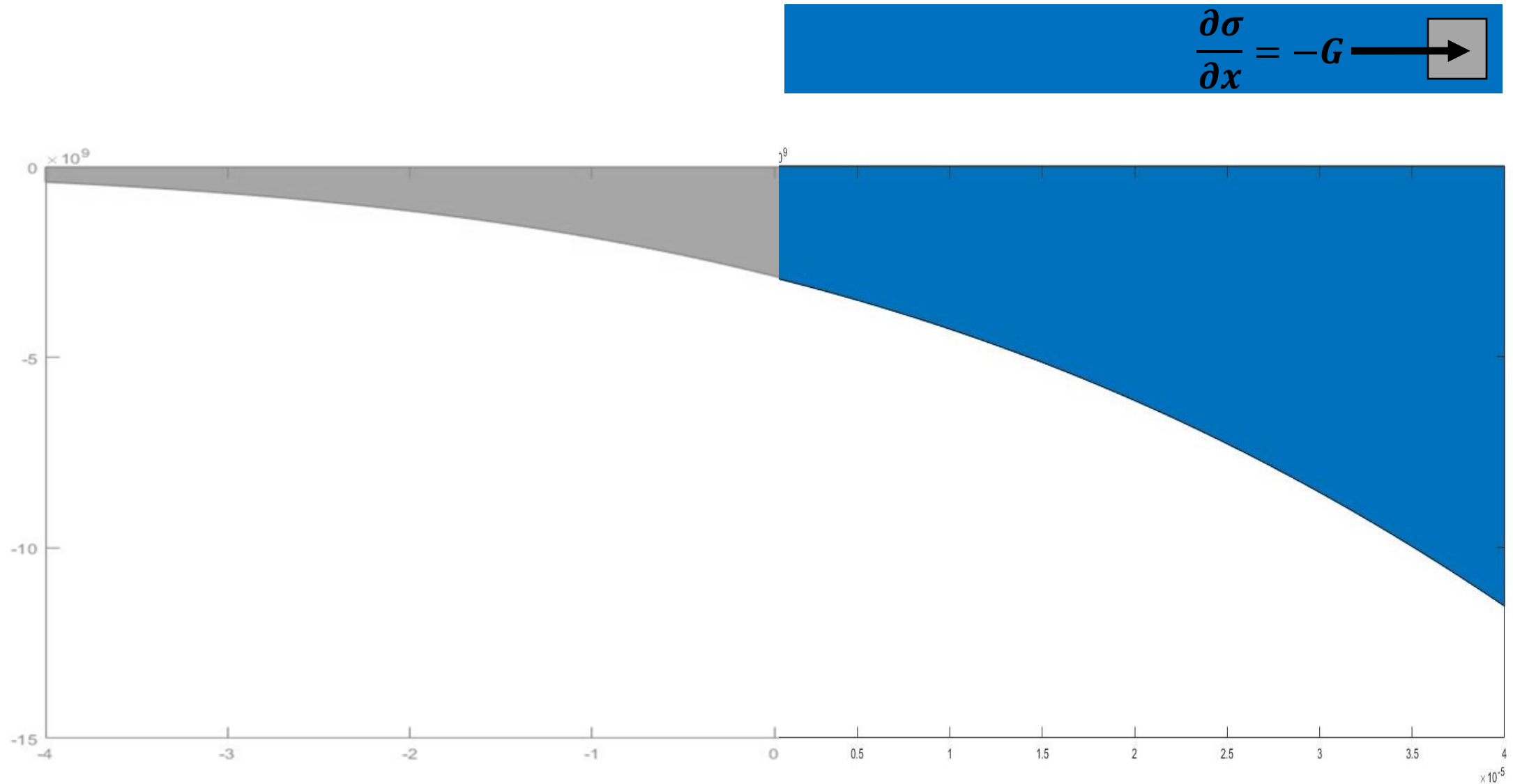
A diagram illustrating a single-segment line with boundary conditions. It consists of a blue horizontal bar. On the left end of the bar is a gray square. A black arrow points from this square to the left-hand side of the equation $\frac{\partial \sigma}{\partial x} = -G$. To the right of this equation is another instance of the same equation, $\frac{\partial \sigma}{\partial x} = -G$. A black arrow points from the right-hand side of this second equation to a gray square on the right end of the bar.

$$\frac{\partial \sigma}{\partial x} = -G \quad \frac{\partial \sigma}{\partial x} = -G$$

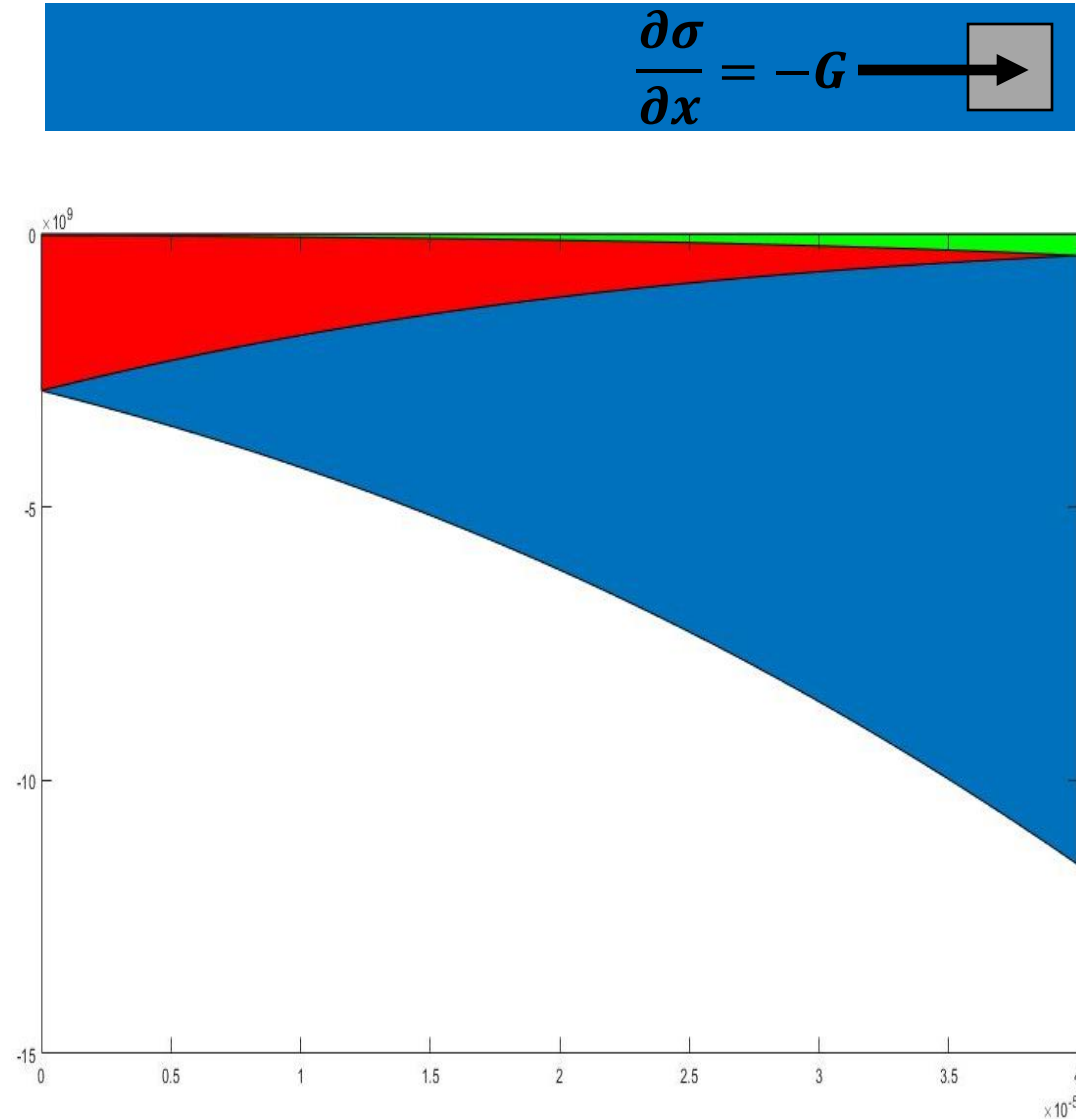
Single-segment line: solution by reflections



