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SRAM Dynamic Stability Verification by Reachability Analysis with Consideration of Threshold Voltage Variation

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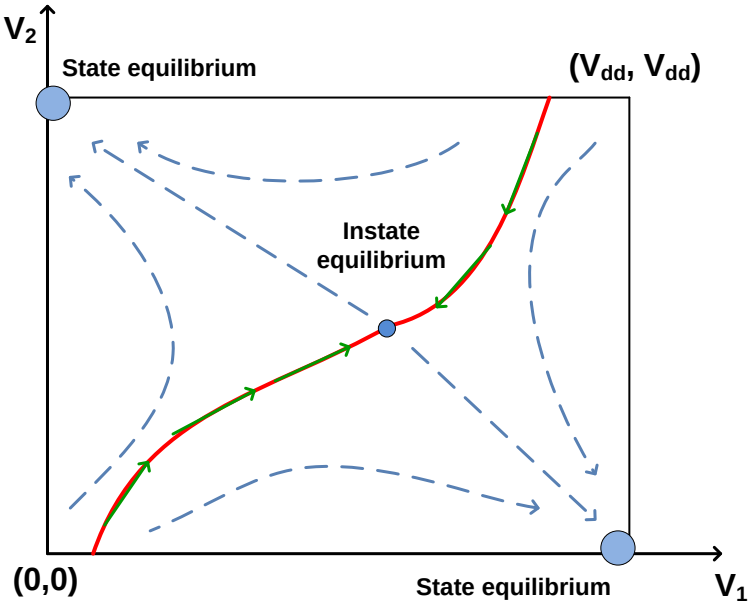
Outline

- SRAM failure analysis
- SRAM nonlinear dynamics
- Verification by reachability analysis
- Experimental results
- Summary

SRAM Failure Analysis

- Becomes difficult as technology scales down
  - Process variations, mismatch among transistors cause failures
- Nonlinear dynamics of SRAM
- Physical mechanisms of failures
  - Separatrix: boundary separating two stable regions in state-variable space*

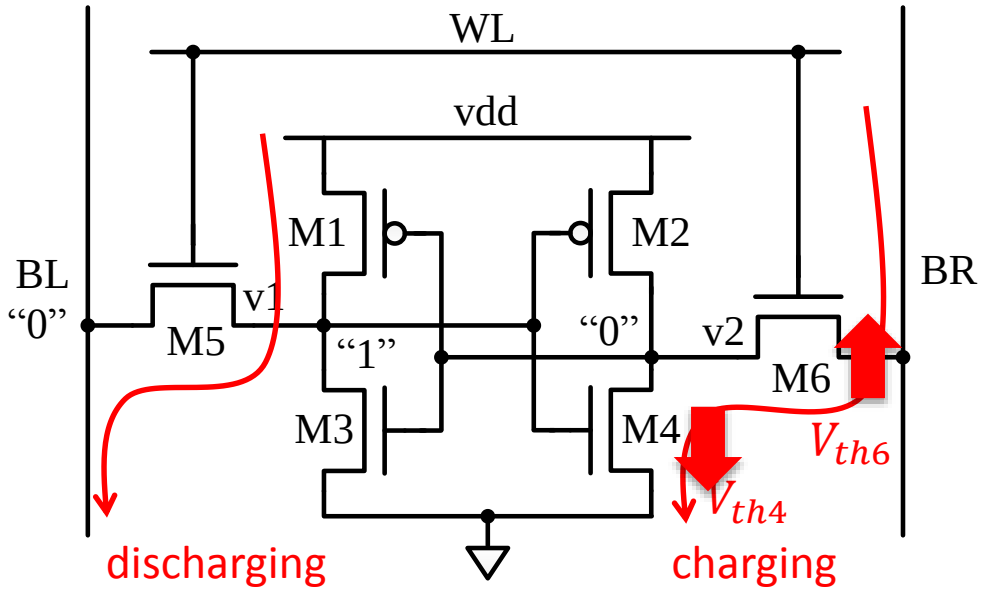
State-variable space

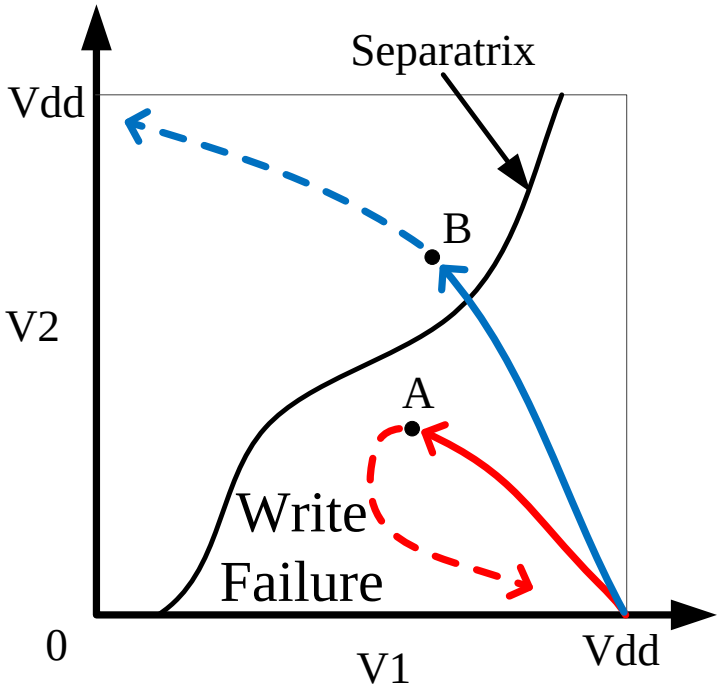


* W. Dong and et.al. *ICCAD*, 2008

Write Failure Analysis

- Initial state (v1,v2) = (vdd, 0)
- Target state (v1, v2) = (0, vdd)

