International Symposium on Physical Design

Recent Progress in the Analysis of Electromigration and Stress Migration in Large Multisegment Interconnects

Nestor Evmorfopoulos[1], Mohammad Shohel[2], Olympia Axelou[1], Pavlos Stoikos[1], Vidya A. Chhabria[2], Sachin S. Sapatnekar[2]

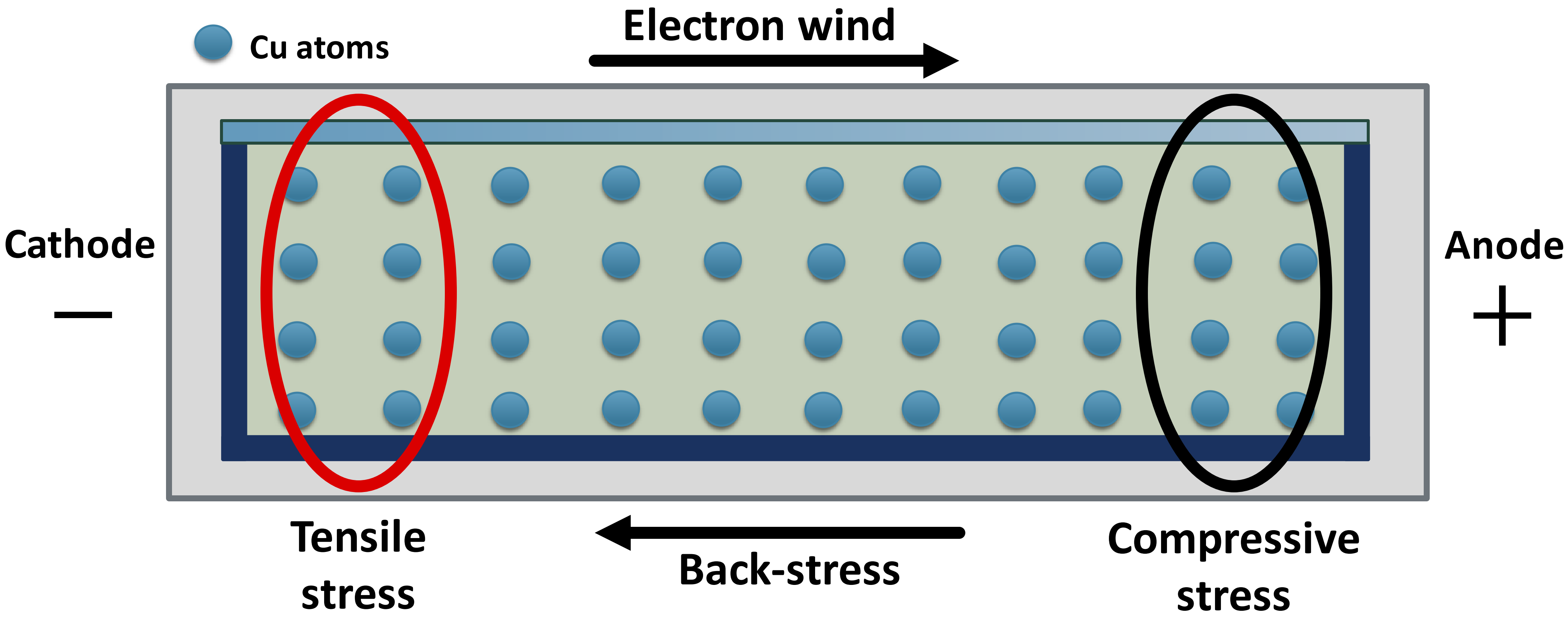
[1]Univ. of Thessaly, Volos, Greece
[2]Univ. of Minnesota, MN, USA

UNIVERSITY OF THESSALY

UNIVERSITY OF MINNESOTA

Outline

EM/Stress Migration Background



EM/Stress Migration Background

Physics-based modeling

(Korhonen et al., JAP 1993)

Stress

[given params]

Current density

$$\frac{\partial \sigma}{\partial t} = \frac{\partial}{\partial x}\left[\kappa\left(\frac{\partial \sigma}{\partial x} + \beta j\right)\right]$$

Atomic flux due to Back-stress

Atomic flux due to Electron wind

- **Diffusion-type Partial Differential Equation**
- **Must hold in every segment of the interconnect**

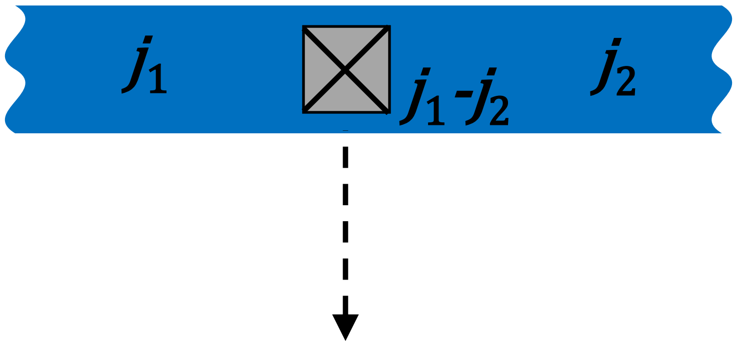
EM/Stress Migration Background

Boundary Conditions

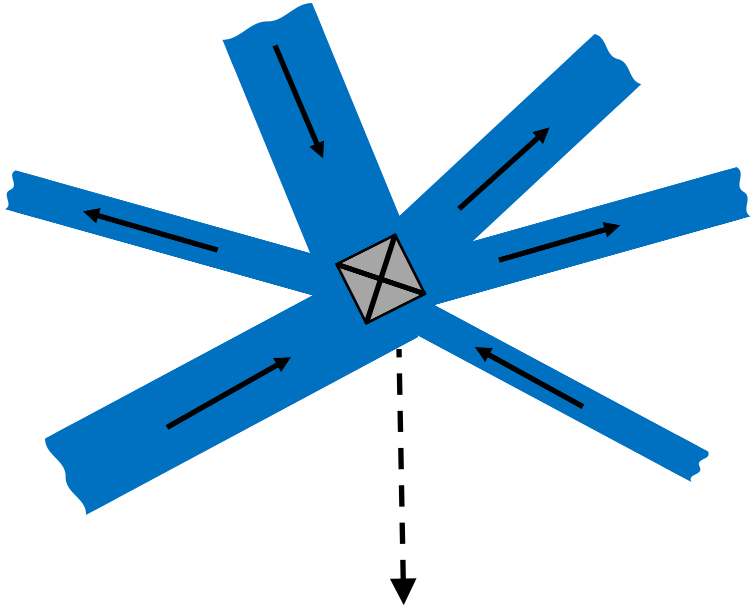
Blocking terminal

$$\frac{\partial \sigma}{\partial x} + \beta j = 0$$

Intermediate point of multisegment line



Junction of N segments with varying cross-sections a_i



Stress continuity

$$\sigma_1 = \sigma_2$$

$$\sigma_1 = \cdots = \sigma_N$$

Conservation of atomic flux

$$\frac{\partial \sigma_1}{\partial x} + \beta j_1 = \frac{\partial \sigma_2}{\partial x} + \beta j_2$$

$$\sum_{i=1}^{N\_out} a_i \left(\frac{\partial \sigma_i}{\partial x} + \beta j_i \right) = \sum_{i=1}^{N\_in} a_i \left(\frac{\partial \sigma_i}{\partial x} + \beta j_i \right)$$

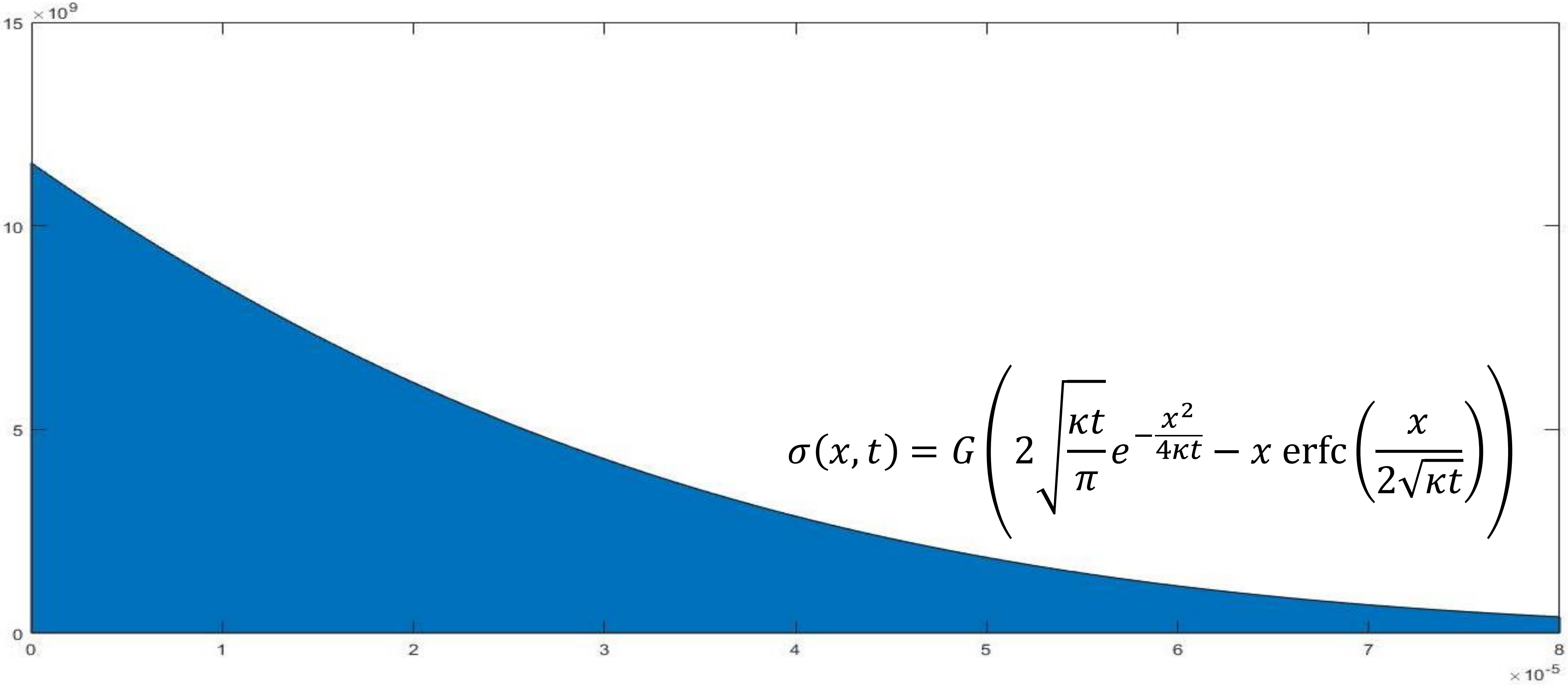
EM/Stress Migration Analyses

- If tensile stress exceeds critical value $\Rightarrow$ void is nucleated
  (further electron wind leads to void growth, and eventual open circuit)
- Steady-state analysis
  - Solution for $t \rightarrow \infty$ (when transients have settled and driving forces reach equilibrium)
  - Can take very long time to reach (much longer than the expected chip lifetime)
  - Used to identify wires where critical stress is never attained ($\Rightarrow$ immortal wires)
  - Quite mature
- Transient analysis
  - Stress evolution over time
  - Identifies if potentially mortal wires from steady-state analysis are susceptible to EM ($\Rightarrow$ can attain critical stress) during chip lifetime