Single-segment line: solution by reflections



Single-segment line: solution by reflections

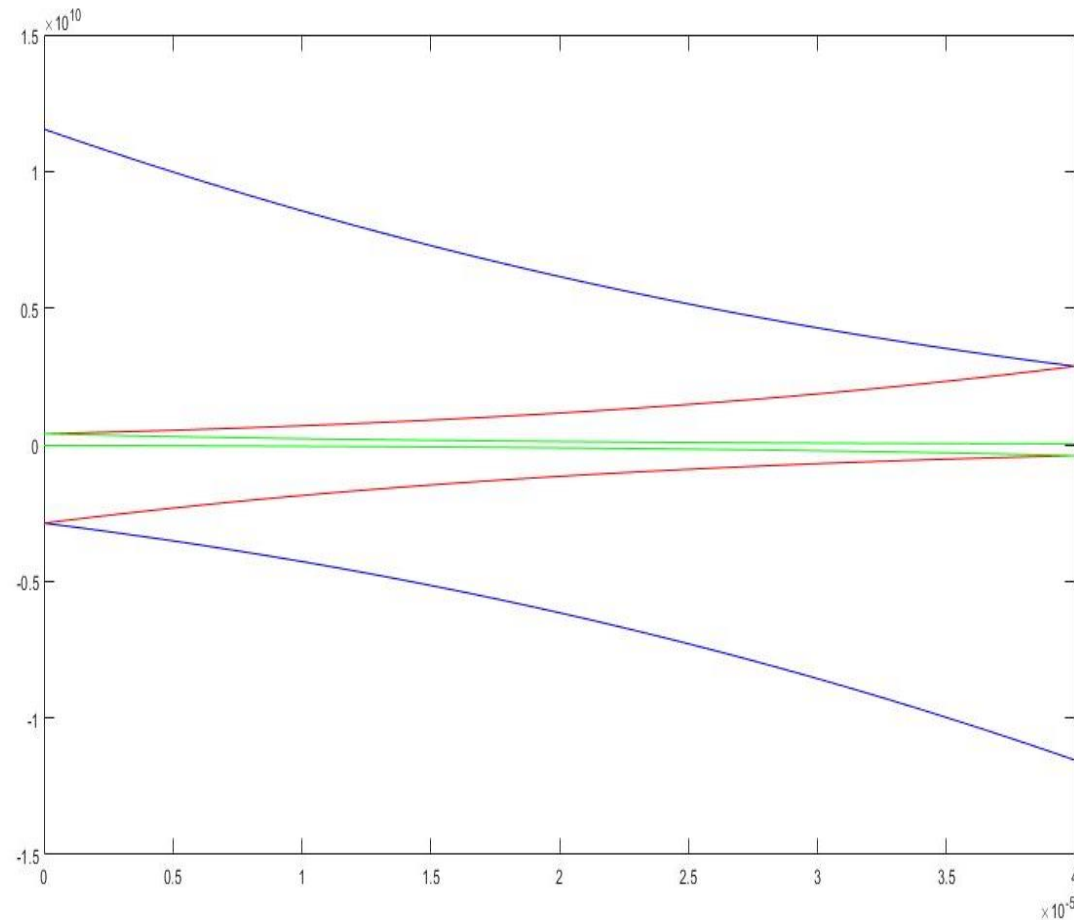


Single-segment line: solution by reflections



A blue rectangular box containing a diagram of a single-segment line. On the left, a gray square represents a boundary, with a black arrow pointing to the right towards the equation $\frac{\partial \sigma}{\partial x} = -G$. In the center, the equation $\frac{\partial \sigma}{\partial x} = -G$ is written. On the right, another black arrow points to the right from the equation towards a second gray square representing a boundary.

$$\frac{\partial \sigma}{\partial x} = -G \qquad \frac{\partial \sigma}{\partial x} = -G$$

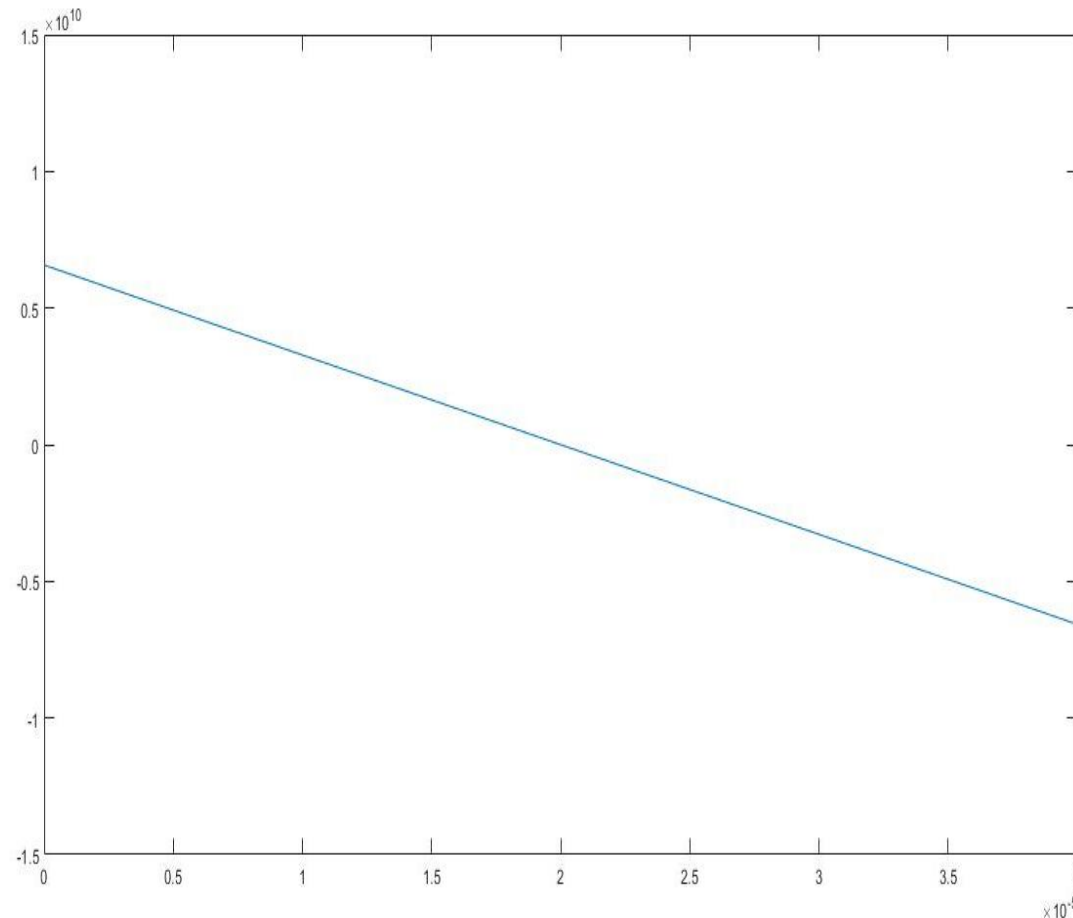


Single-segment line: solution by reflections



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$$\frac{\partial \sigma}{\partial x} = -G \quad \frac{\partial \sigma}{\partial x} = -G$$