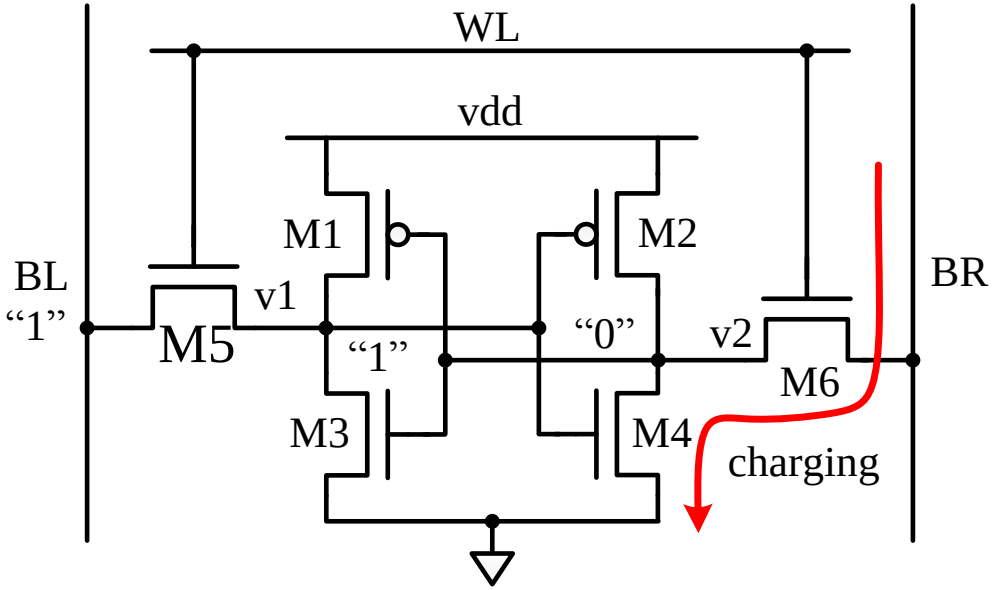


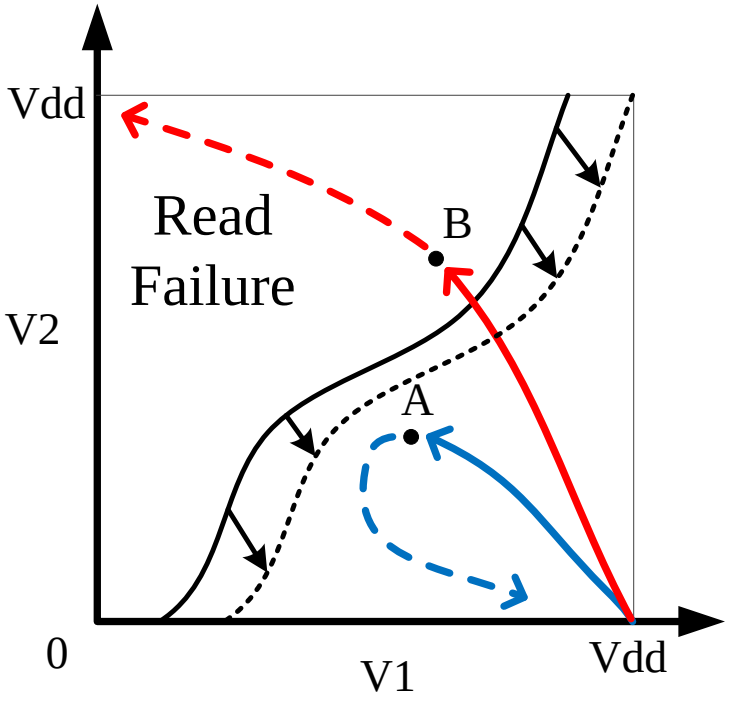


- Threshold voltage variation causes difficulty to move state point to the target state.

Read Failure Analysis

- Internal state is aimed to maintain regardless perturbation during read operation.

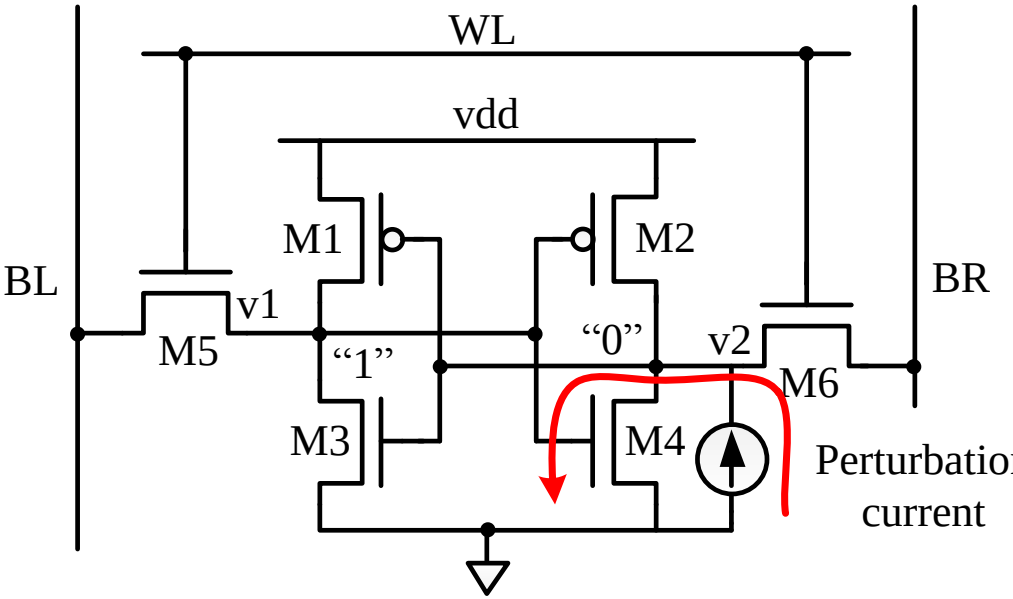


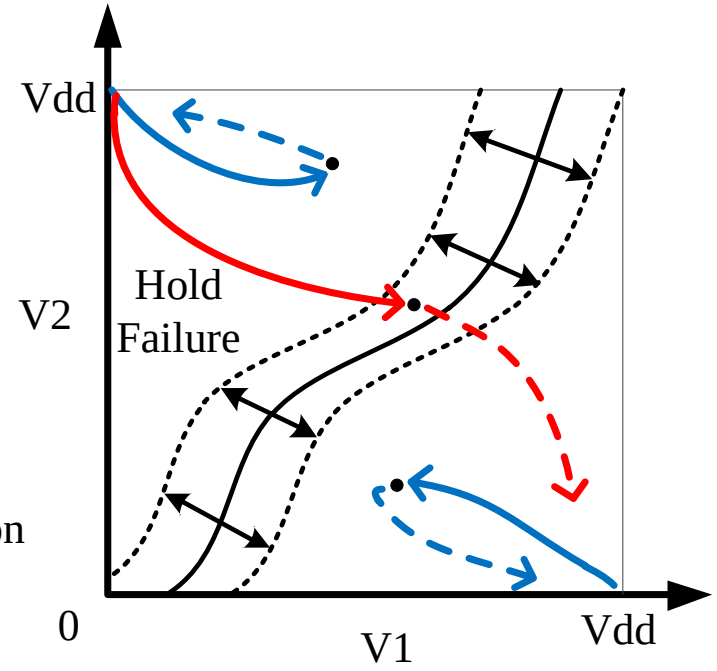


- Mismatch between M4 and M6
- Mismatch among M1-4

Hold Failure Analysis

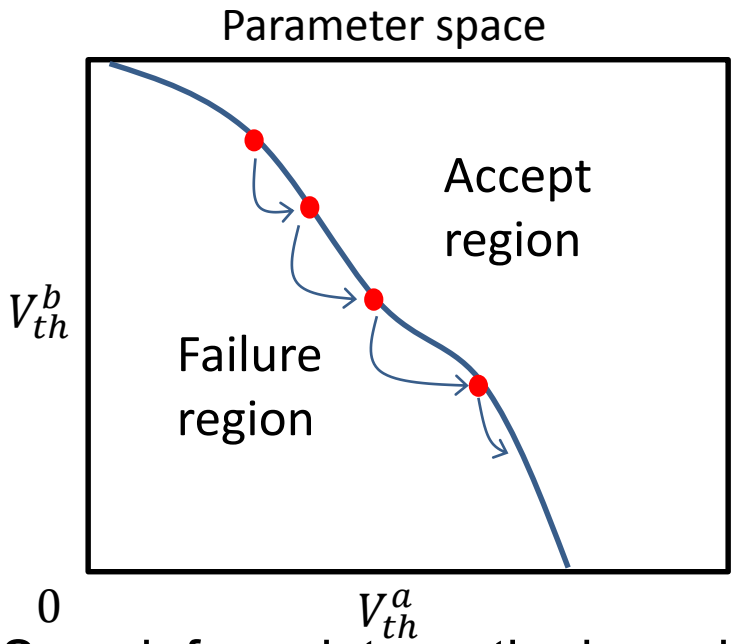
- Hold failure happens when the SRAM fails to retain the stored data.