Single-segment line: solution by reflections

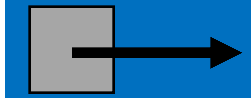
Single-segment line: solution by reflections

$$\frac{\partial \sigma}{\partial x} = -\beta j \equiv -G$$

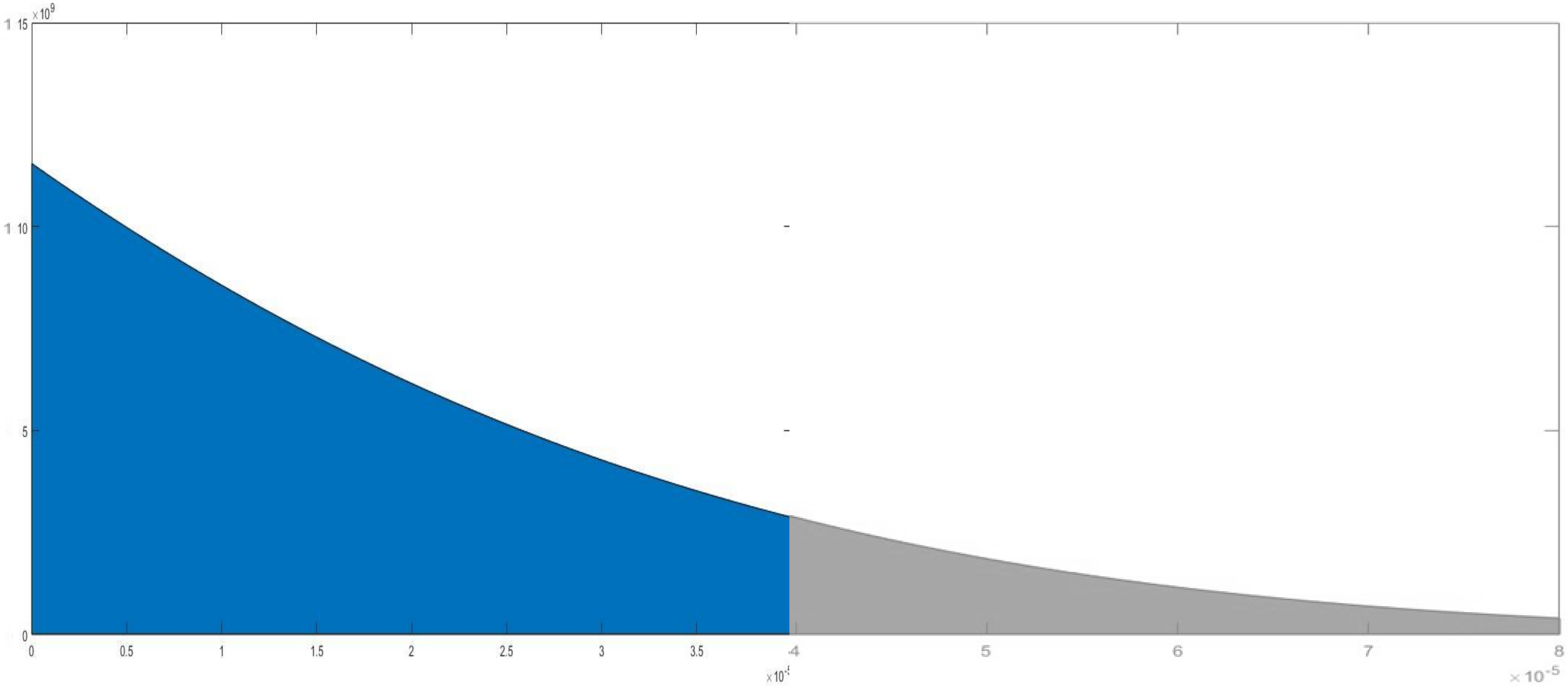


$$\sigma(x,t) = G\left(2\sqrt{\frac{\kappa t}{\pi}}e^{-\frac{x^2}{4\kappa t}} - x\,\mathrm{erfc}\left(\frac{x}{2\sqrt{\kappa t}}\right)\right)$$

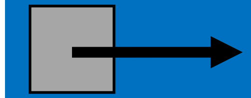
Single-segment line: solution by reflections



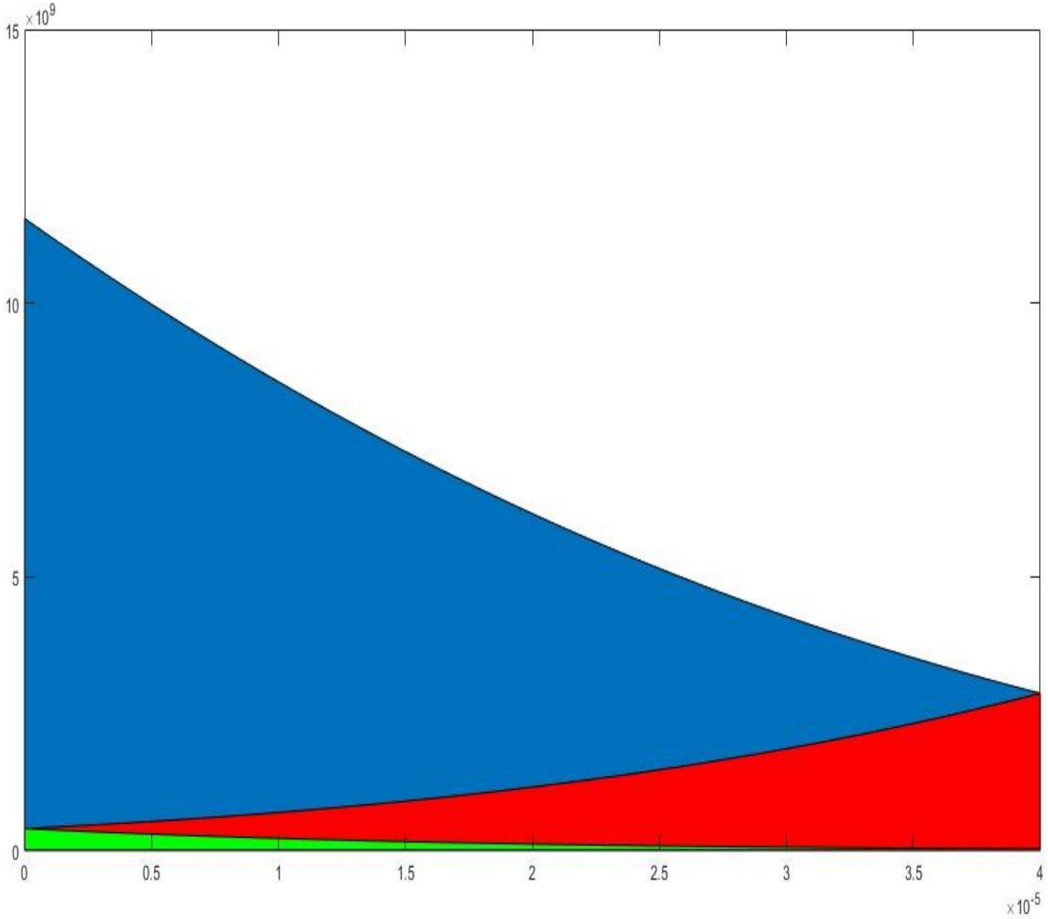
$$\frac{\partial \sigma}{\partial x} = -G$$