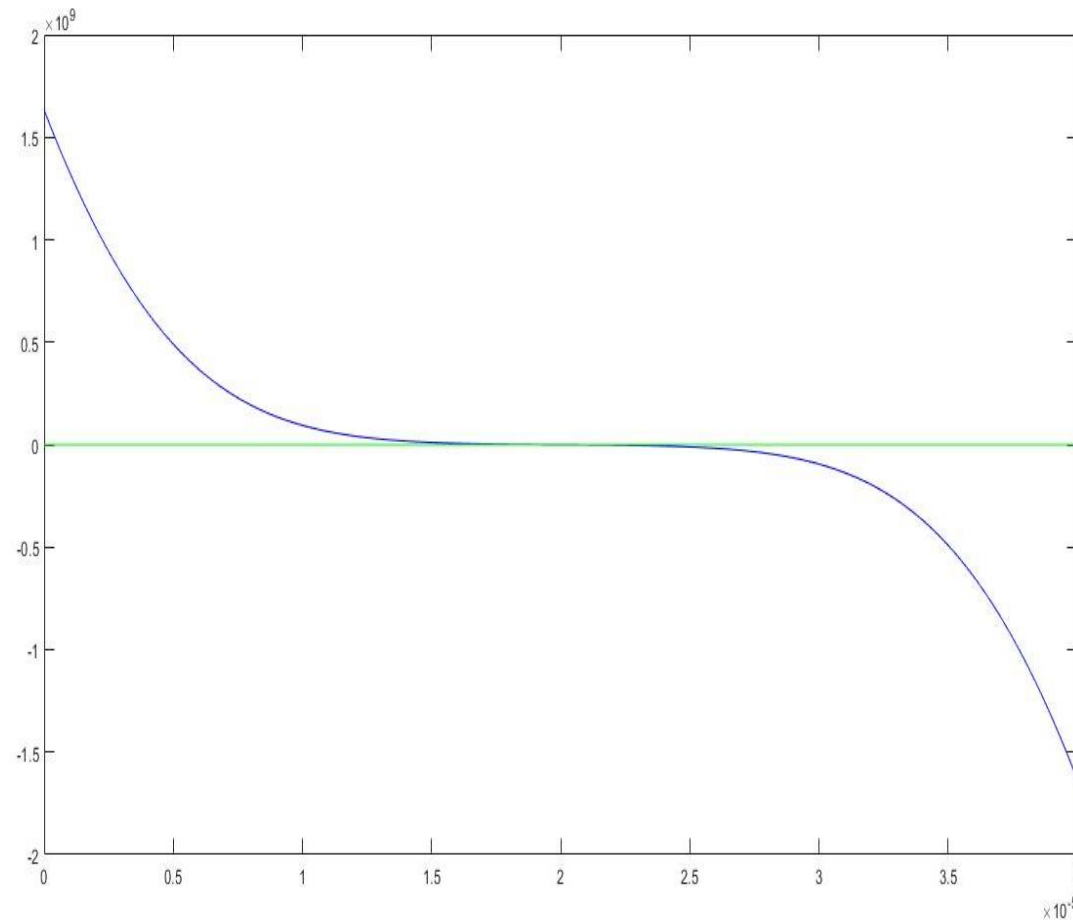


Single-segment line: solution by reflections



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$$\frac{\partial \sigma}{\partial x} = -G \qquad \frac{\partial \sigma}{\partial x} = -G$$

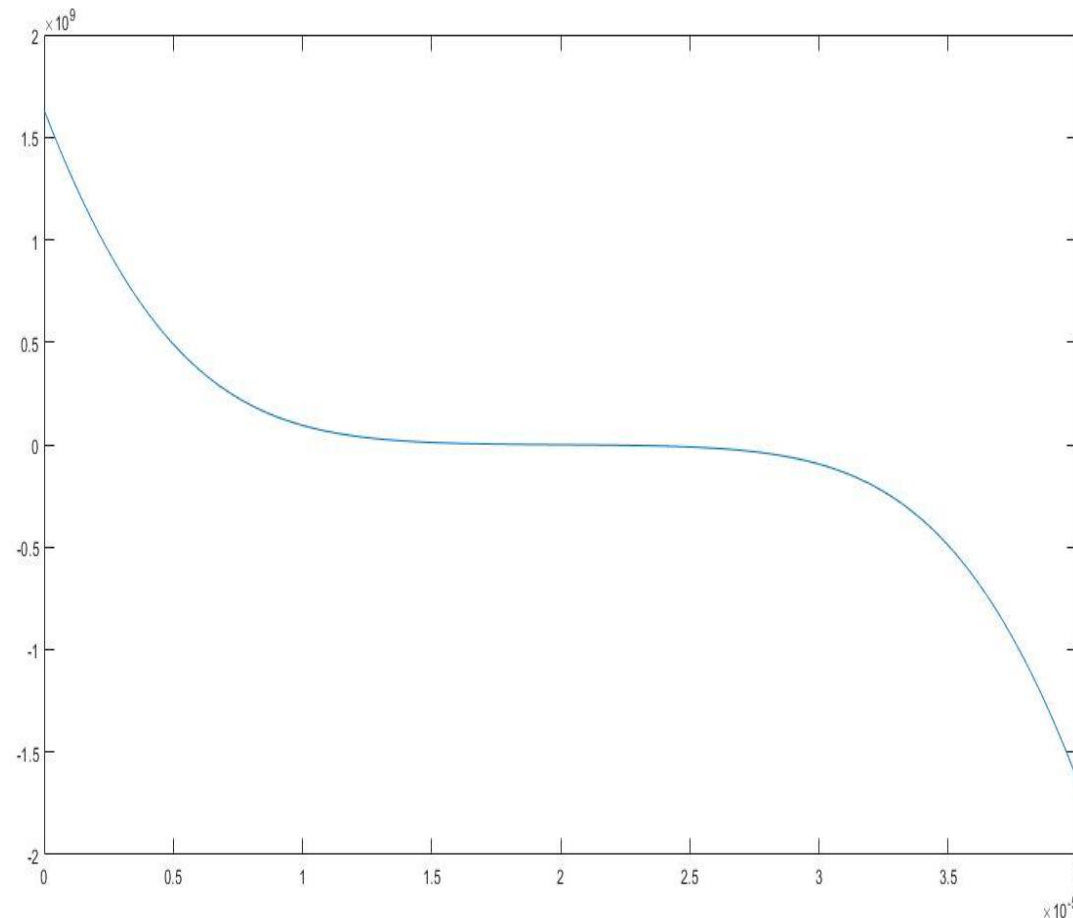


Single-segment line: solution by reflections

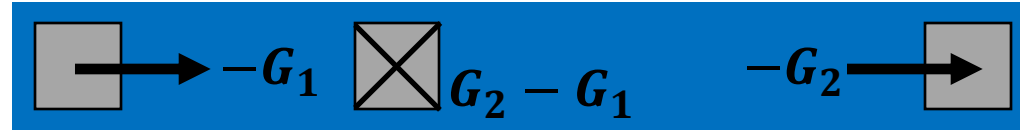


A blue rectangular box containing a diagram of a single-segment line. On the left, a gray square represents a boundary, with a black arrow pointing right towards the equation $\frac{\partial \sigma}{\partial x} = -G$. In the center, the equation $\frac{\partial \sigma}{\partial x} = -G$ is repeated. On the right, a black arrow points right from the equation towards another gray square representing a boundary.

$$\frac{\partial \sigma}{\partial x} = -G \quad \frac{\partial \sigma}{\partial x} = -G$$