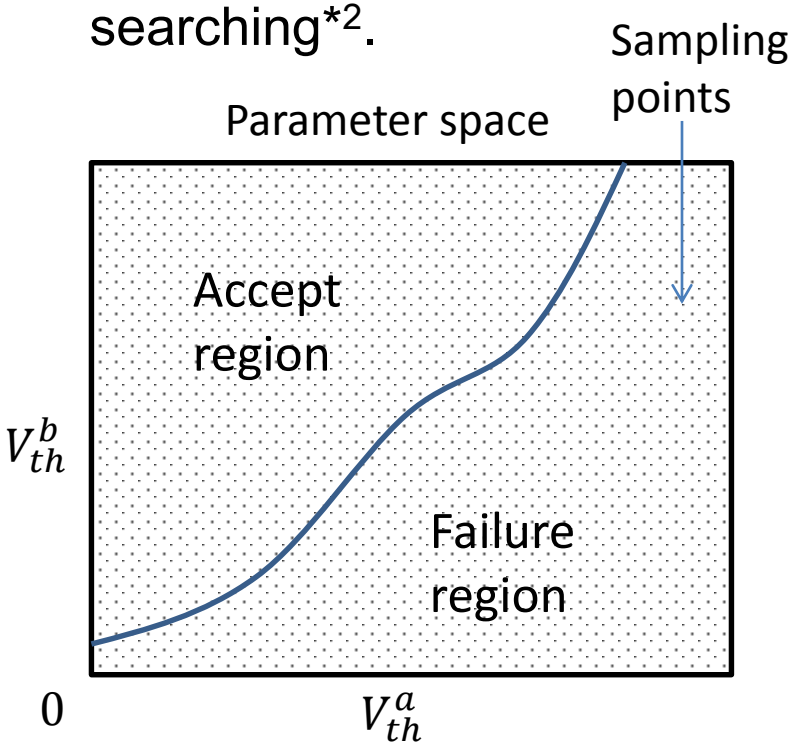
- Threshold variations in M1-4 affect the position of seperatrix.

Previous Work



- Search for points on the boundary of failure region in the parameter space*[1].
- Confined in 2-D space, i.e. only two parameters considered.

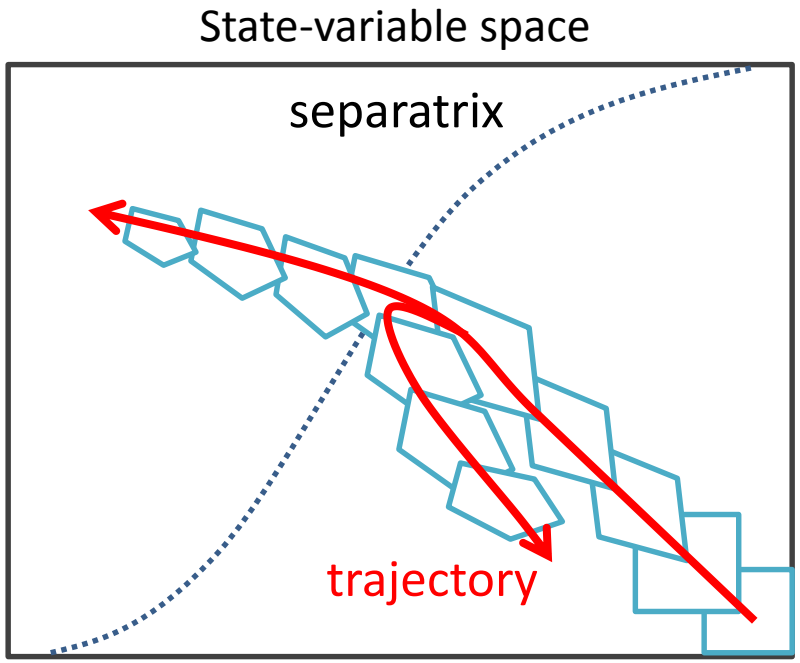
- Statistical method: sampling within the region for searching*[2].



*[1] W. Dong and et.al. *ICCAD*, 2008

*[2] D. E. Khalil and et.al. IEEE Tran. on VLSI, Dec 2008

Motivation

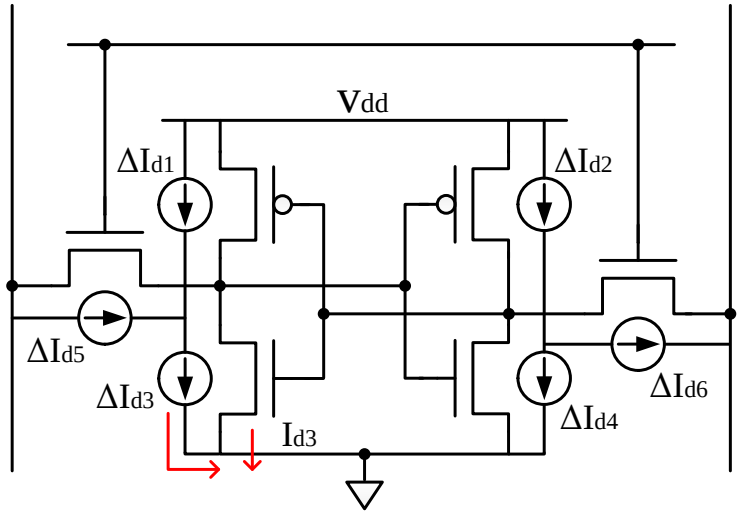


- Fast verification of SRAM nonlinear dynamics by reachability analysis
  - Variations from multiple sources considered at the same time