Single-segment line: solution by reflections

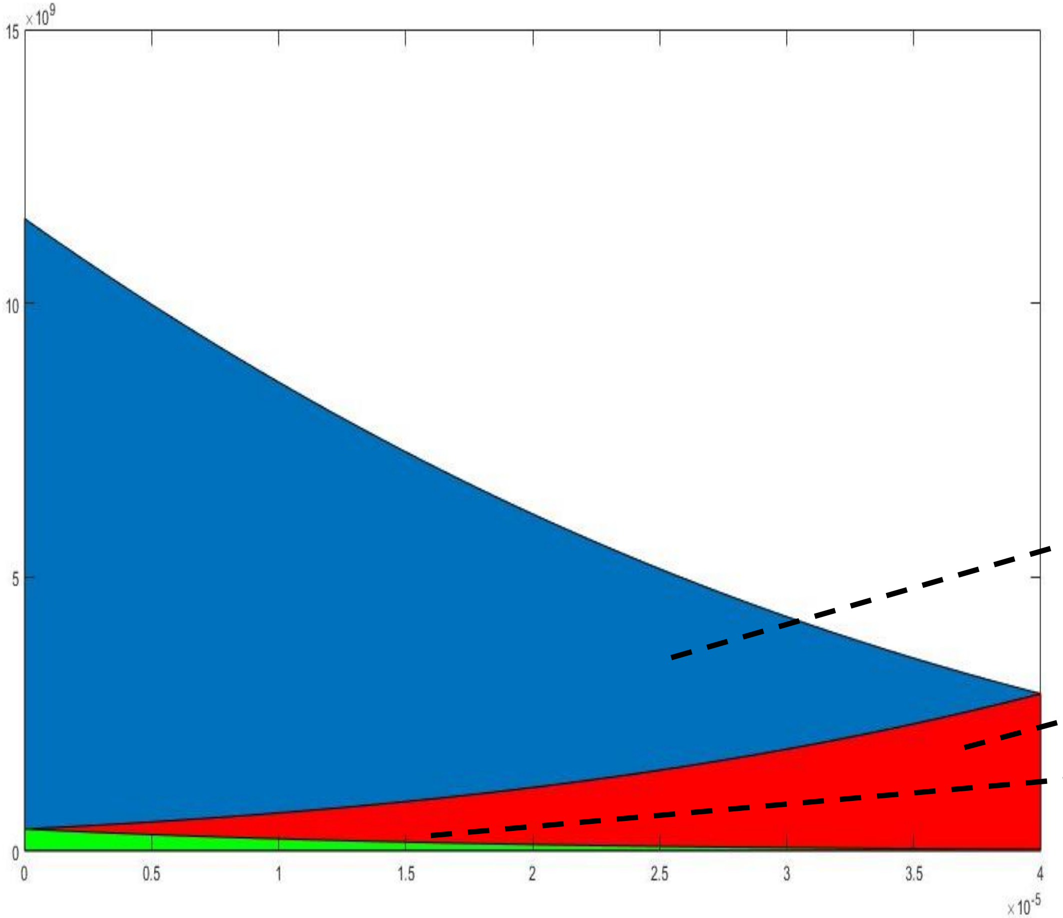


$$\frac{\partial \sigma}{\partial x} = -G$$



Single-segment line: solution by reflections

$$\frac{\partial \sigma}{\partial x} = -G$$



Stress-Wave Analogy

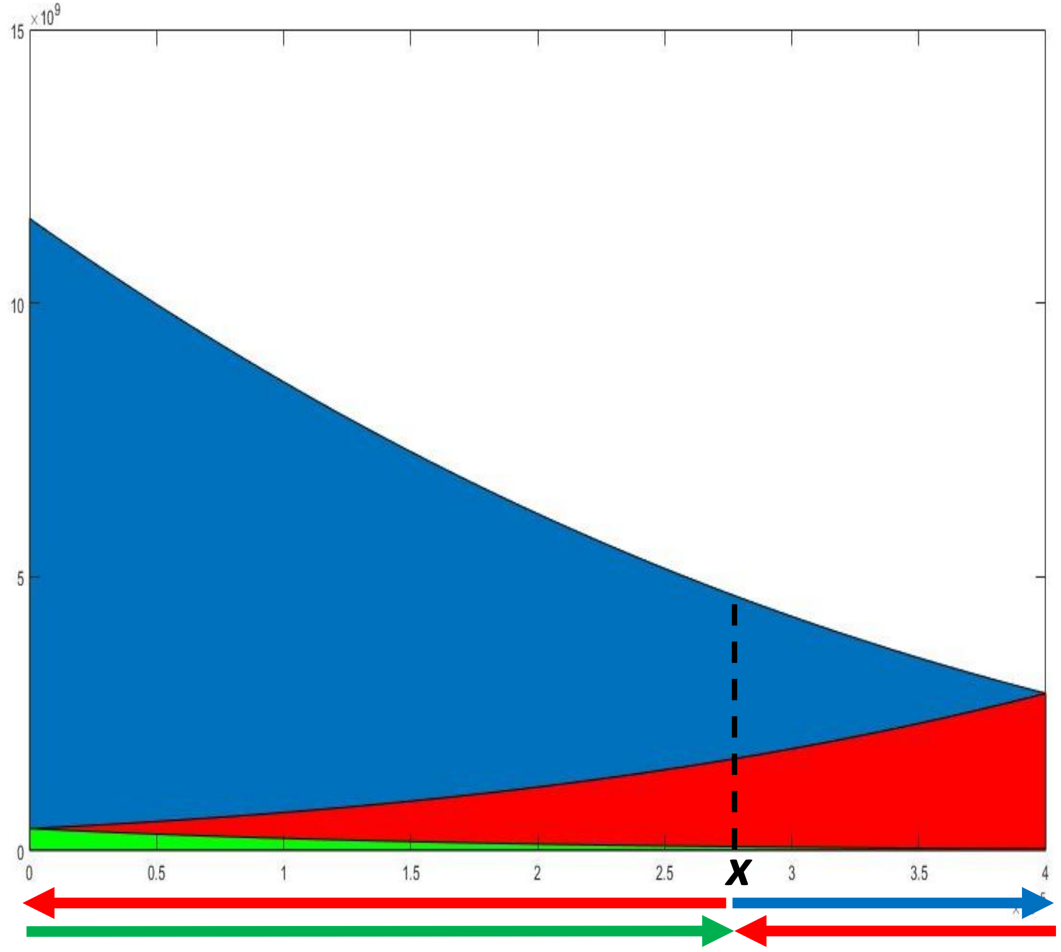
Fundamental component

Reflection (1st order)

Reflection (2nd order)

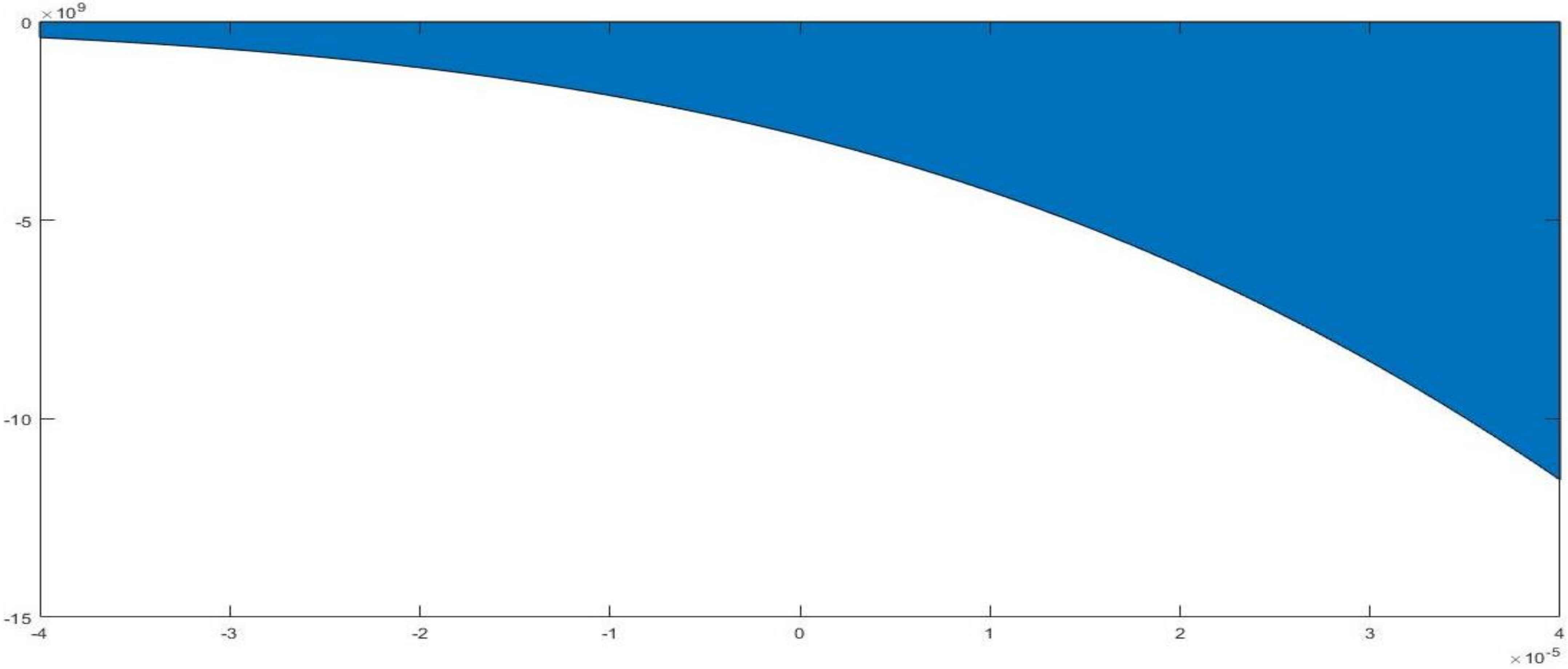
Single-segment line: solution by reflections