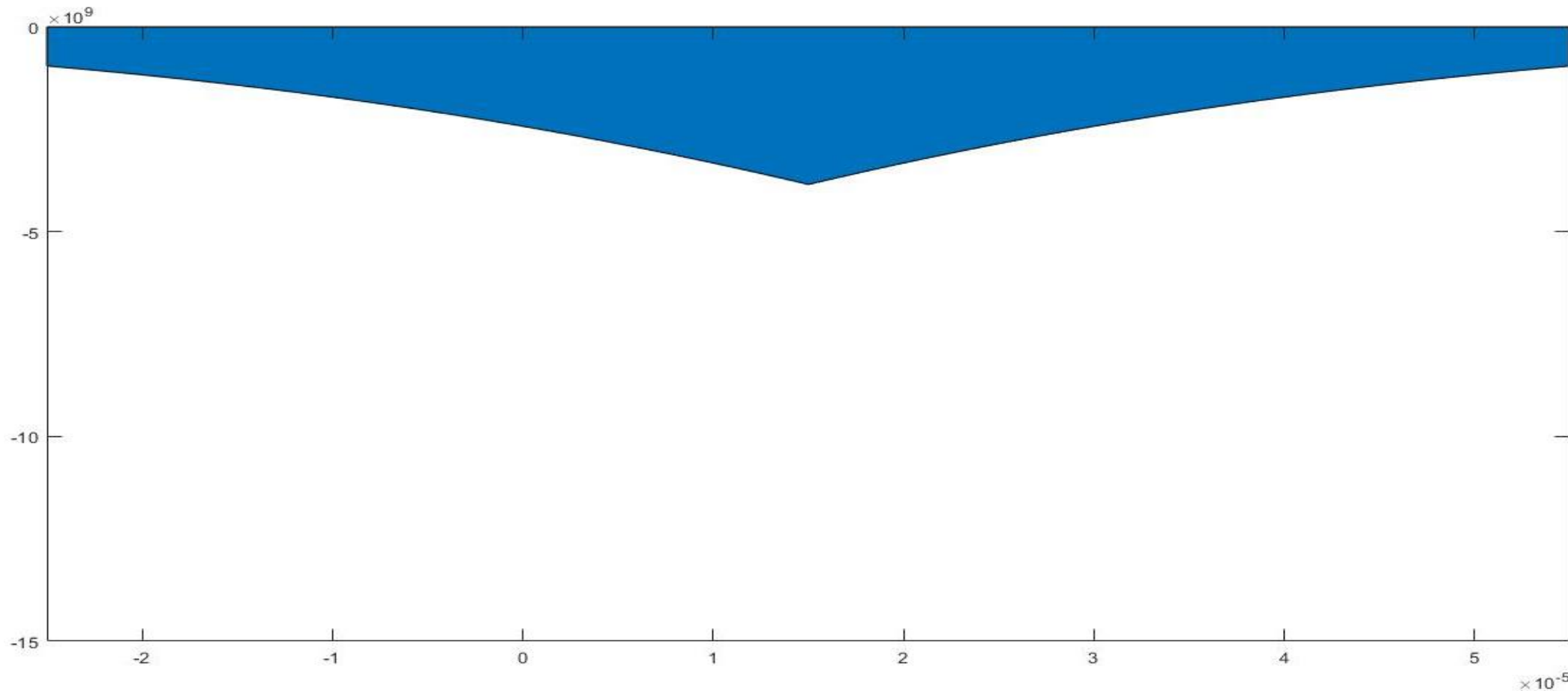


Two-segment line: solution by reflections

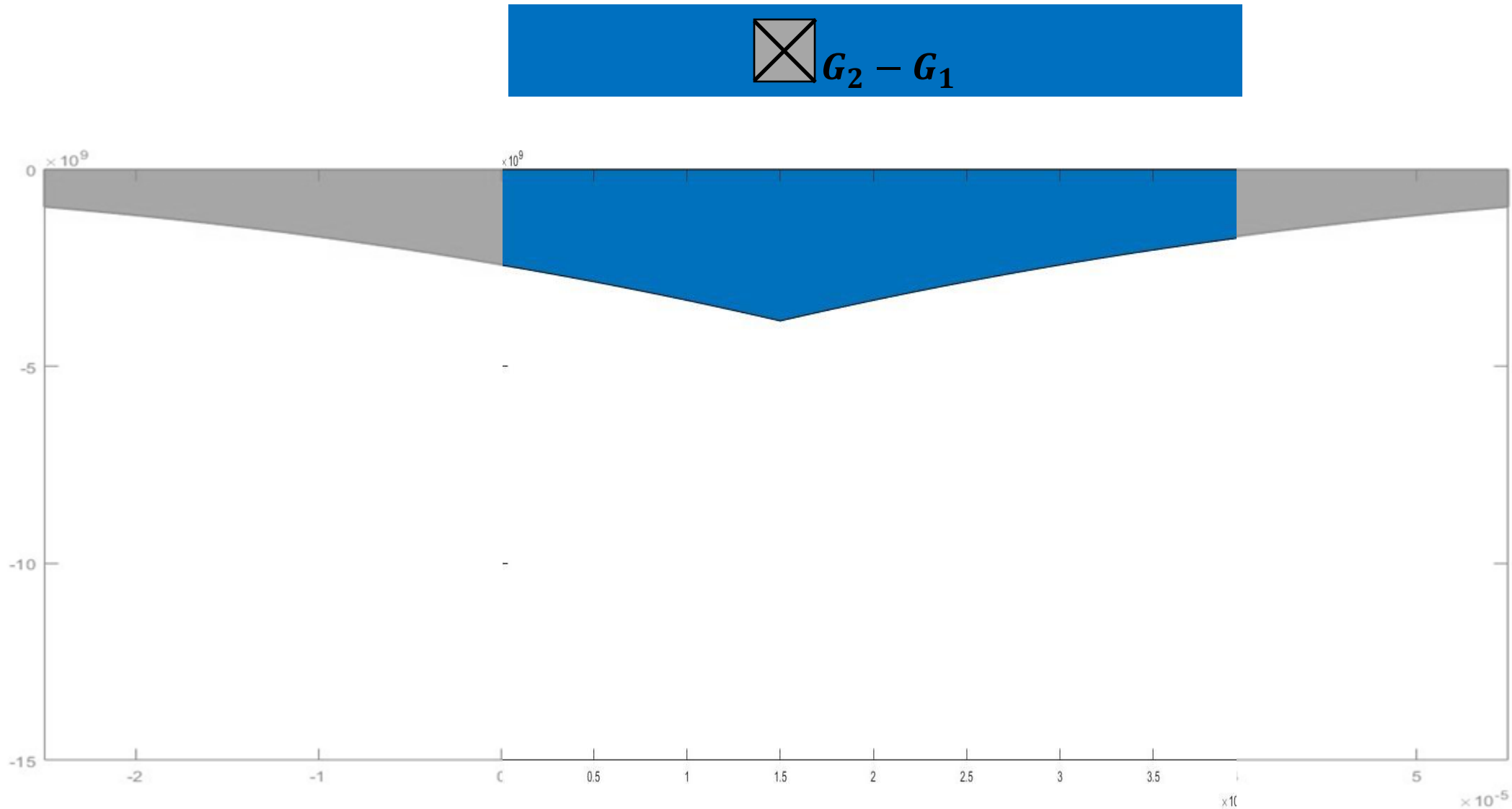


Two-segment line: solution by reflections