- Threshold variation is modeled as ad-hoc current source at input

$$I_d + \Delta I_d = \frac{1}{2} k \frac{W}{L} \left[V_{gs} - (V_{th} + \Delta V_{th}) \right]^2$$

$$\Delta I_d \approx -k \frac{W}{L} (V_{gs} - V_{th}) \Delta V_{th}$$



SRAM Nonlinear Dynamics

- One nominal point on the trajectory:

$$\dot{x} = f\big(z(t)\big), \quad z^T(t) = [x^T, u^T].$$

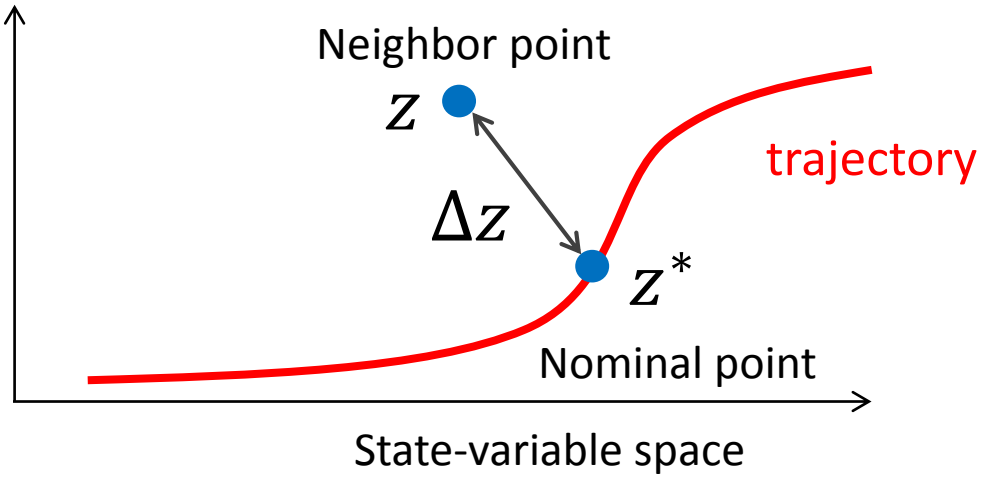
- One operating point in the neighborhood of the nominal point:

$$\dot{x} = f(z^*) + \frac{\partial f}{\partial z}\Big|_{z=z^*}(z - z^*) + \frac{1}{2}(z - z^*)^T \cdot \frac{\partial^2 f}{\partial z^2}\Big|_{z=\xi} \cdot (z - z^*),$$

1st order Taylor term

2nd order Residue

mean value theorem



SRAM Nonlinear Dynamics (cnt'd)

$$\dot{x} = f(z^*) + \frac{\partial f}{\partial z}\Big|_{z=z^*}(z - z^*) + \underline{\frac{1}{2}(z - z^*)^T \cdot \frac{\partial^2 f}{\partial z^2}\Big|_{z=\xi} \cdot (z - z^*)}$$

Linearization error L

$$\dot{x} = f(x^*, u^*) + \frac{\partial f}{\partial x}\Big|_{x=x^*}(x - x^*) + \frac{\partial f}{\partial u}\Big|_{u=u^*}(u - u^*) + L$$

$$\dot{x} = \underline{f(x^*, u^*)} + A(x - x^*) + B(u - u^*) + L$$

$$A = \frac{\partial f}{\partial x}\Big|_{x=x^*}$$

$$B = \frac{\partial f}{\partial u}\Big|_{u=u^*}$$

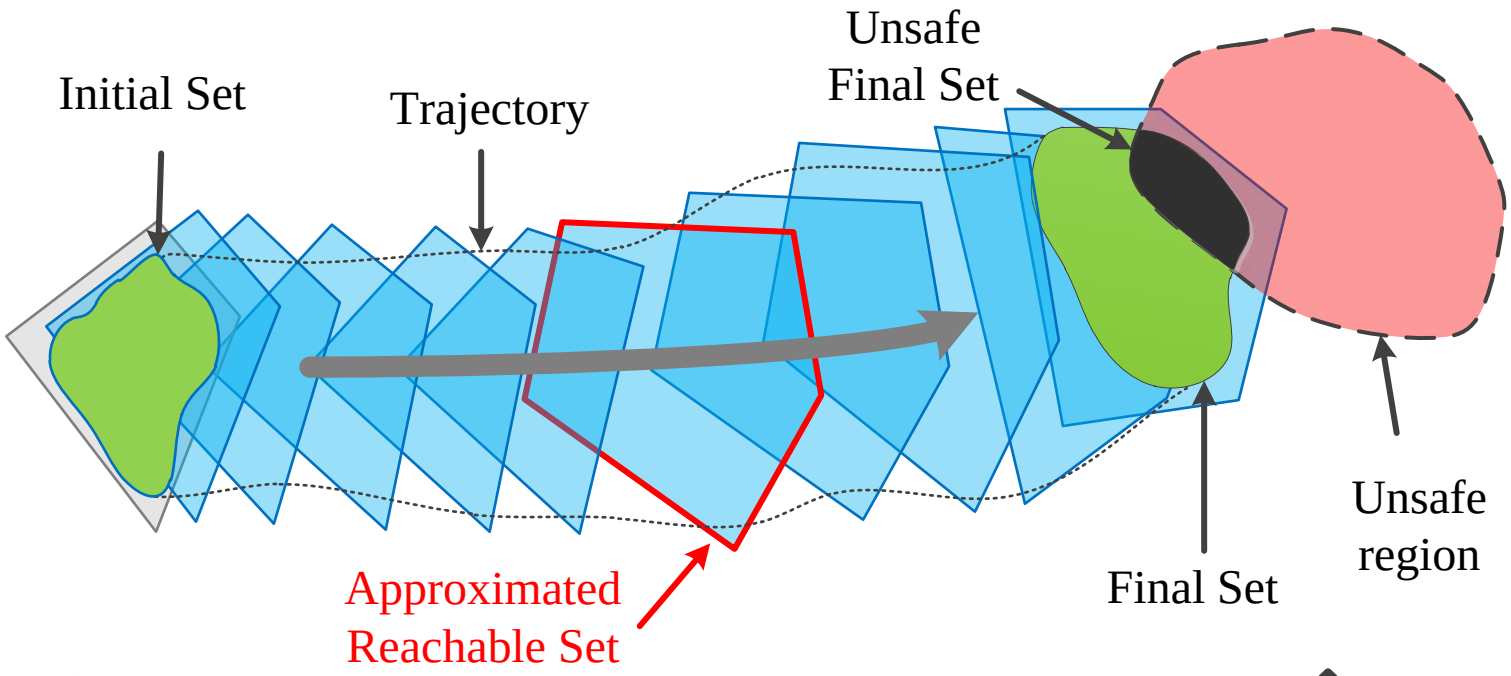
Nonlinear dynamics of nominal point

Nonlinear dynamics for nominal point

$$\begin{cases} \dot{x^*} = f(x^*, u^*) \\ (\dot{x - x^*}) = A(x - x^*) + B(u - u^*) + L \end{cases}$$