$$\frac{\partial \sigma}{\partial x} = -G$$

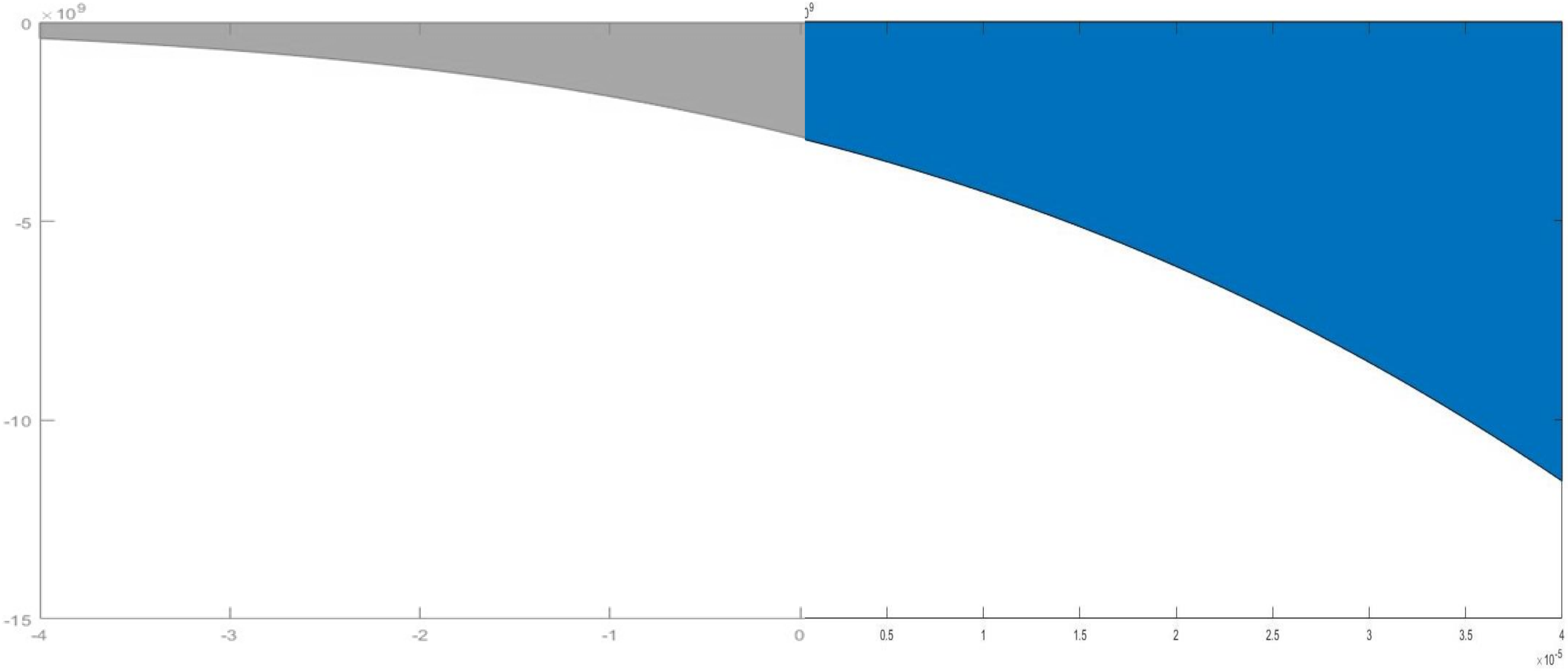


Single-segment line: solution by reflections

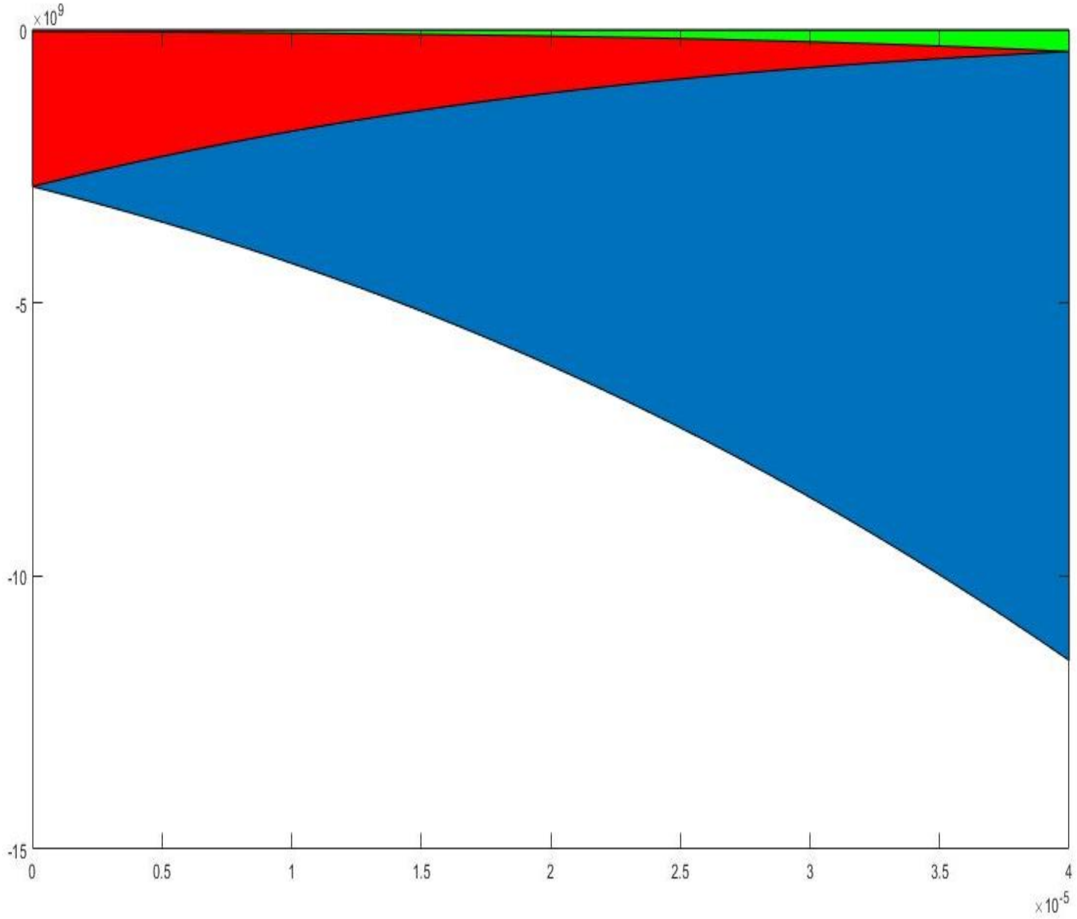
Single-segment line: solution by reflections



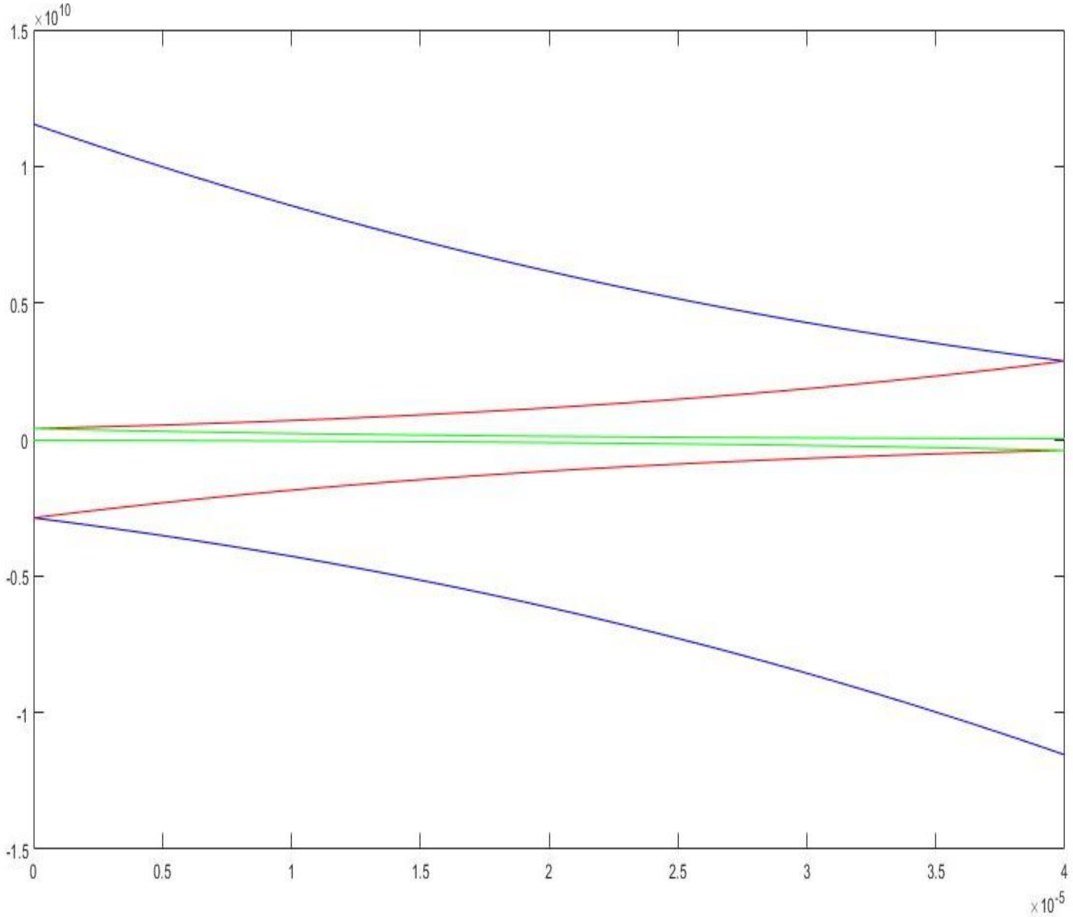
Single-segment line: solution by reflections



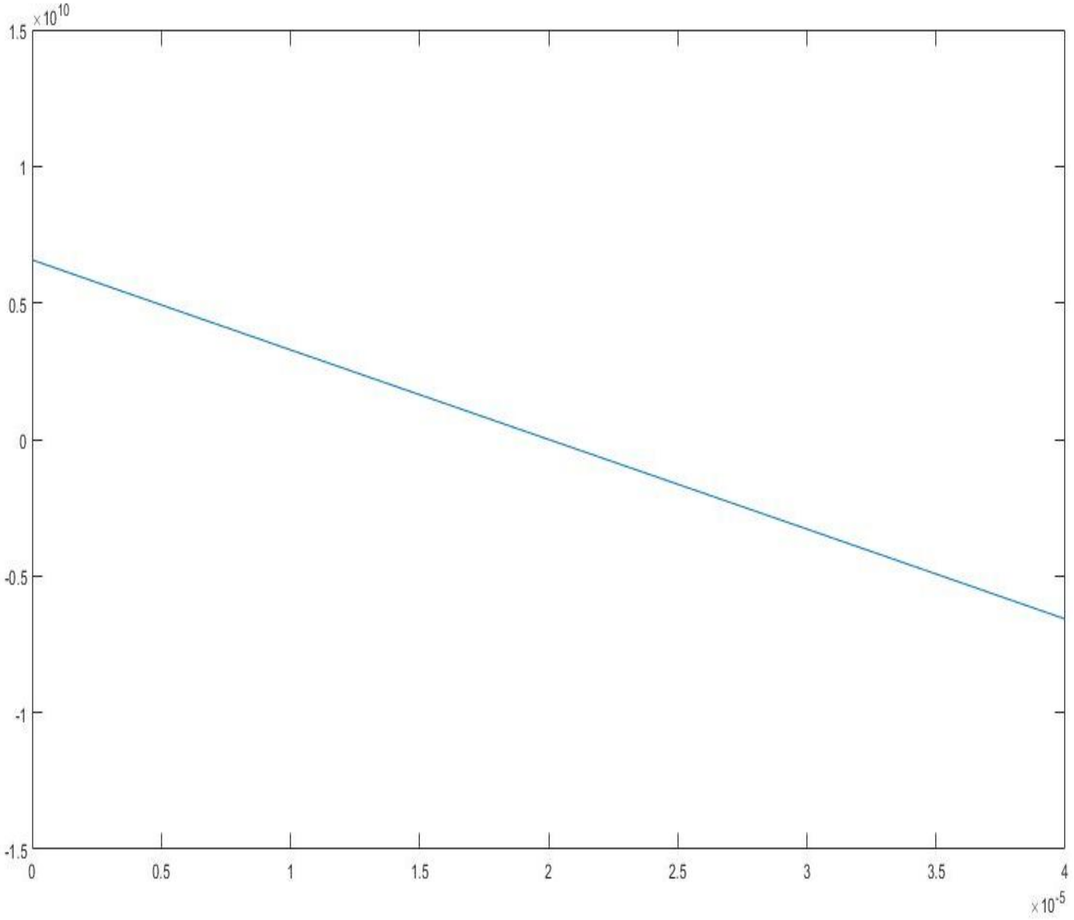
Single-segment line: solution by reflections



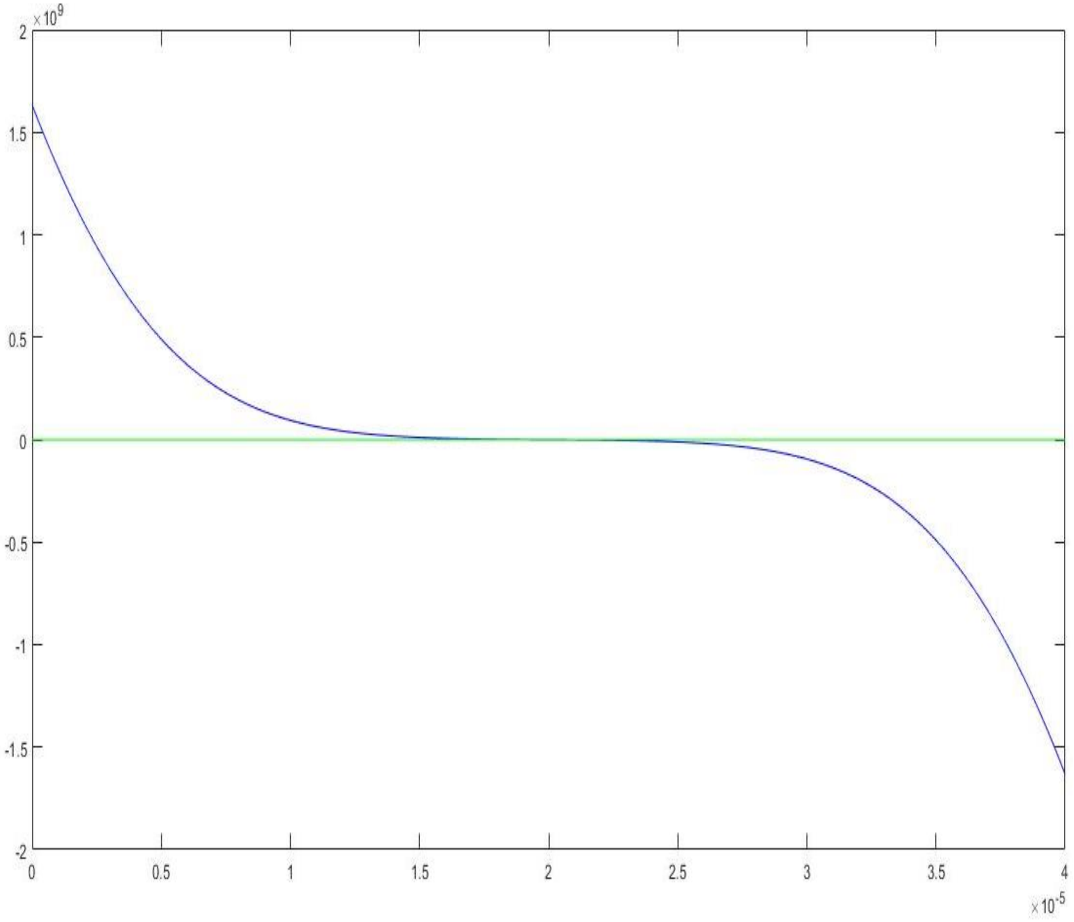
Single-segment line: solution by reflections



