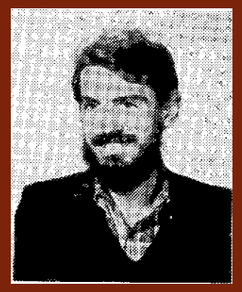
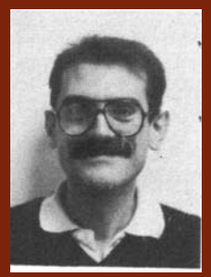
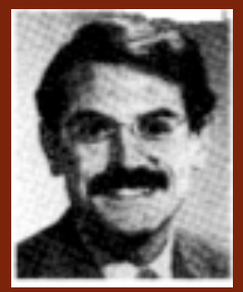
RESEARCH LABORATORY OF ELECTRONICS
Massachusetts Institute of Technology

A Brief History of Timing

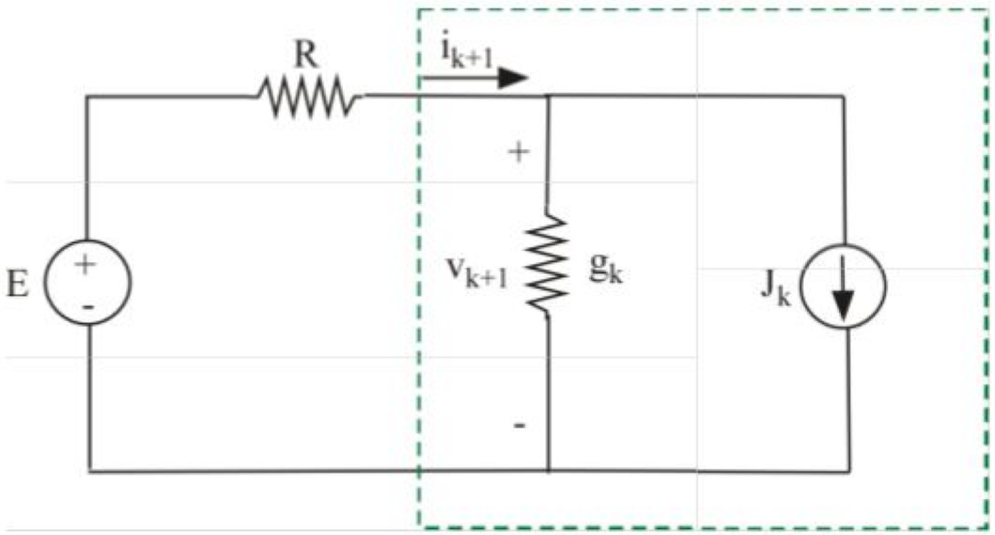
Technical Content: Too Many to name

Presenter: J. White

VAX in Time

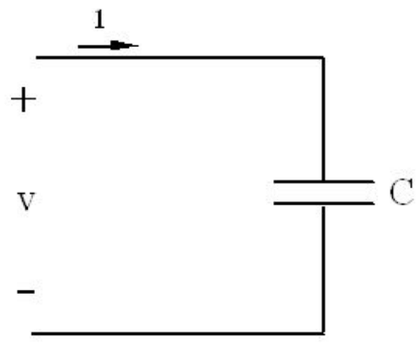
- In 1983:
  - 0.1Mflop computers, flop > 10*MemoryRef
  - BIG CIRCUIT: 1000 Transistors
  - Exploitable Properties
    - Multi-rate, no bipolar, MOS unidirectional, hierarchy
  - Key Techniques
    - Decomposition, multirate integration, simplified models
- In 2019
  - 100 Gigaflops or more, MemoryRef > 10 Flop
  - BIG CIRCUIT: 1,000,000,000 Transistors
  - Exploitable Properties
    - Multi-rate, no bipolar, MOS unidirectional, HIERARCHY
  - Key Techniques
    - Multi-rate, no bipolar, SIMD (GPU),REUSE