Linear dynamics for distance from nominal point

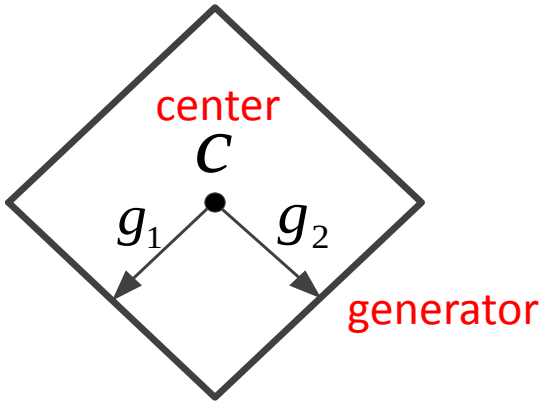
Reachability Analysis: Zonotope



Zonotope

Set of points in n-dimensional polygon with generator g_i

$$z = c + \sum_{i=1}^{e} \beta_i g_i \ , -1 \leq \beta_i \leq 1$$



Reachability Analysis: Linear Multi-Step

$$\dot{x^*} = f(x^*, u^*)$$ Transient simulation

$$(\dot{x} - \dot{x^*}) = A(x - x^*) + B(u - u^*) + L$$

Calculation of generators

$$\dot{G} = AG \oplus BU \oplus L,$$
$$G = [g_1, g_2, \ldots, g_n]$$

Generator matrix

Backward Euler with h time-step

$$\dot{G_k} = (I - hA)^{-1} G_{k-1} \oplus hBU_k \oplus hL_k$$

Initial state solution — Input solution — Linearization error

$$x = x^* + \sum_{i=1}^{n} \beta_i g_i \ , -1 \le \beta_i \le 1$$

Zonotope for state vector

$$u = u^* + \sum_{i=1}^{m} \alpha_i \overrightarrow{\Delta I_d^i}, -1 \le \alpha_i \le 1$$

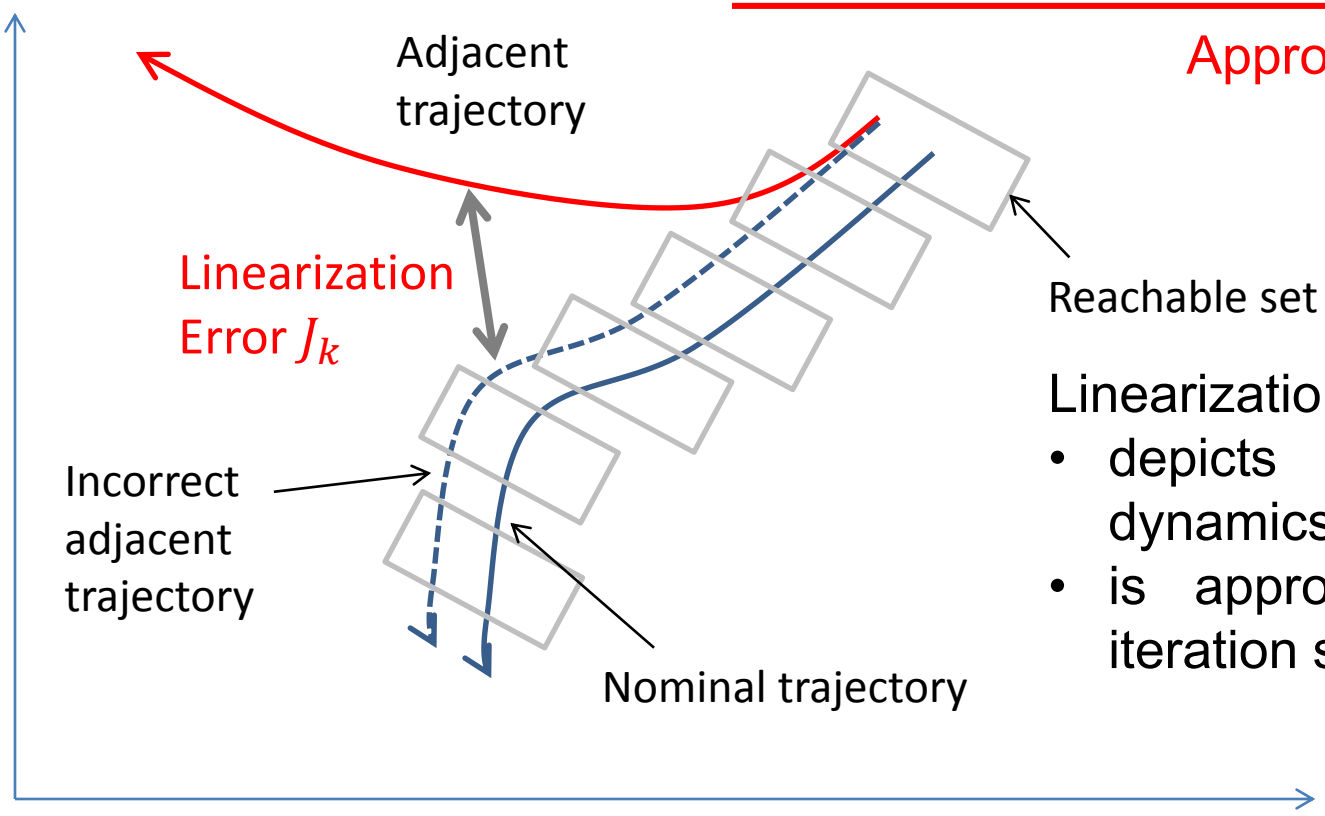
Zonotope for noise current vector with given variation range

Linearization Error

$$J_k = \frac{1}{2}(z - z^*)^T \cdot \frac{\partial^2 f_k}{\partial z^2}|_{z=\xi} \cdot (z - z^*)$$

$$L_k = \frac{1}{2} max|z - z^*|^T \cdot \max\left|\frac{\partial^2 f_k}{\partial z^2}|_{z=\xi}\right| \cdot max|z - z^*|$$