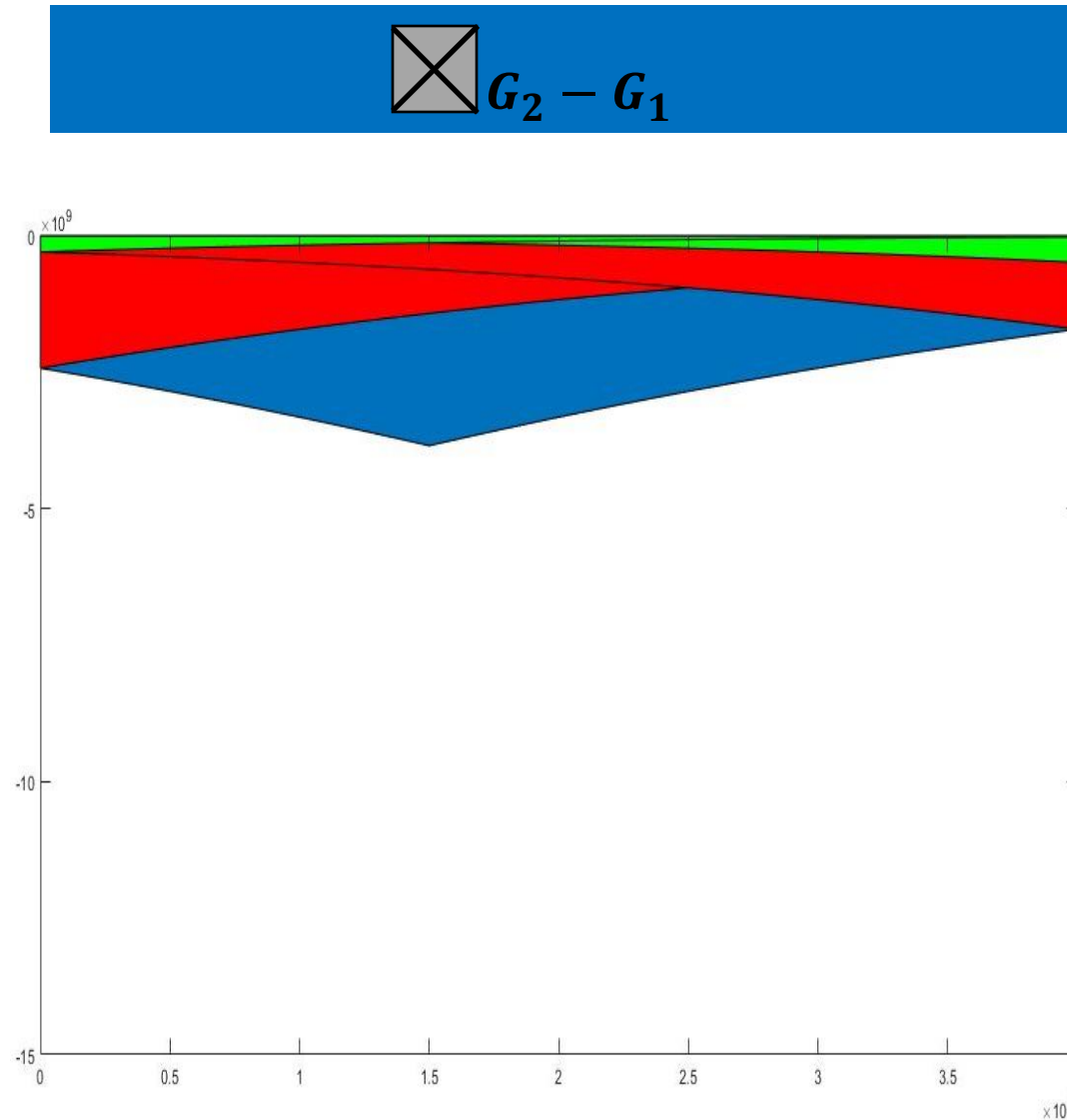

$$\square G_2 - G_1$$



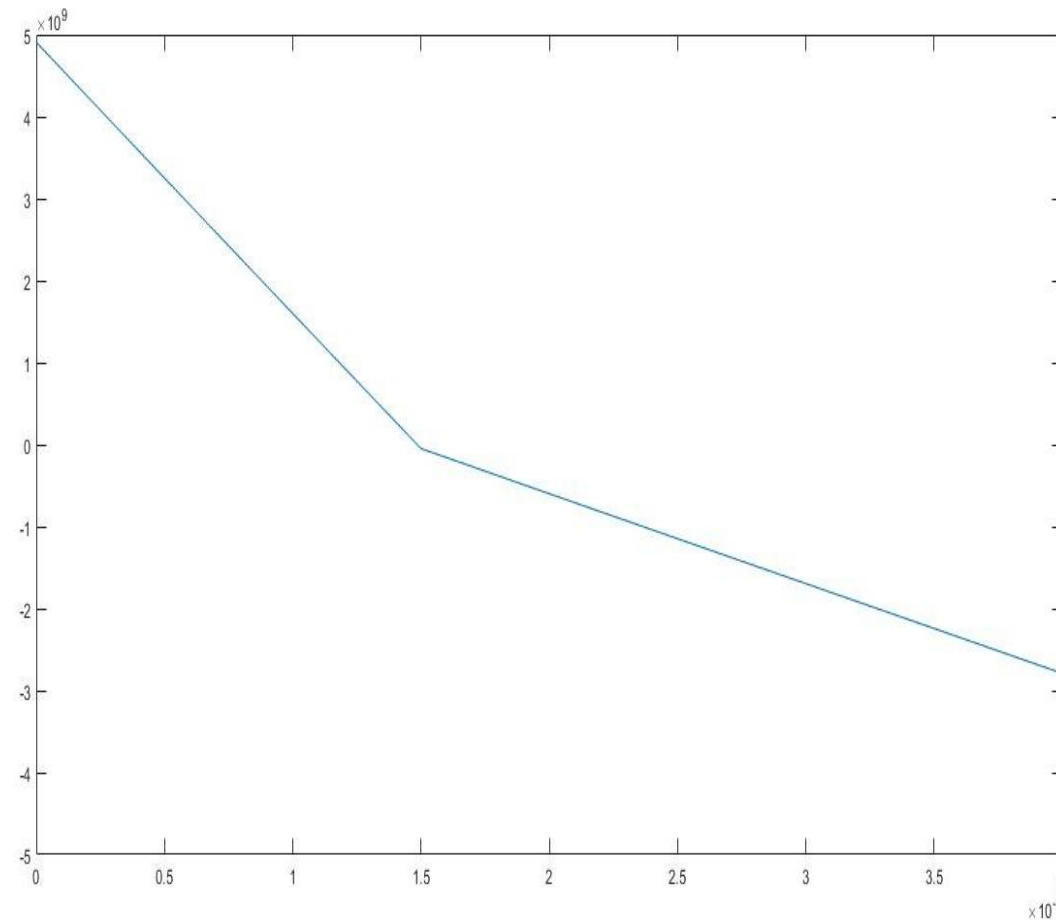
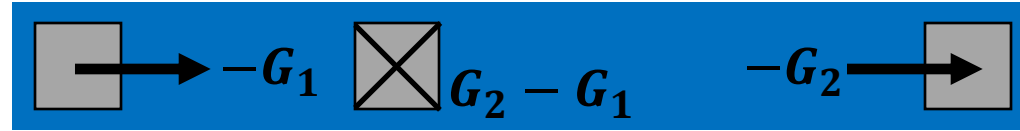
Two-segment line: solution by reflections