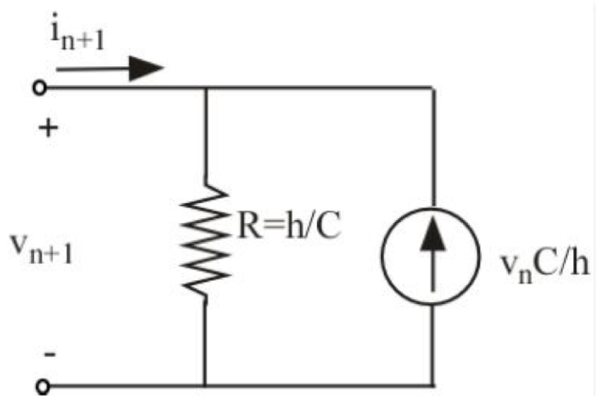
Bad Companions



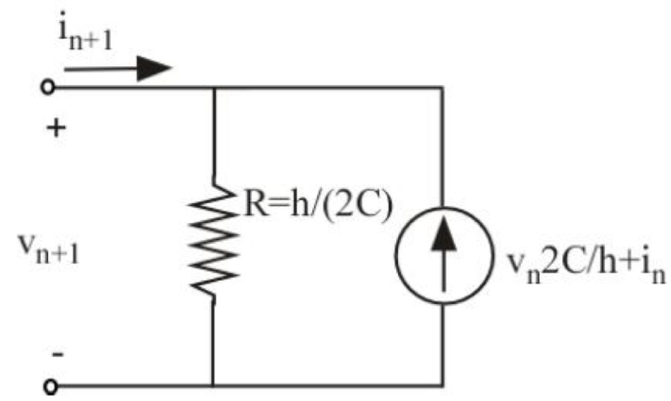
$$g_k = \frac{I_s}{V_t} e^{V_k / V_t}$$

$$J_k = I_s(e^{Vk/Vt} -1) - V_k g_k$$





Backward Euler companion model at t=nh



Trapezoidal companion model at t=nh

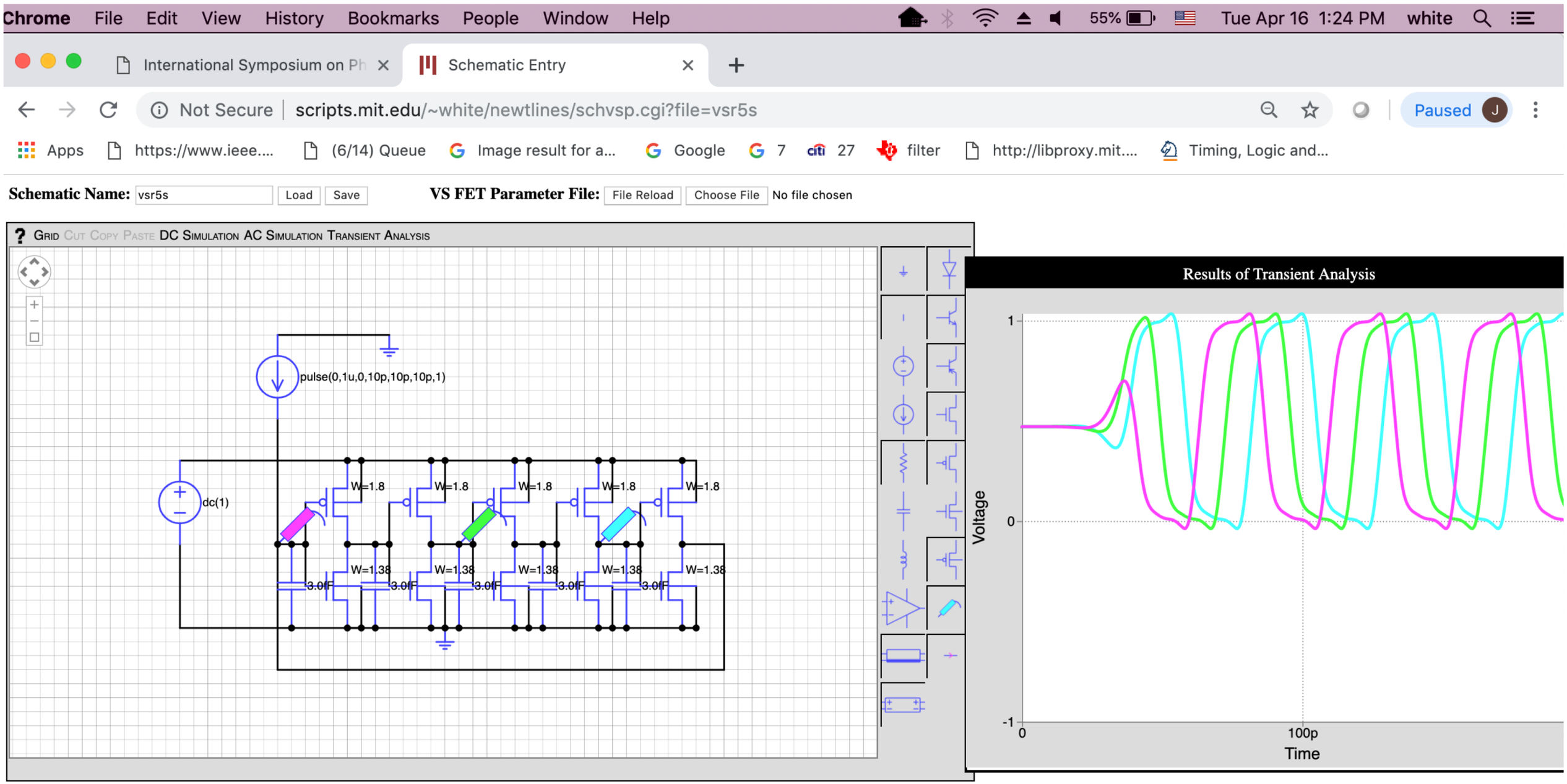
N models * N methods = N^2 work