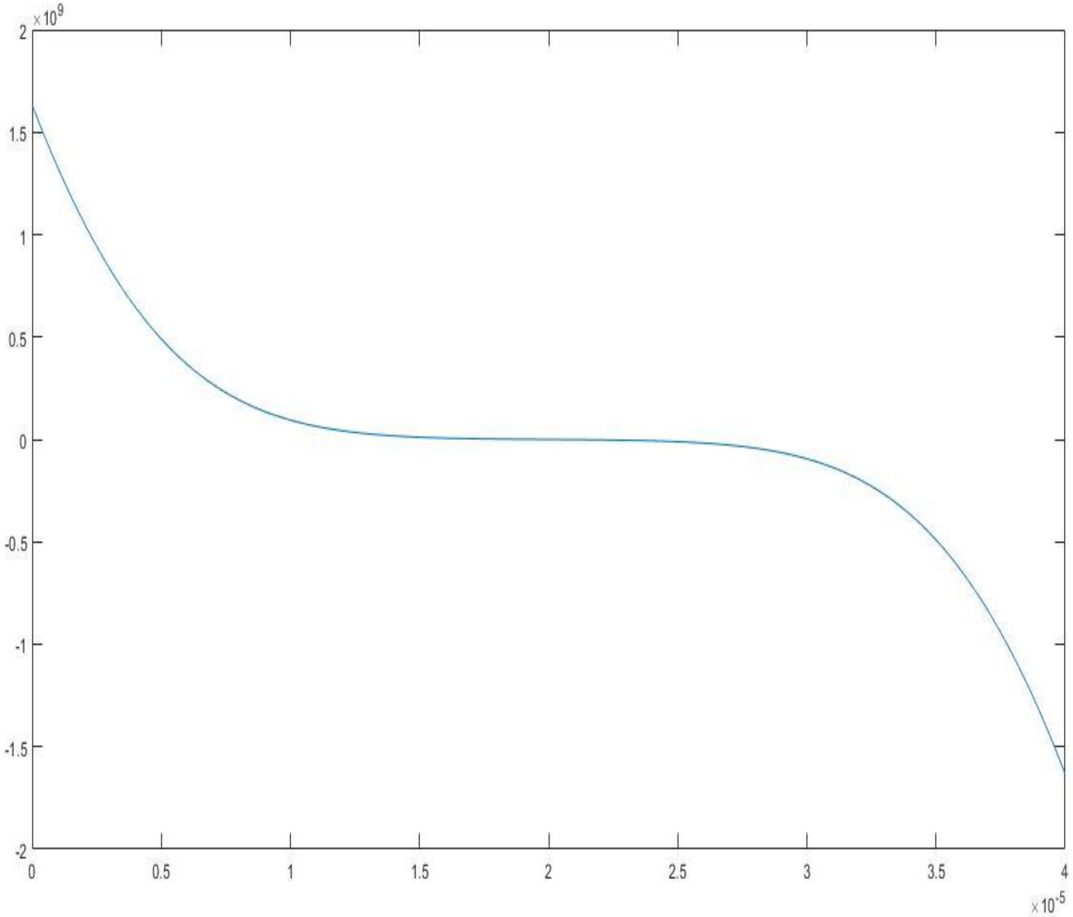
Single-segment line: solution by reflections



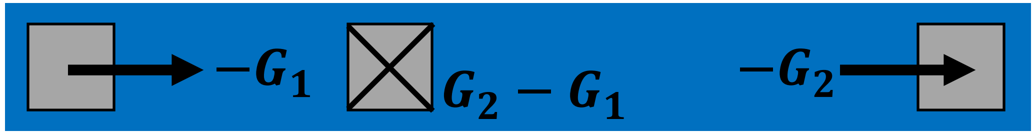
Single-segment line: solution by reflections



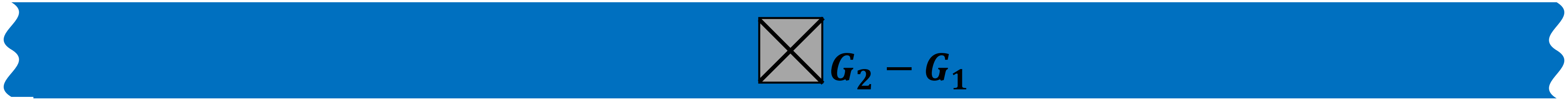
Single-segment line: solution by reflections