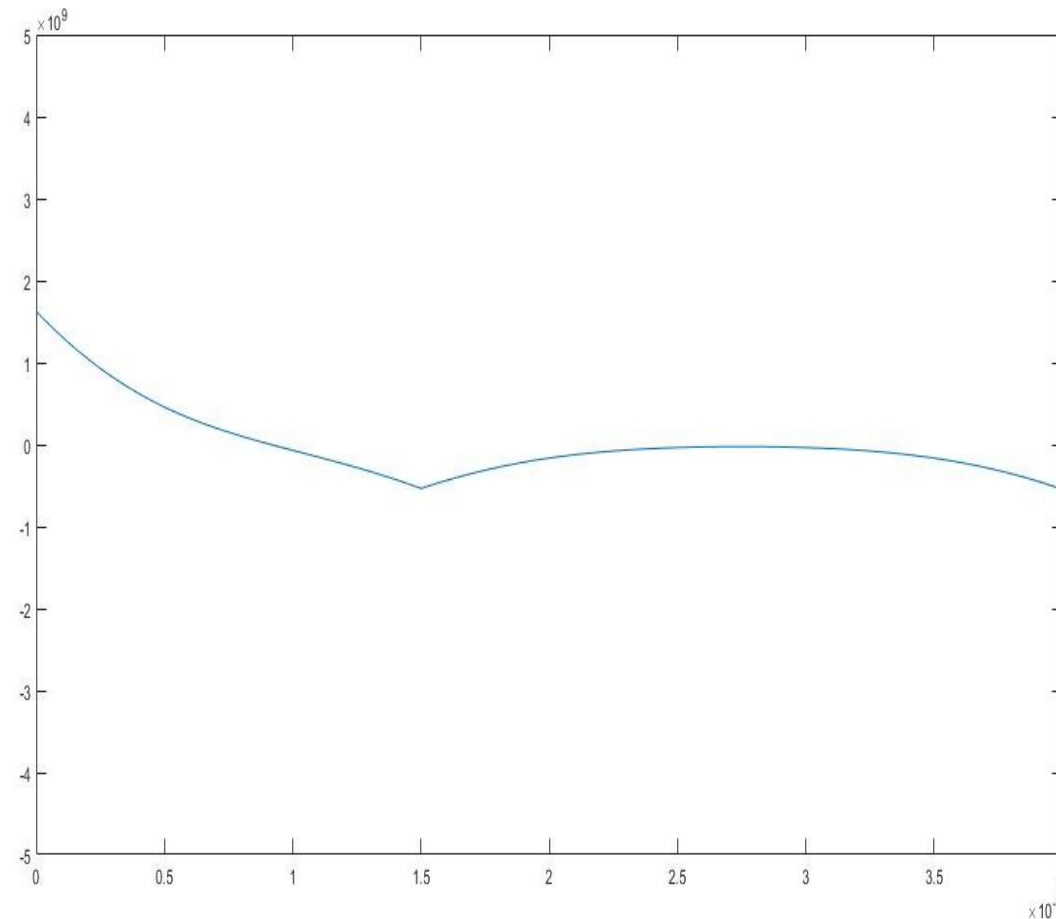
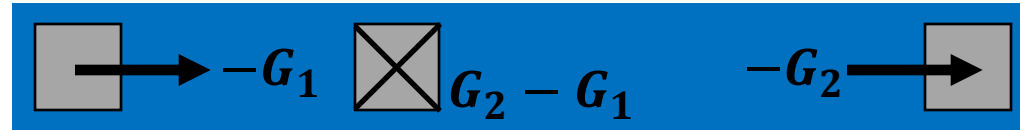
Two-segment line: solution by reflections



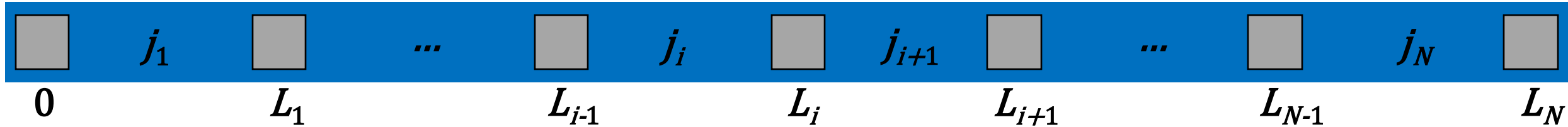
Two-segment line: solution by reflections



Two-segment line: solution by reflections



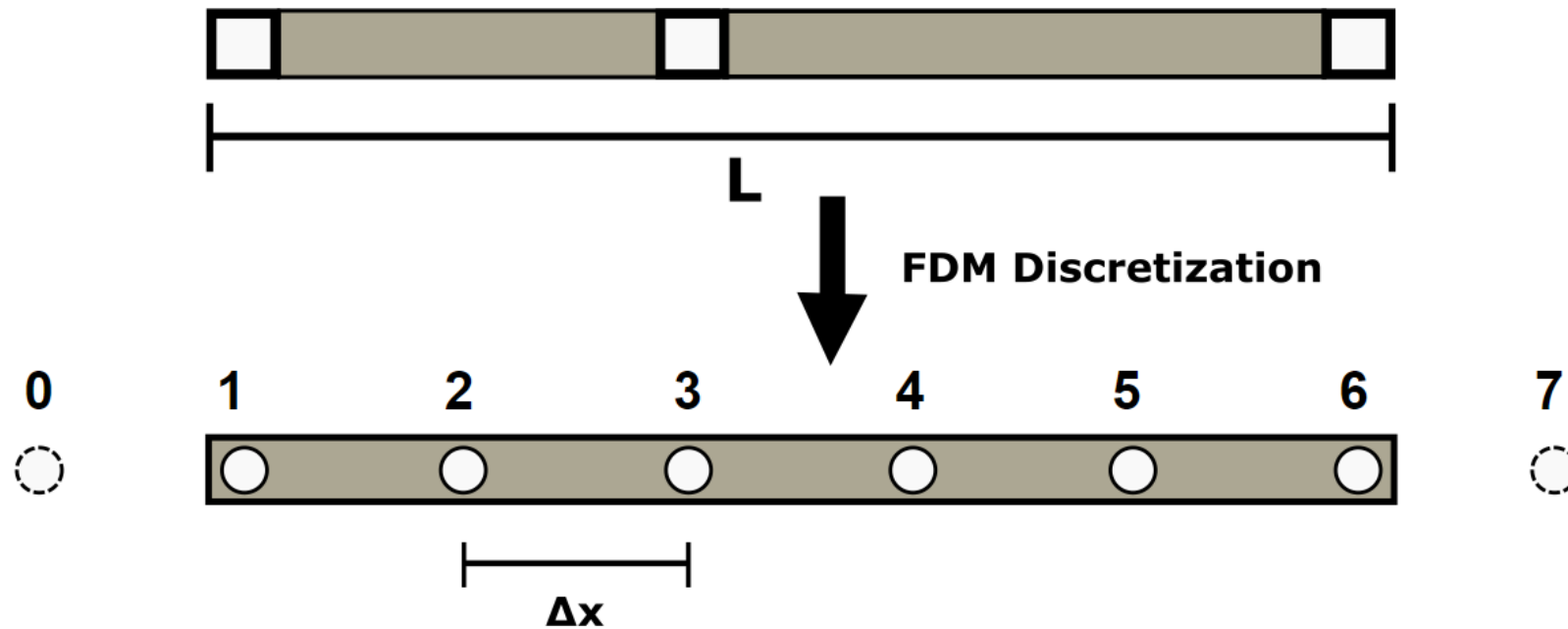
Multisegment line: solution by reflections