$$\frac{dq(\mathbf{v},t)}{dt} = i(\mathbf{v},\mathbf{u},t) \ ,$$

$$\mathbf{v}_{t+\Delta t} = \mathbf{v}_t + \Delta t f(\mathbf{v}_{t+\Delta t}, \mathbf{u}_{t+\Delta t}, t+\Delta t) \Rightarrow F(\mathbf{v}) = 0$$

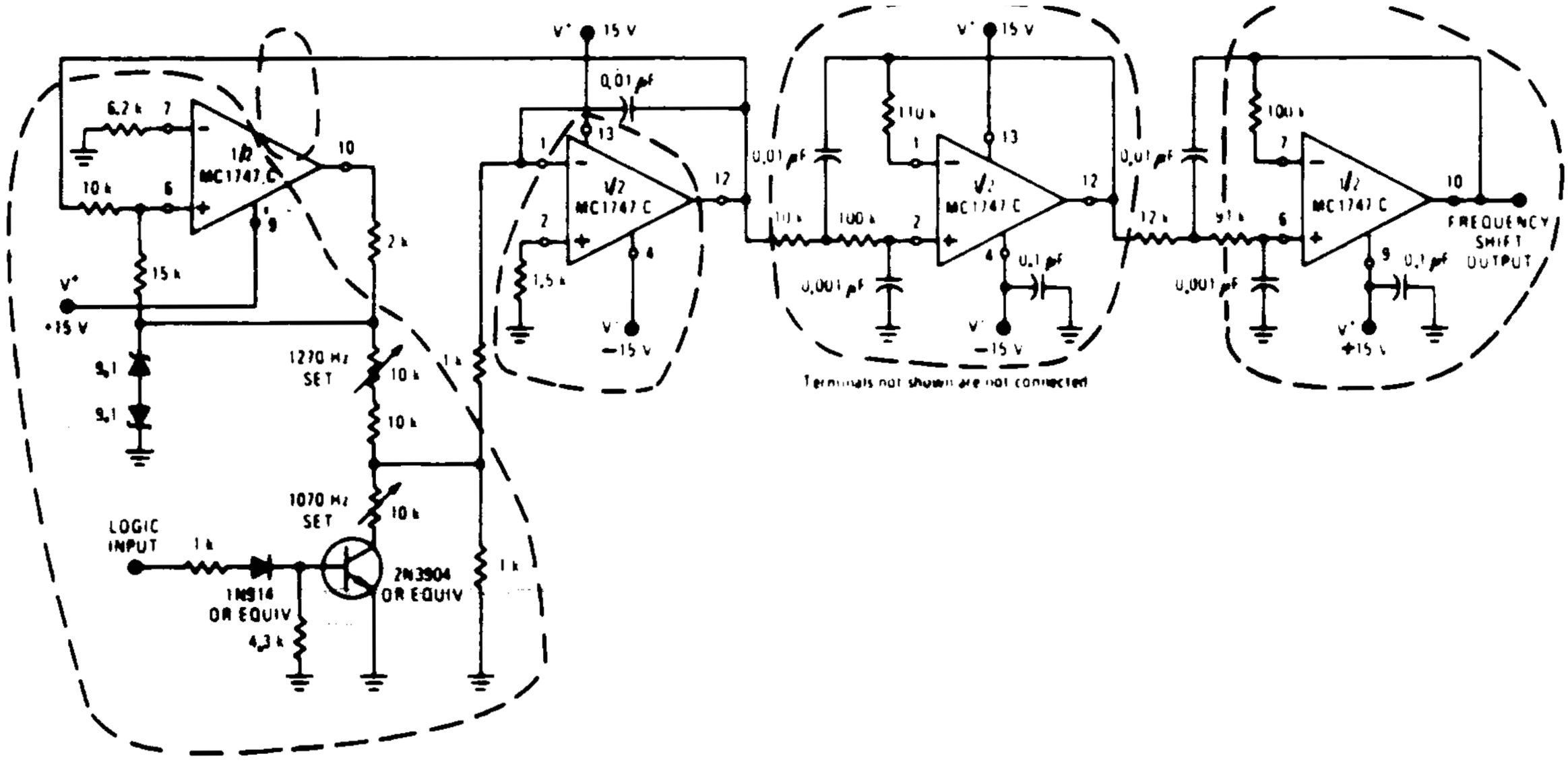
$$\mathbf{v}^{i+1} = \mathbf{v}^i - J_F^{-1}(\mathbf{v}^i)F(\mathbf{v}^i) \Rightarrow \quad J_F(\mathbf{v}_i)\mathbf{x} = F(\mathbf{v}_i).$$