Approximation

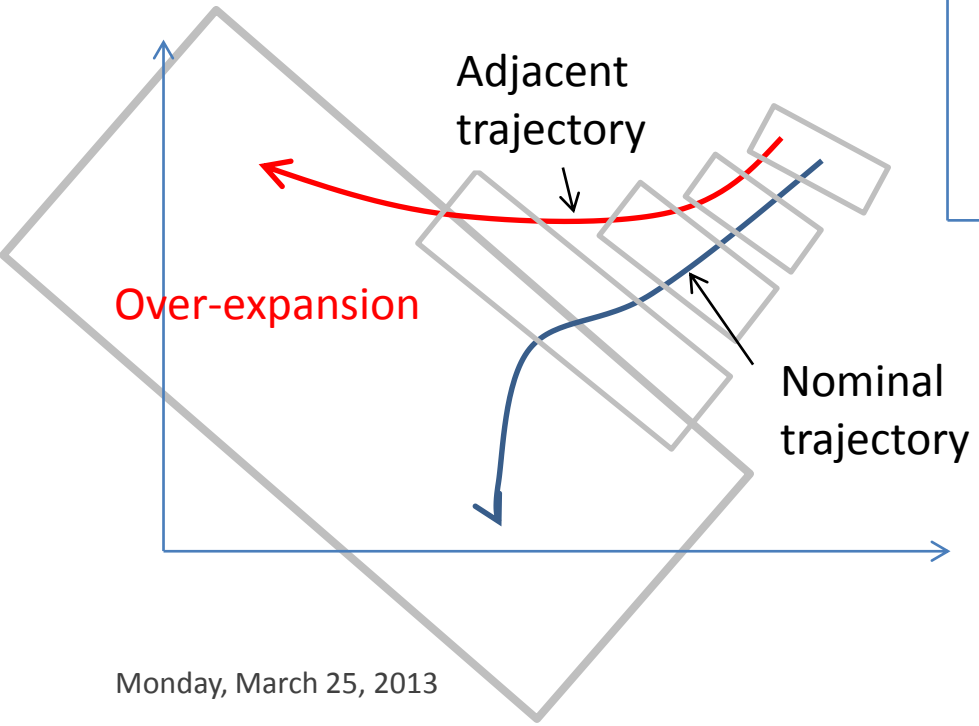


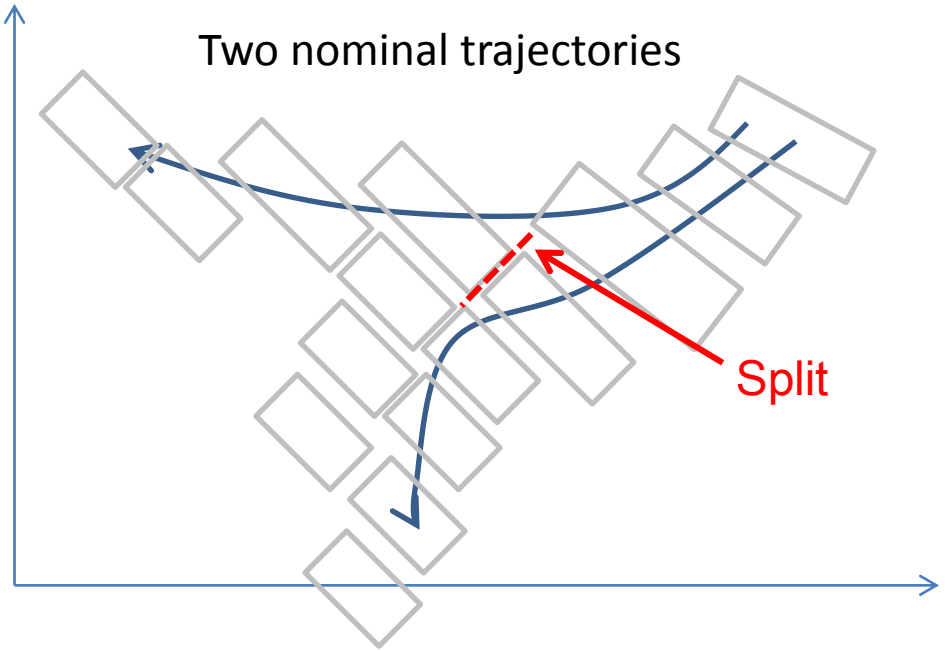
Linearization error

- depicts the nonlinear dynamics.
- is approximated at each iteration step.

Reachable Set Refinement

- Reachable set is over-expanded without refinement.
- Zonotope is split into smaller ones confine linearization error in each set.





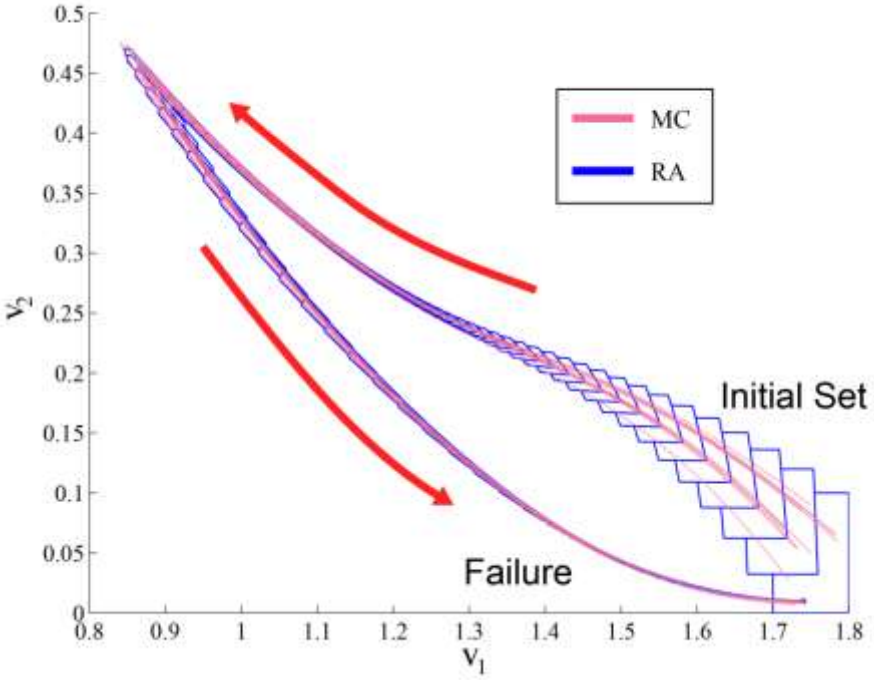
if $\mathbf{IH}(hL_k) \nsubseteq [-\varepsilon, \varepsilon]$,
$\Rightarrow$ *split set*

- A new trajectory is created.

Experimental Results

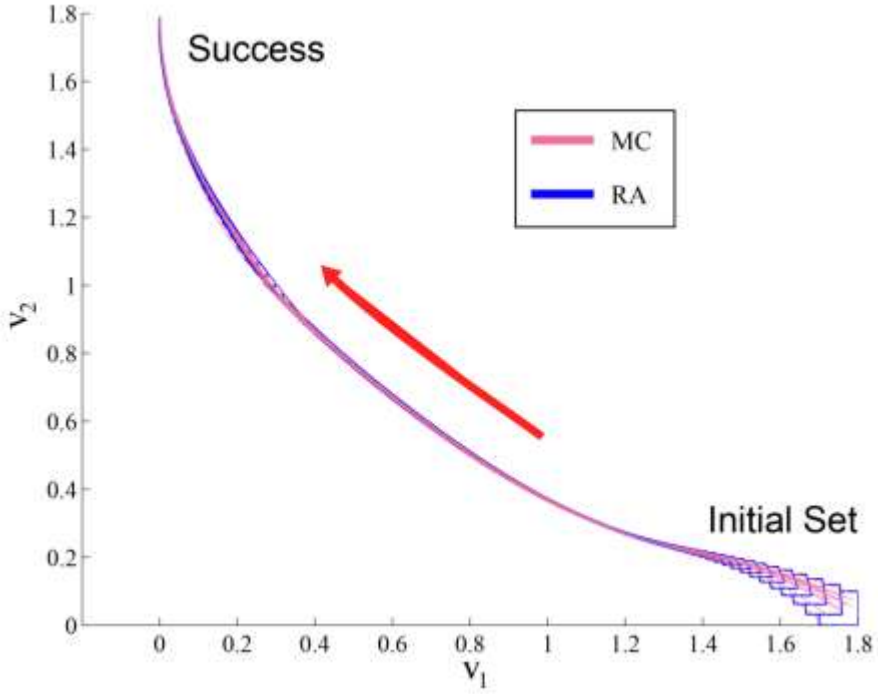
- Implemented in Matlab 7.12 and C
- Platform
  - Core i5 3.2GHz processor
  - 8GB memory
- Simulation parameters of SRAM
  - Initial state $v_1 \in [1.7,1.8], v_2 \in [0,0.1]$
  - Variation range $\Delta I_d = \delta k \frac{W}{L} \left(V_{gs} - V_{th}\right)V_{th}, \delta = 1\%, 5\%, 10\%$

Write Operation



- Write pulse is set as 50ns with relative threshold variation of 5% in each transistor.
- Write operation fails.