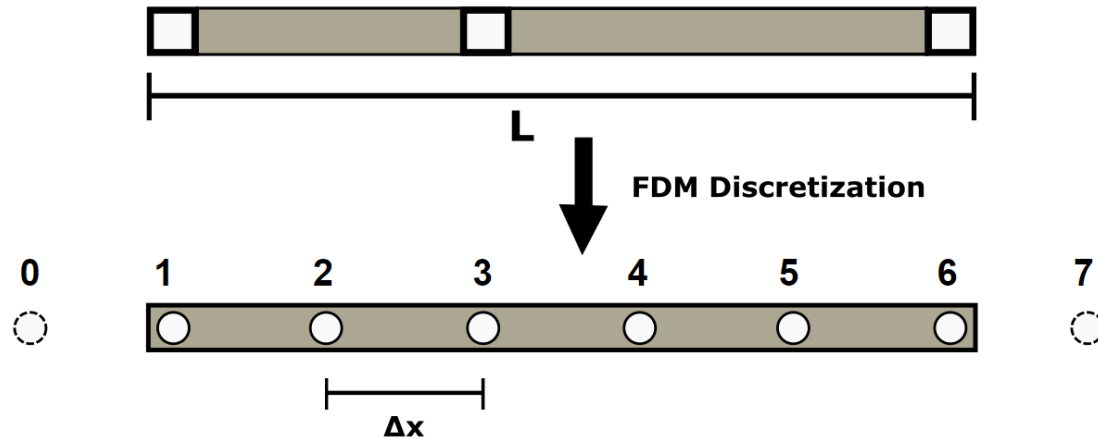
$$\begin{aligned} \sigma(x, t) \approx & G_1 [g(x, t) + g(2L_N - x, t) + g(2L_N + x, t)] \\ & - G_N [g(L_N - x, t) + g(L_N + x, t) + g(3L_N - x, t)] \\ & + \sum_{i=1}^{N-1} \frac{G_{i+1} - G_i}{2} [g(|L_i - x|, t) + g(L_i + x, t) \\ & + g(L_i + 2L_N - x, t) + g(-L_i + 2L_N - x, t) \\ & + g(-L_i + 2L_N + x, t)] \end{aligned}$$

$$g(x, t) = 2 \sqrt{\frac{\kappa t}{\pi}} e^{-\frac{x^2}{4\kappa t}} - x \operatorname{erfc}\left(\frac{x}{2\sqrt{\kappa t}}\right)$$

Multisegment line: semi-analytical solution



Multisegment line: semi-analytical solution



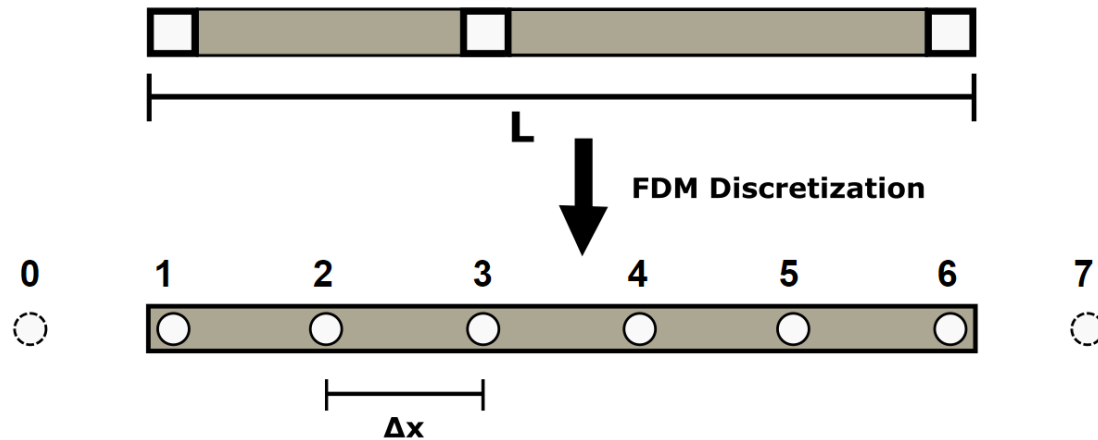
$$\dot{\boldsymbol{\sigma}}(t) = \mathbf{A}\boldsymbol{\sigma}(t) + \mathbf{B}\mathbf{j}$$

Stress at discrete spatial points

Current densities of line segments

$$\begin{bmatrix} \dot{\sigma}_1 \\ \dot{\sigma}_2 \\ \vdots \\ \dot{\sigma}_{n-1} \\ \dot{\sigma}_n \end{bmatrix} = \frac{\kappa}{\Delta x^2} \begin{bmatrix} -1 & 1 & & & \\ 1 & -2 & 1 & & \\ & & \ddots & & \\ & & & 1 & -2 & 1 \\ & & & 1 & -2 & 1 \\ & & & & 1 & -1 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_{n-1} \\ \sigma_n \end{bmatrix} + \mathbf{B} \begin{bmatrix} j_1 \\ \vdots \\ j_N \end{bmatrix}$$

Multisegment line: semi-analytical solution



Eigendecomposition

$$\mathbf{A} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^{-1}$$

$$\boldsymbol{\sigma}(t) = \mathbf{V} \left(\int_0^t e^{\mathbf{\Lambda}(t-\tau)} d\tau \right) \mathbf{V}^{-1} \mathbf{B}\mathbf{j}$$

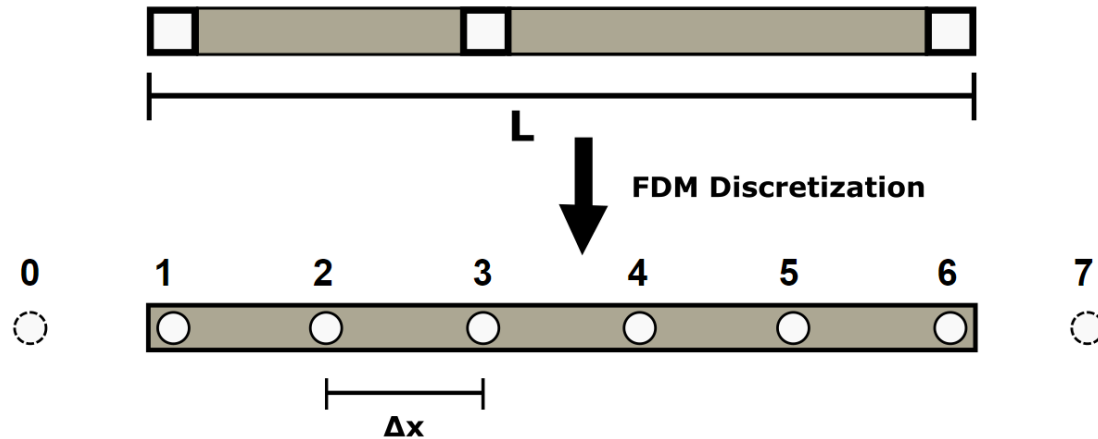
$$\dot{\boldsymbol{\sigma}}(t) = \mathbf{A}\boldsymbol{\sigma}(t) + \mathbf{B}\mathbf{j}$$

$$\boldsymbol{\sigma}(t) = \int_0^t e^{\mathbf{A}(t-\tau)} \mathbf{B}\mathbf{j} d\tau$$

For each eigenvalue λ_j :

$$\int_0^t e^{\lambda_j(t-\tau)} d\tau = \begin{cases} \frac{e^{\lambda_j t} - 1}{\lambda_j}, & \lambda_j \neq 0 \\ t, & \lambda_j = 0 \end{cases}$$

Multisegment line: semi-analytical solution



$$\mathbf{A} = \frac{\kappa}{\Delta x^2} \begin{bmatrix} -1 & 1 & & & & \\ 1 & -2 & 1 & & & \\ & & \ddots & & & \\ & & & 1 & -2 & 1 \\ & & & & 1 & -1 \end{bmatrix}$$