Circuit Simulators have become a Commodity

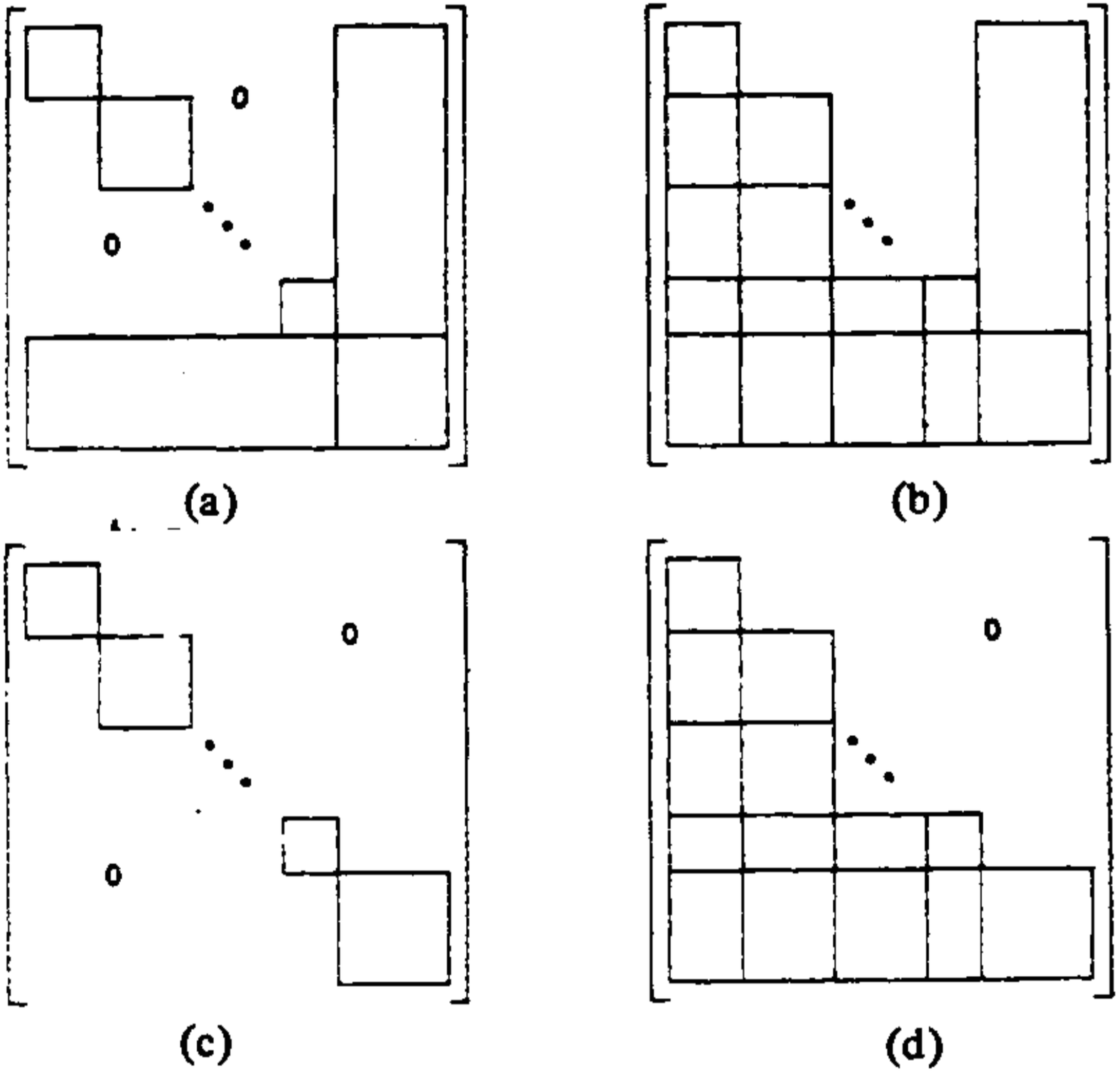


Javascript Simulator – 3 person weeks to write