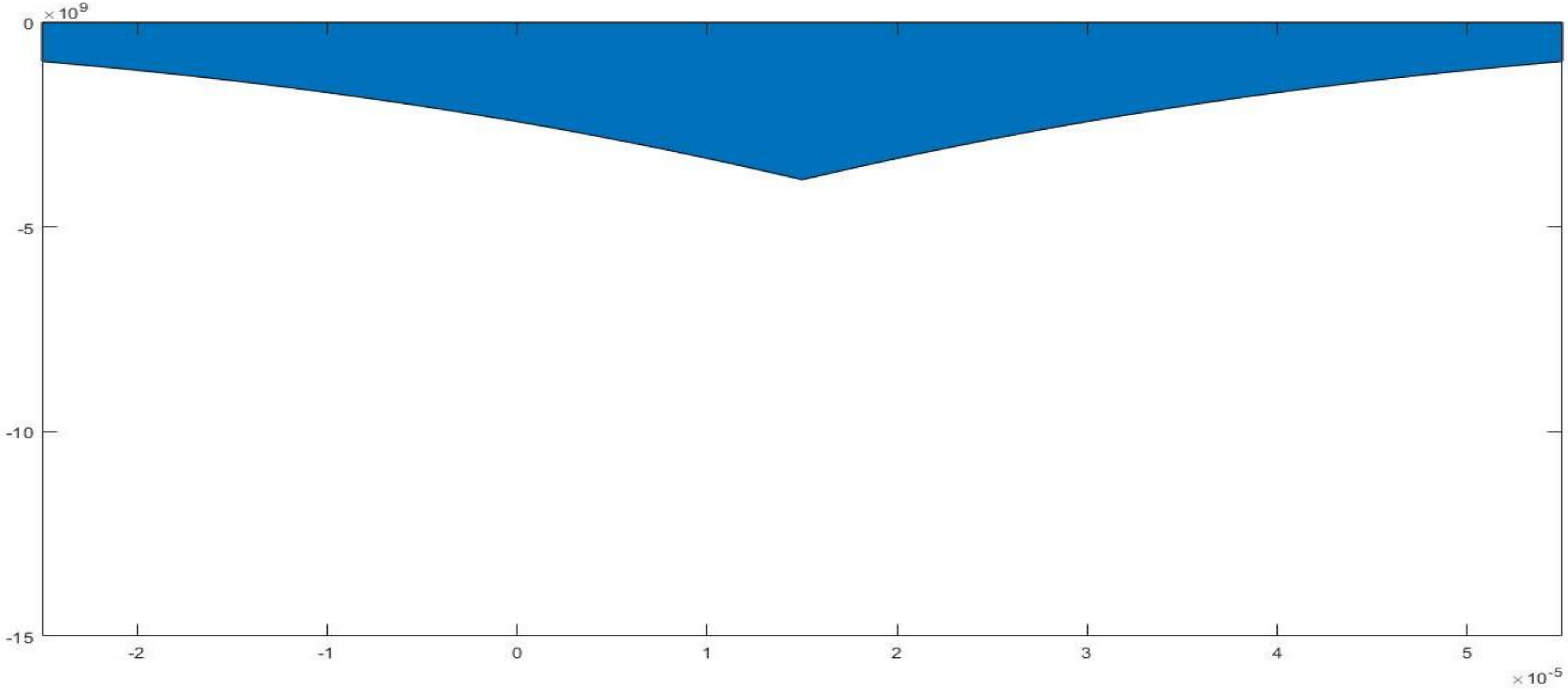


Two-segment line: solution by reflections



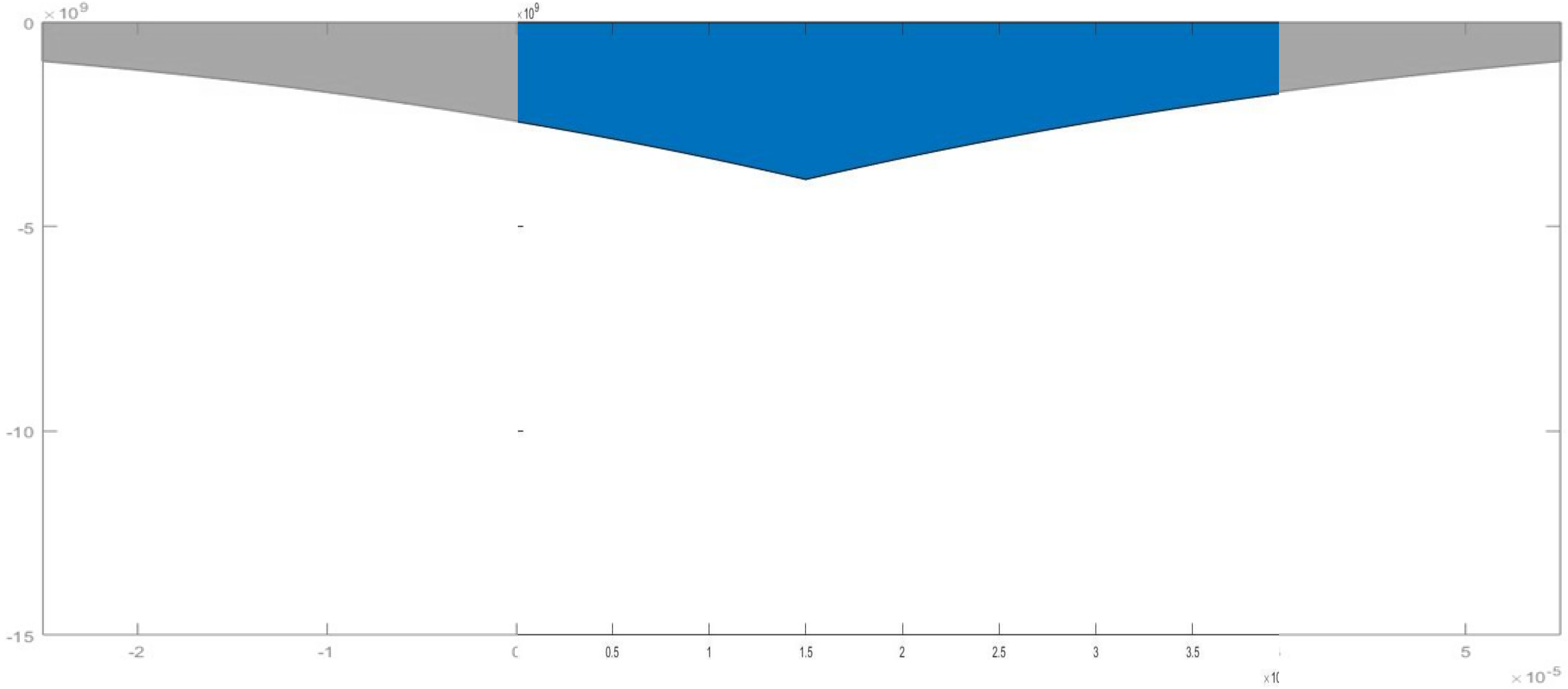
Two-segment line: solution by reflections





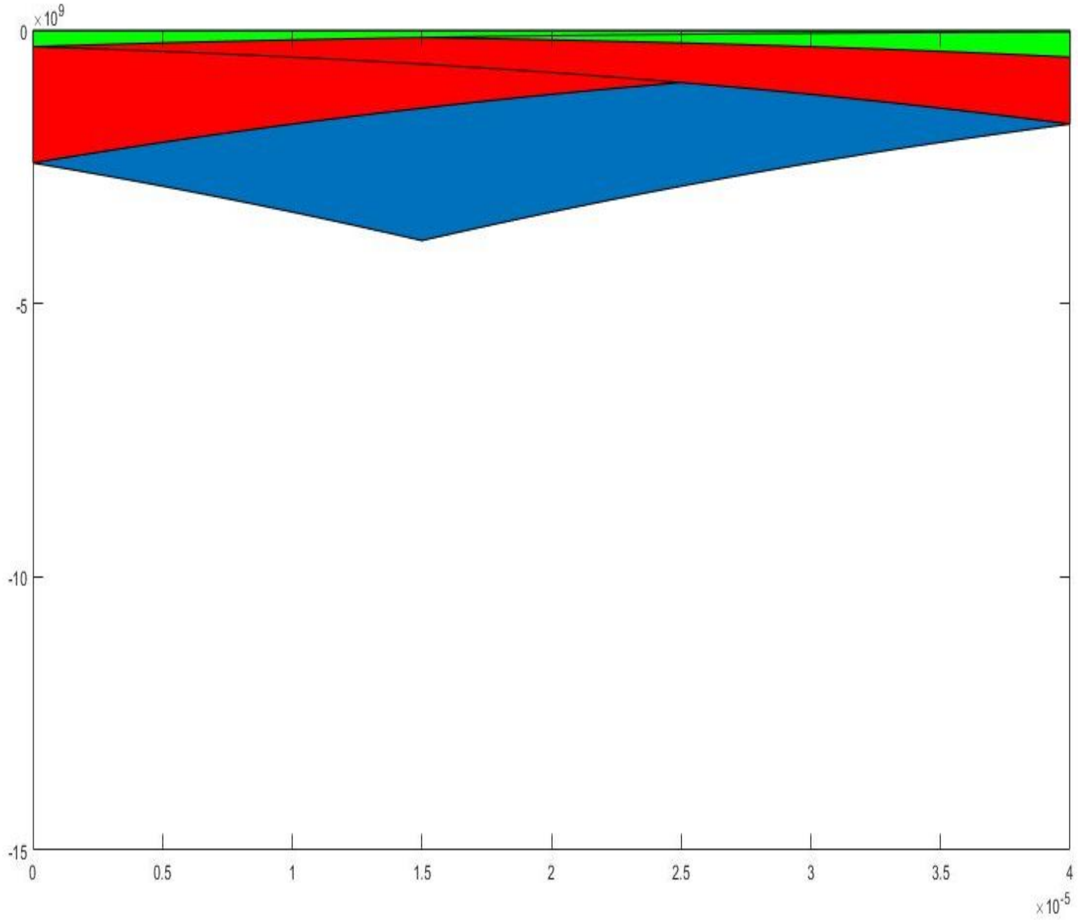
Two-segment line: solution by reflections

$G_2 - G_1$

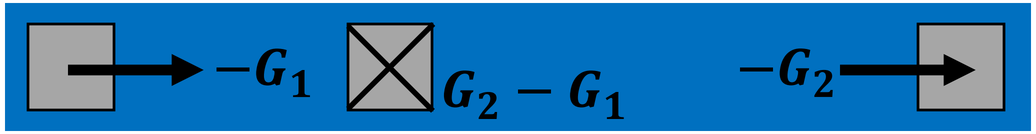


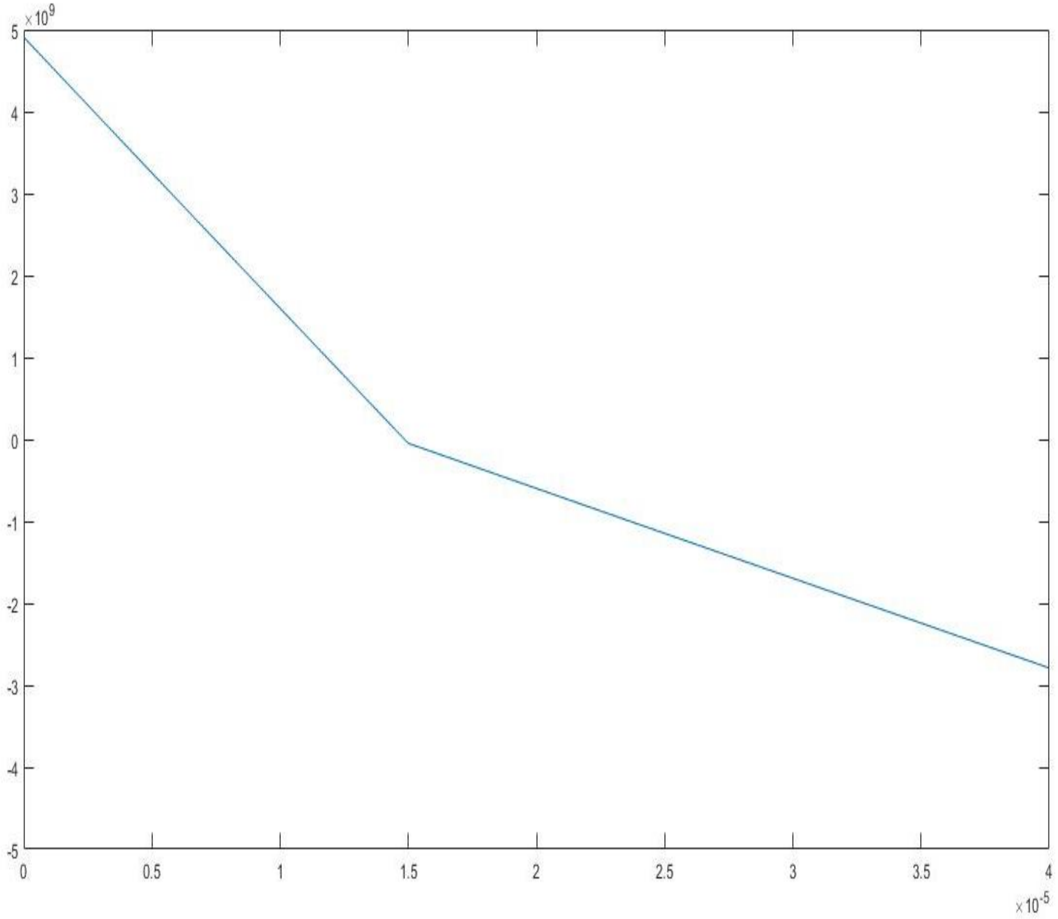
Two-segment line: solution by reflections

$G_2 - G_1$

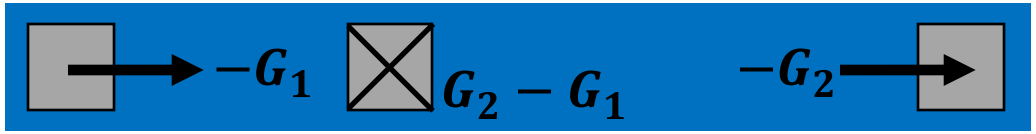


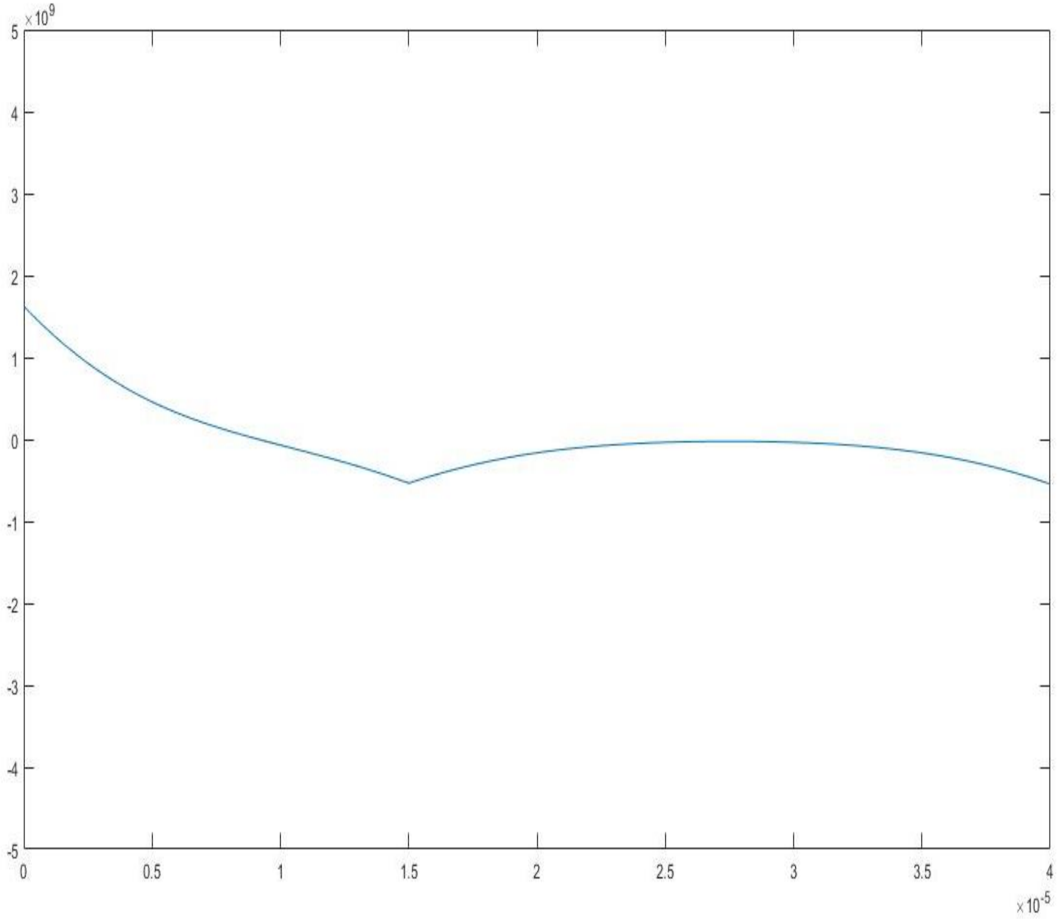
Two-segment line: solution by reflections