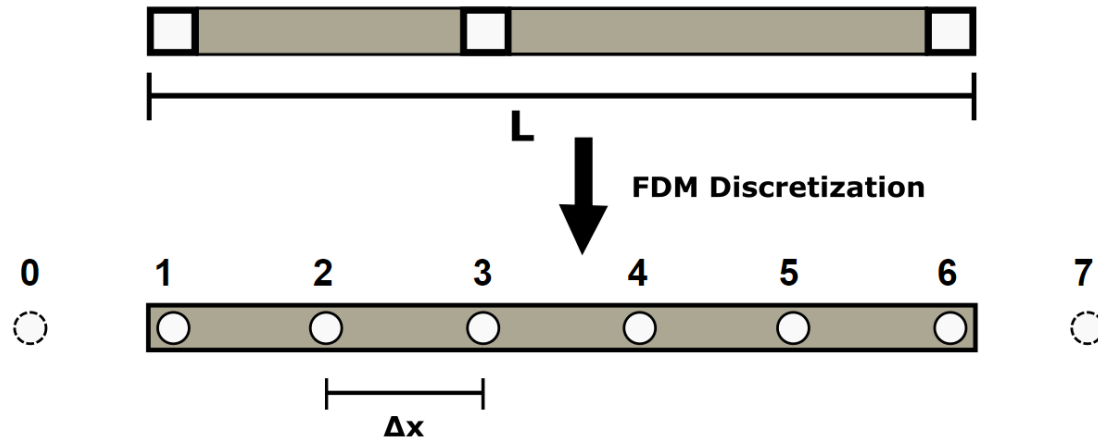
$$\lambda_j = \frac{\kappa}{\Delta x^2} \left(2 \cos \frac{(j-1)\pi}{n} - 2 \right), \quad j = 1, \dots, n$$

$$\mathbf{V}^{-1} \mathbf{r} \equiv \mathbf{V}^T \mathbf{r} \Leftrightarrow \text{DCT-II}(\mathbf{r})$$

$$\mathbf{V} \mathbf{r} \Leftrightarrow \text{IDCT-II}(\mathbf{r})$$

$O(n \log n)$

Multisegment line: semi-analytical solution



$$\mathbf{L}(t) = \begin{bmatrix} t & & \\ & \frac{e^{\lambda_2 t} - 1}{\lambda_2} & \\ & & \ddots \\ & & & \frac{e^{\lambda_n t} - 1}{\lambda_n} \end{bmatrix}$$

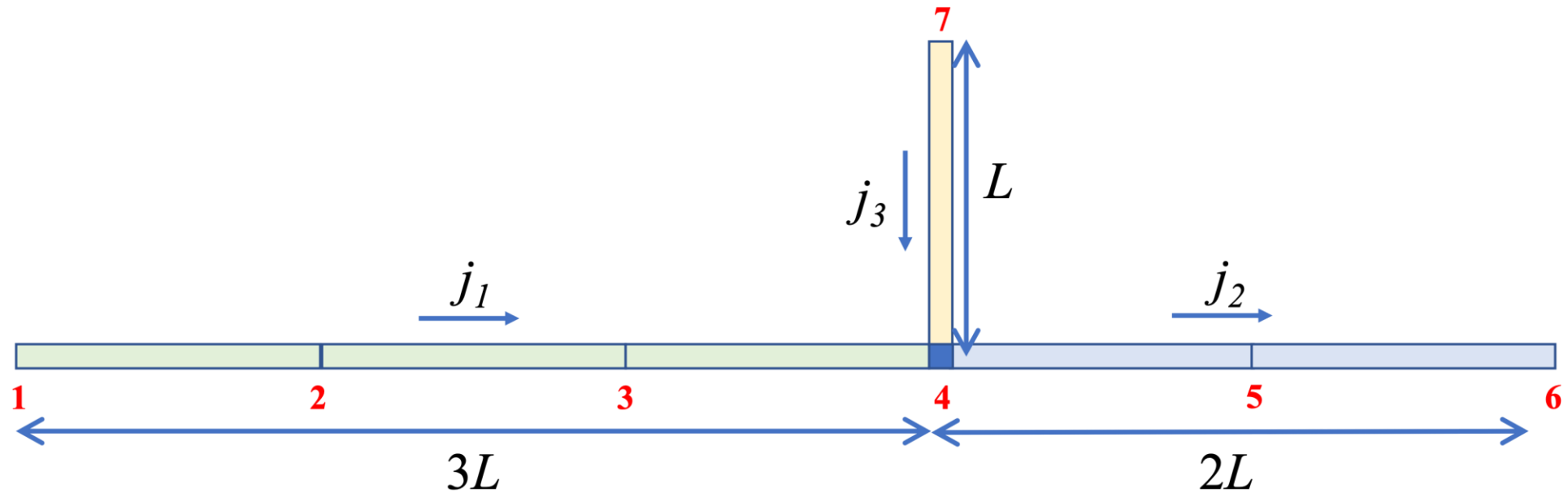
$$\boldsymbol{\sigma}(t) = \mathbf{V} \mathbf{L}(t) \mathbf{V}^T \mathbf{B} \mathbf{j}$$

DCT-II

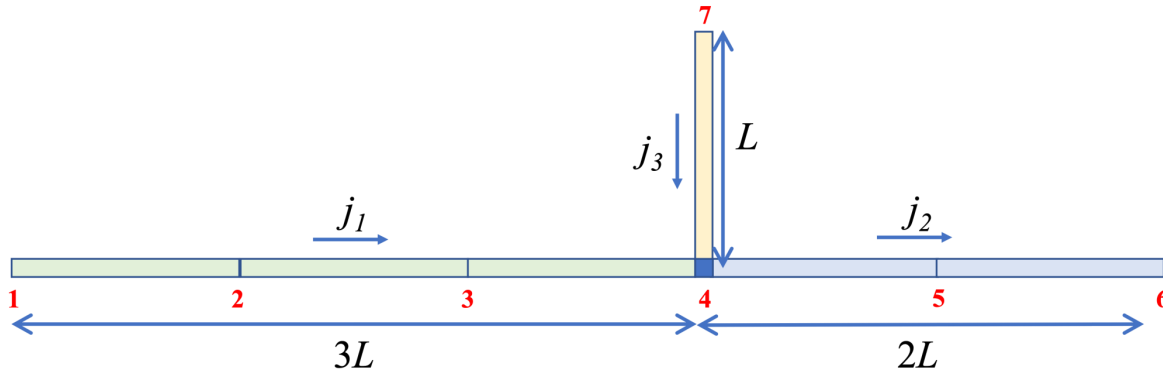
IDCT-II

$$\lambda_j = \frac{\kappa}{\Delta x^2} \left(2 \cos \frac{(j-1)\pi}{n} - 2 \right)$$

Interconnect tree: semi-analytical solution



Interconnect tree: semi-analytical solution



Conservation of atomic flux over the *entire* tree relates stress at discretized points

Solve for any σ_j and substitute

$$\dot{\sigma}(t) = A\sigma(t) + B\mathbf{j}$$

$$\sigma(t) = \int_0^t e^{A(t-\tau)} B\mathbf{j} d\tau$$

$(n-1) \times (n-1)$
nonsingular A ?

$$\sigma(t) = (e^{At} - I) A^{-1} B\mathbf{j}$$

sparse
linear solve

matrix-exponential
vector product

Conclusion and Future Challenges

Recent EM/Stress solutions for multisegment structures

- Reflections and stress-wave analogy for multisegment lines
→ analytical calculation of stress via closed-form expression
- Semi-analytical solutions for multisegment lines and interconnect trees
→ discretize only space, able to calculate stress at any future time directly

Future challenges and perspectives

- Extend reflections framework and stress-wave analogy to interconnect trees
- Adapt analytical and semi-analytical solutions to growth phase
- Handling of time-varying (or frequency-specific) input current densities
- Design connections (lifetime-driven P&R, segment widths, P/G stripe spacing)
- Adoption of physics-based models and emerging solutions in industrial flows

Thank you!

Questions / Comments

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