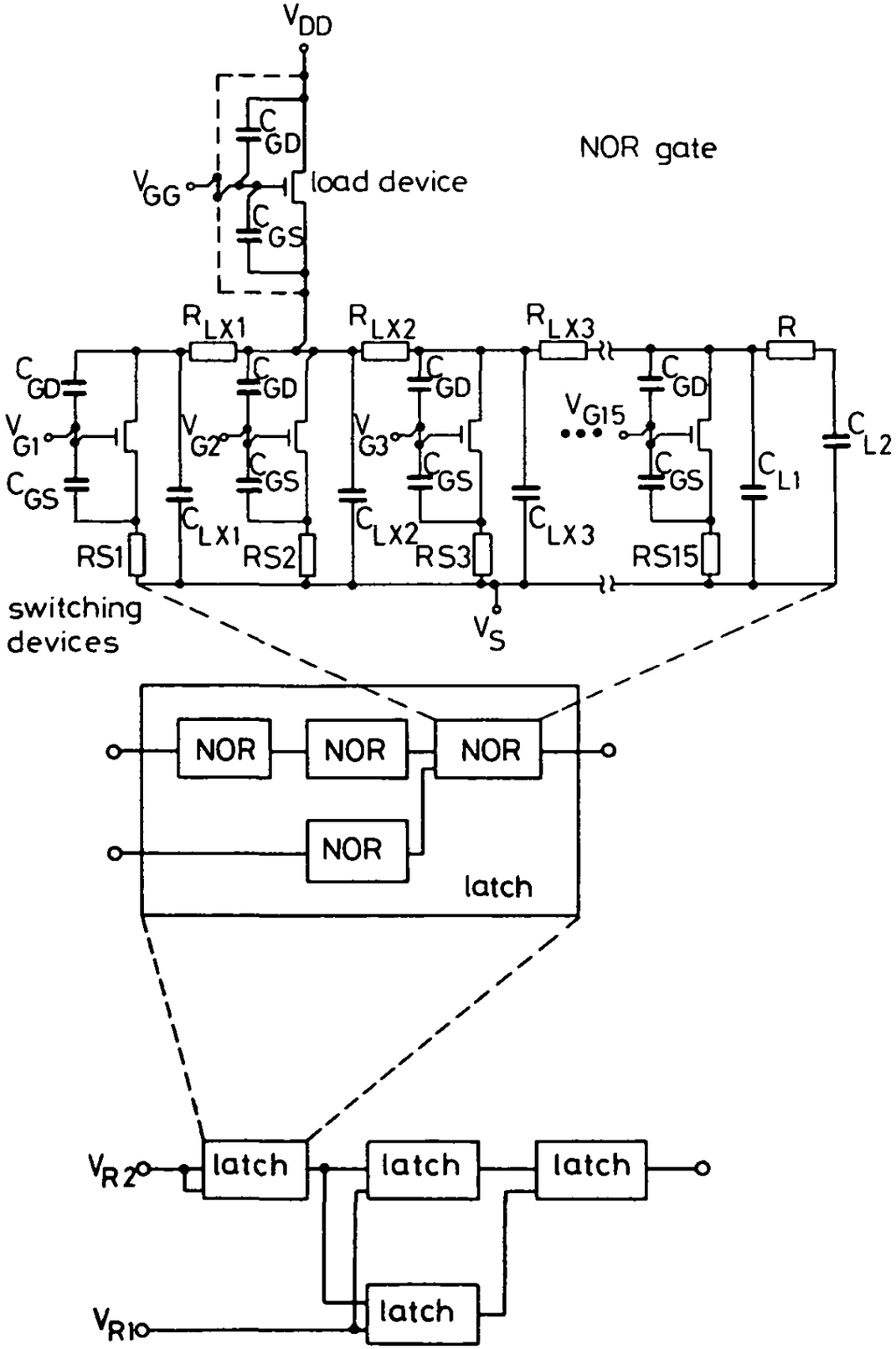
Tears for Fears



BBD, BLBD,BD, LBD, ……



Exploit hierarchy



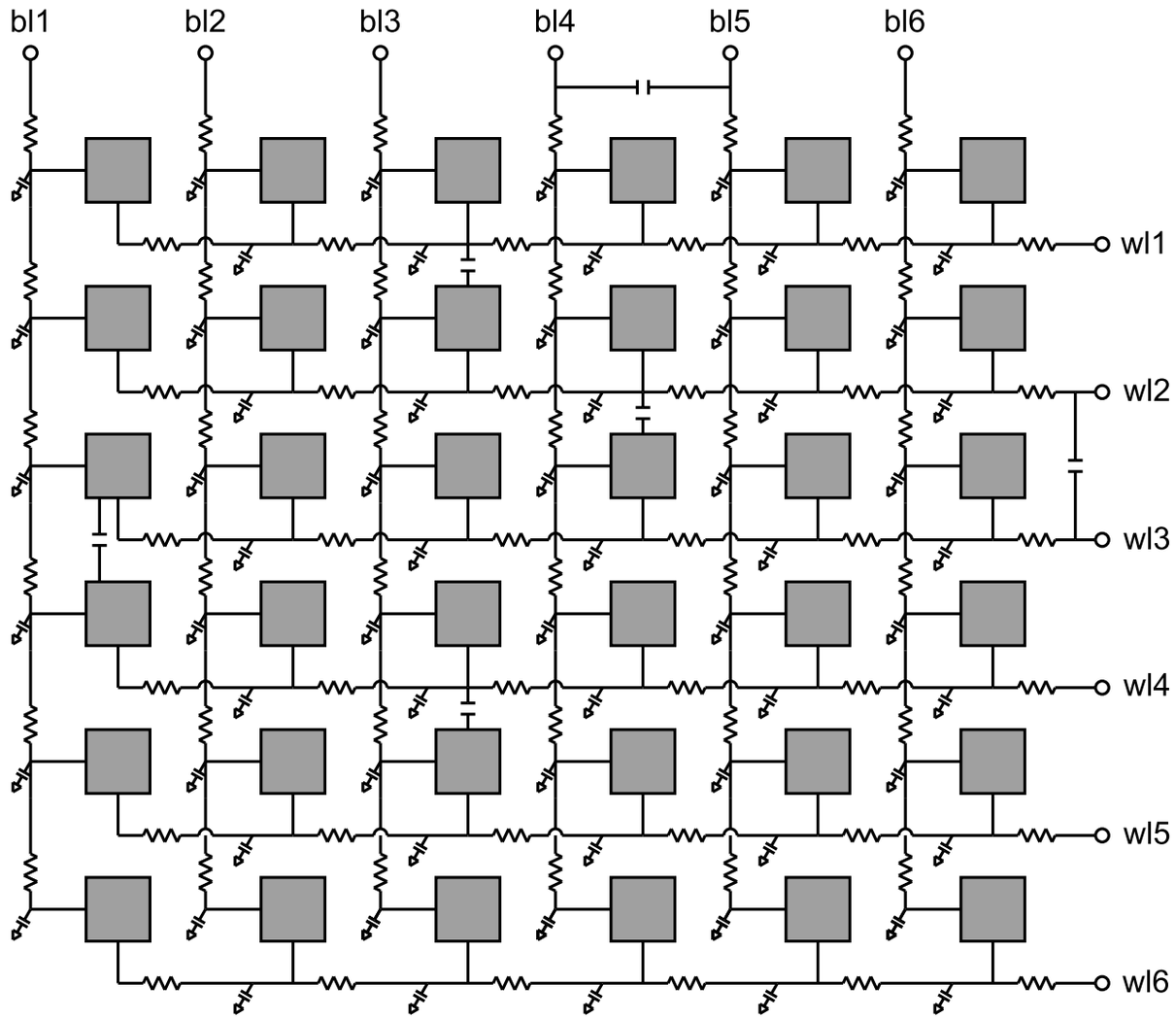


Fig. 1 *Example of integrated circuit with hierarchical structure*

Don't be Too Explicit

(a) Gauss–Jacobi integration algorithm:

$$[\mathbf{I} - h\mathbf{D}]\mathbf{x}_{k+1} = [\mathbf{I} + h(\mathbf{L} + \mathbf{U})]\mathbf{x}_k.$$

$$\mathbf{x}_{k+1} = \mathbf{M}_{GJ}(h)\mathbf{x}_k.$$

where I is the identity matrix and

$$\mathbf{M}_{GJ}(h) = [\mathbf{I} - h\mathbf{D}]^{-1}[\mathbf{I} + h(\mathbf{L} + \mathbf{U})]$$

(b) Gauss-Seidel integration algorithm:

$$[\mathbf{I} - h(\mathbf{D} + \mathbf{L})]\mathbf{x}_{k+1} = [\mathbf{I} + h\mathbf{U}]\mathbf{x}_k$$

$$\mathbf{x}_{k+1} = \mathbf{M}_{GS}(h)\mathbf{x}_k$$

where

$$\mathbf{M}_{GS}(h) = [\mathbf{I} - h(\mathbf{D} + \mathbf{L})]^{-1}[\mathbf{I} + h\mathbf{U}]$$

The Wrath of Kahan

(c) Modified symmetric Gauss–Seidel integration algorithm:
Let:

$$\mathbf{A}_{\mathrm{L}} = \mathbf{L} + \tfrac{1}{2}\mathbf{D} \qquad \mathbf{A}_{\mathrm{U}} = \mathbf{U} + \tfrac{1}{2}\mathbf{D}$$

orward step:

$$\left[\mathbf{I} - \frac{h}{4}(2\mathbf{L} + \mathbf{D})\right]\mathbf{x}_{k+1/2} = \left[\mathbf{I} + \frac{h}{4}(\mathbf{D} + 2\mathbf{U})\right]\mathbf{x}_k$$

$$\left[\mathbf{I} - \frac{h}{2}\mathbf{A}_{\mathrm{L}}\right]\mathbf{x}_{k+1/2} = \left[\mathbf{I} + \frac{h}{2}\mathbf{A}_{\mathrm{U}}\right]\mathbf{x}_k$$

$$\mathbf{x}_{k+1/2} = \left[\mathbf{I} - \frac{h}{2}\mathbf{A}_{\mathrm{L}}\right]^{-1}\left[\mathbf{I} + \frac{h}{2}\mathbf{A}_{\mathrm{U}}\right]\mathbf{x}_k$$

ackward step:

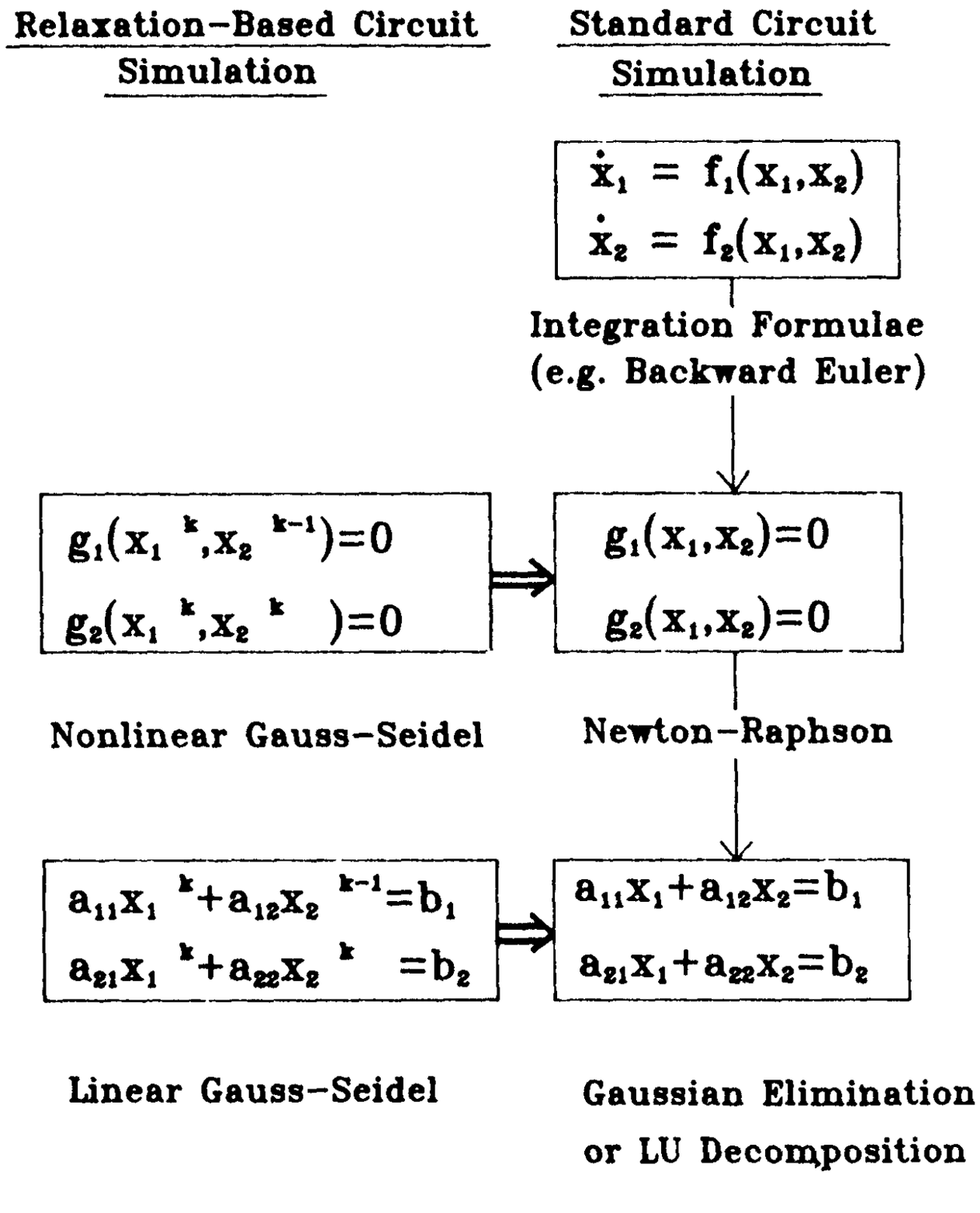
$$\left[\mathbf{I} - \frac{h}{4}(\mathbf{D} + 2\mathbf{U})\right]\mathbf{x}_{k+1} = \left[\mathbf{I} + \frac{h}{2}(2\mathbf{L} + \mathbf{D})\right]\mathbf{x}_{k+1/2}$$

$$\mathbf{x}_{k+1} = \left[\mathbf{I} - \frac{h}{2}\mathbf{A}_{\mathrm{U}}\right]^{-1}\left[\mathbf{I} + \frac{h}{2}\mathbf{A}_{\mathrm{L}}\right]\mathbf{x}_{k+1/2}$$

ombining (22) and (24) we obtain:

$$\mathbf{x}_{k+1} = \mathbf{M}_{\mathrm{S}}(h)\mathbf{x}_k$$

The Missing Iteration



Who Shot W.R.?

SPICE2 VS RELAX2.2 (DIRECT) VS RELAX2.2(WR) ON INDUSTRIAL CIRCUITS

Circuit	Devices	SPICE2	RELAX2.2(DIRECT)	RELAX2.2(WR)
uP Control	232	1400s*	90s*	45s*
Cmos Memory	621	10400s*	995s*	308s*
4-bit Counter	259	4300s*	540s*	299s*
Inverter Chain	250	439s**	98s**	38s**
Digital Filter	1082	18000s*	1800s*	520s*
Encode-Decode	3295	115,000s*	5000s*	1500s*

Theorem 1.5.1: If, in addition to the assumptions of Theorem 1.2.1, the WR iteration equations are solved using a stable and consistent multistep method with a fixed timestep h , for a finite number of points, then there exists an $h' > 0$ such that the sequences $\{ x^k(n) \}$ generated by the Gauss-Seidel or Gauss-Jacobi discretized WR algorithm will converge for all $0 < h < h'$.