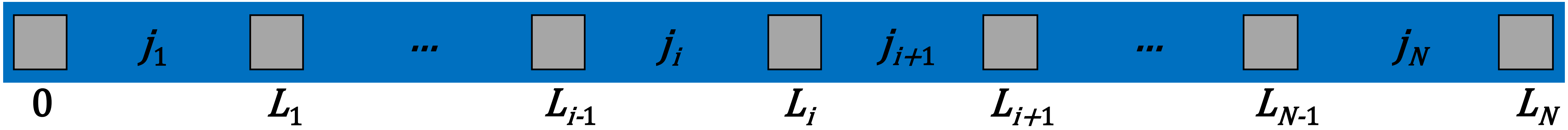


Two-segment line: solution by reflections





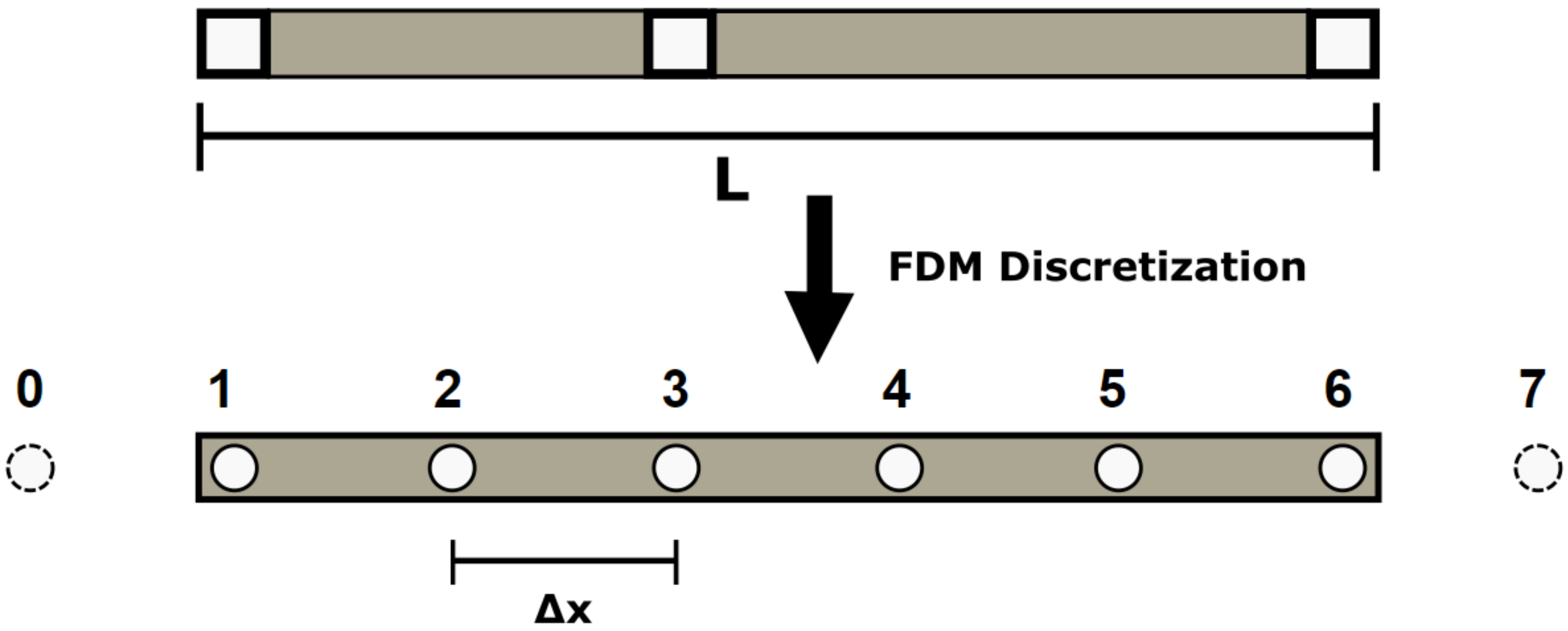
Multisegment line: solution by reflections



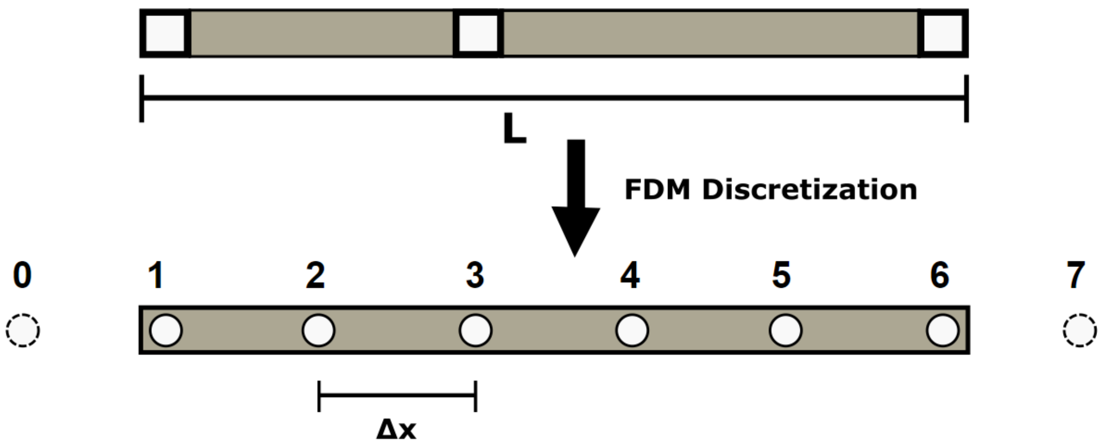
$$
\begin{aligned}
\sigma(x,t) \approx\ & G_1\left[g(x,t) + g(2L_N - x, t) + g(2L_N + x, t)\right] \\
& - G_N\left[g(L_N - x, t) + g(L_N + x, t) + g(3L_N - x, t)\right] \\
& + \sum_{i=1}^{N-1} \frac{G_{i+1} - G_i}{2}\left[g(|L_i - x|, t) + g(L_i + x, t)\right. \\
& \qquad + g(L_i + 2L_N - x, t) + g(-L_i + 2L_N - x, t) \\
& \qquad \left. + g(-L_i + 2L_N + x, t)\right]
\end{aligned}
$$

$$
g(x,t) = 2\sqrt{\frac{\kappa t}{\pi}}\, e^{-\frac{x^2}{4\kappa t}} - x\,\mathrm{erfc}\left(\frac{x}{2\sqrt{\kappa t}}\right)
$$

Multisegment line: semi-analytical solution



Multisegment line: semi-analytical solution



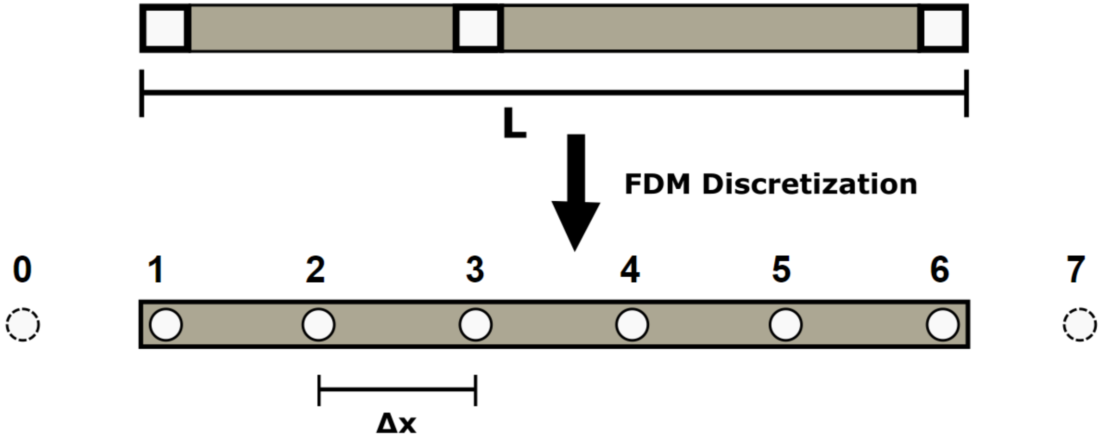
$$\dot{\boldsymbol{\sigma}}(t) = \mathbf{A}\boldsymbol{\sigma}(t) + \mathbf{B}\mathbf{j}$$

Stress at discrete spatial points

Current densities of line segments

$$\begin{bmatrix} \dot{\sigma}_1 \\ \dot{\sigma}_2 \\ \vdots \\ \dot{\sigma}_{n-1} \\ \dot{\sigma}_n \end{bmatrix} = \frac{\kappa}{\Delta x^2} \begin{bmatrix} -1 & 1 & & & \\ 1 & -2 & 1 & & \\ & & \ddots & & \\ & & 1 & -2 & 1 \\ & & & 1 & -1 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_{n-1} \\ \sigma_n \end{bmatrix} + \mathbf{B} \begin{bmatrix} j_1 \\ \vdots \\ j_N \end{bmatrix}$$