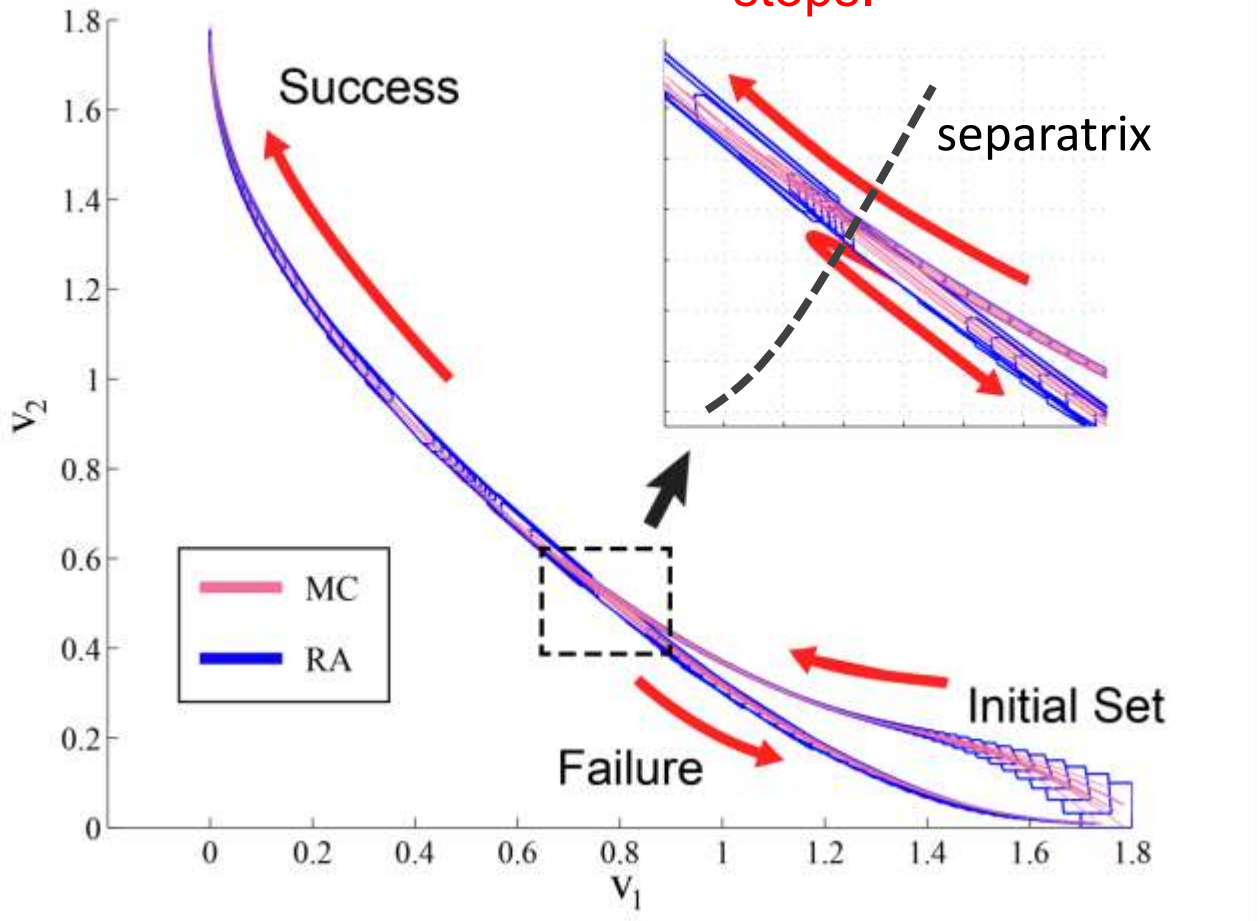
- Write pulse is set as 70ns.
- Write operation succeeds.



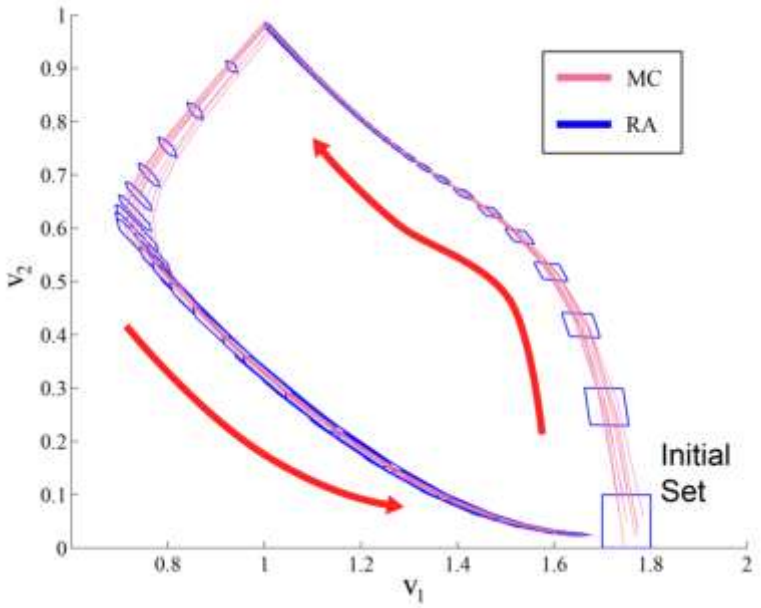
Write Operation

- Write pulse is set as 60ns.
- Write operation fails.

Trajectory splits near separatrix after write pulse stops.

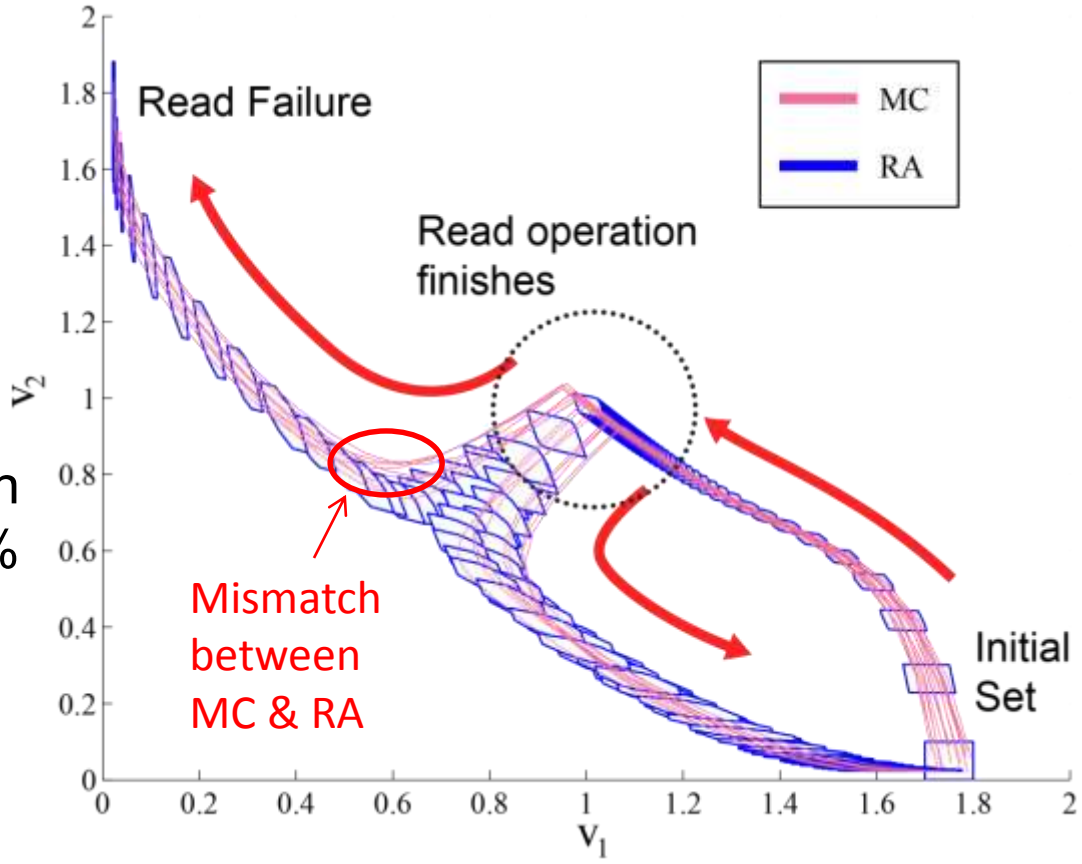


Read Operation



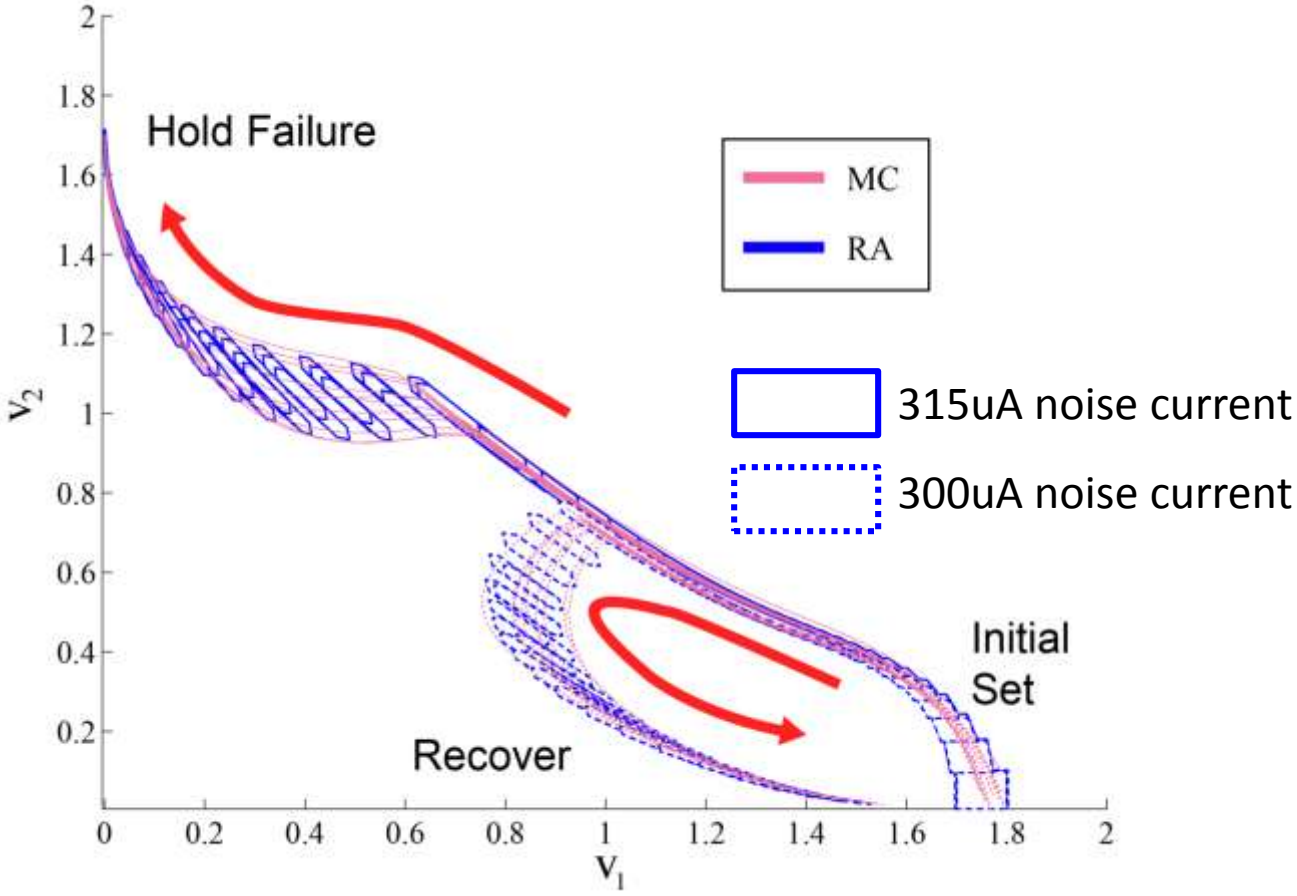
- Relative threshold variation in each transistor is set as 1% with 50ns read pulse.
- Read operation succeeds.

- Relative threshold variation in each transistor is set as 5%.
- Read operation fails.



Hold Operation

- Injected noise current lasts 12.5ns.
- Relative threshold variation in each transistor is set as 5%



Performance

Pulse(ns)	Threshold Variation	Reachability Analysis (s)	Monte Carlo (s)	Speedup
50	1%	12.71	5635.57	443.38X
	5%	13.13	5817.71	443.01X
	10%	12.63	6078.68	481.24X
60	1%	52.70	6224.09	118.09X
	5%	52.58	6535.35	124.29X
	10%	52.68	6387.28	121.25X
70	1%	13.72	5931.76	432.32X
	5%	14.43	6245.45	432.73X
	10%	13.21	6348.54	480.45X

Monte- Carlo setup with 500 samples are considered

Summary

- Introduced SRAM failure mechanisms in the state space.
- Presented reachability analysis for nonlinear continuous systems.
- Proposed reachability-based verification for SRAMs with consideration of threshold voltage variations.
- Reachability verification for SRAMs achieved good speed and precision, and can be extended for optimization

Thank you!

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