Multisegment line: semi-analytical solution



$$\dot{\boldsymbol{\sigma}}(t) = \mathbf{A}\boldsymbol{\sigma}(t) + \mathbf{Bj}$$

$$\boldsymbol{\sigma}(t) = \int_0^t e^{\mathbf{A}(t-\tau)}\,\mathbf{Bj}\,d\tau$$

Eigendecomposition

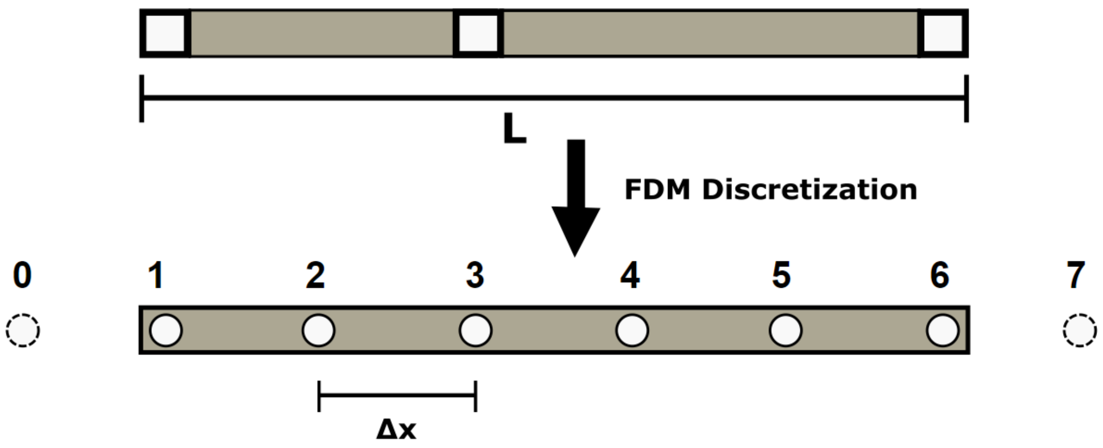
$$\mathbf{A} = \mathbf{V\Lambda V}^{-1}$$

$$\boldsymbol{\sigma}(t) = \mathbf{V}\left(\int_0^t e^{\boldsymbol{\Lambda}(t-\tau)}\,d\tau\right)\mathbf{V}^{-1}\mathbf{Bj}$$

For each eigenvalue λ_j:

$$\int_0^t e^{\lambda_j(t-\tau)}\,d\tau = \begin{cases} \dfrac{e^{\lambda_j t} - 1}{\lambda_j}, & \lambda_j \neq 0 \\ t, & \lambda_j = 0 \end{cases}$$

Multisegment line: semi-analytical solution



$$\lambda_j = \frac{\kappa}{\Delta x^2}\left(2\cos\frac{(j-1)\pi}{n} - 2\right), \quad j = 1, \dots, n$$

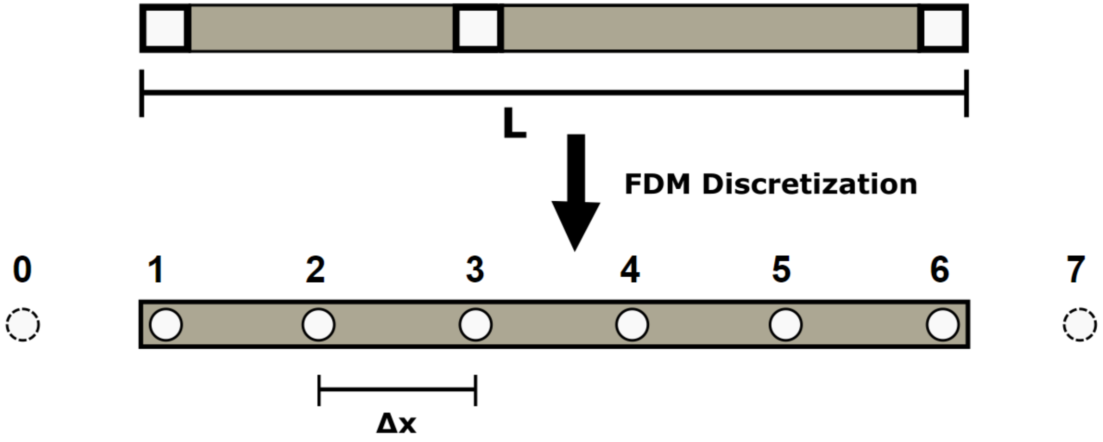
$$\mathbf{A} = \frac{\kappa}{\Delta x^2}\begin{bmatrix} -1 & 1 & & & \\ 1 & -2 & 1 & & \\ & & \ddots & & \\ & & 1 & -2 & 1 \\ & & & 1 & -1 \end{bmatrix}$$

$$\mathbf{V}^{-1}\mathbf{r} \equiv \mathbf{V}^T\mathbf{r} \Leftrightarrow \mathrm{DCT}\text{–}II(\mathbf{r})$$

$$\mathbf{V}\,\mathbf{r} \Leftrightarrow \mathrm{IDCT}\text{–}II(\mathbf{r})$$

$O(n\log n)$

Multisegment line: semi-analytical solution

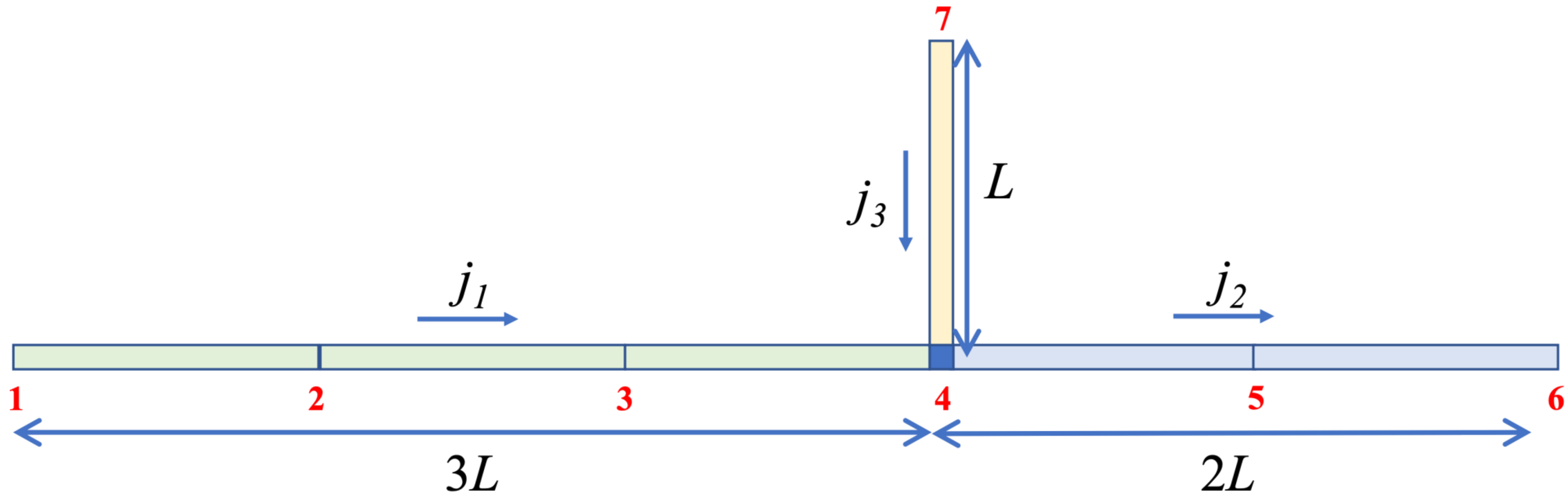


$$\boldsymbol{\sigma}(t) = \underbrace{\mathbf{V}\mathbf{L}(t)\underbrace{\mathbf{V}^T\mathbf{B}\mathbf{j}}_{\text{DCT}-II}}_{\text{IDCT}-II}$$

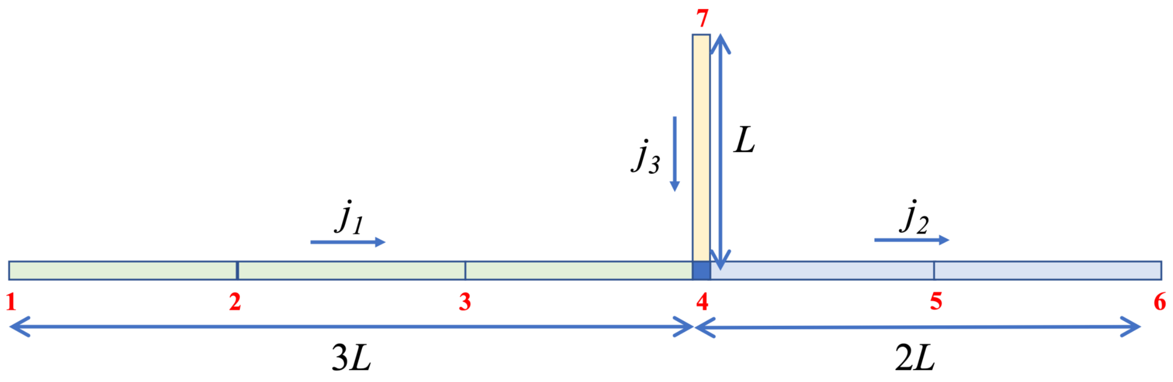
$$\mathbf{L}(t) = \begin{bmatrix} t & & & \\ & \dfrac{e^{\lambda_2 t}-1}{\lambda_2} & & \\ & & \ddots & \\ & & & \dfrac{e^{\lambda_n t}-1}{\lambda_n} \end{bmatrix}$$

$$\lambda_j = \frac{\kappa}{\Delta x^2}\left(2\cos\frac{(j-1)\pi}{n} - 2\right)$$

Interconnect tree: semi-analytical solution



Interconnect tree: semi-analytical solution



Conservation of atomic flux over the *entire* tree relates stress at discretized points

Solve for any σ_j and substitute

$$\dot{\boldsymbol{\sigma}}(t) = \mathbf{A}\boldsymbol{\sigma}(t) + \mathbf{Bj}$$

$$\boldsymbol{\sigma}(t) = \int_0^t e^{\mathbf{A}(t-\tau)}\,\mathbf{Bj}\,d\tau$$

$(n-1)\times(n-1)$

nonsingular A

?

$$\boldsymbol{\sigma}(t) = \underbrace{\left(e^{\mathbf{A}t} - \mathbf{I}\right)\underbrace{\mathbf{A}^{-1}\mathbf{Bj}}_{\text{sparse linear solve}}}_{\text{matrix-exponential vector product}}$$

Conclusion and Future Challenges

Recent EM/Stress solutions for multisegment structures

- Reflections and stress-wave analogy for multisegment lines
  $\rightarrow$ analytical calculation of stress via closed-form expression
- Semi-analytical solutions for multisegment lines and interconnect trees
  $\rightarrow$ discretize only space, able to calculate stress at any future time directly

Future challenges and perspectives

- Extend reflections framework and stress-wave analogy to interconnect trees
- Adapt analytical and semi-analytical solutions to growth phase
- Handling of time-varying (or frequency-specific) input current densities
- Design connections (lifetime-driven P&R, segment widths, P/G stripe spacing)
- Adoption of physics-based models and emerging solutions in industrial flows

Thank you!

Questions / Comments

nestevmo@uth.gr