Synthesizing a Representative Critical Path for Post-Silicon Delay Prediction

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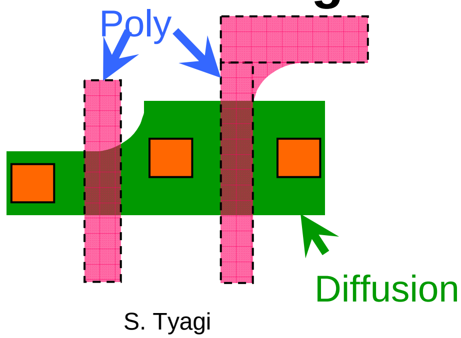
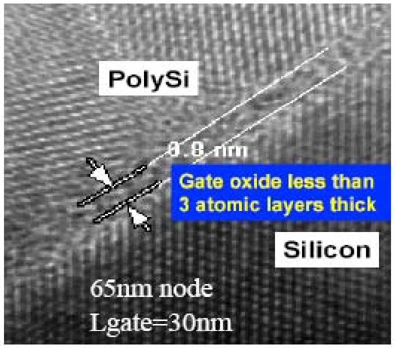
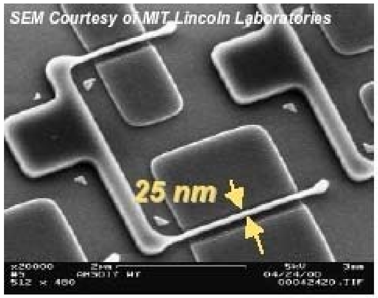
Variations in Digital Circuits

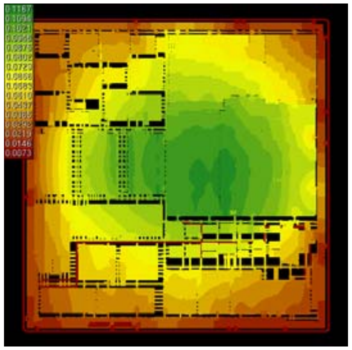
- **Variations in nanoscale technologies**





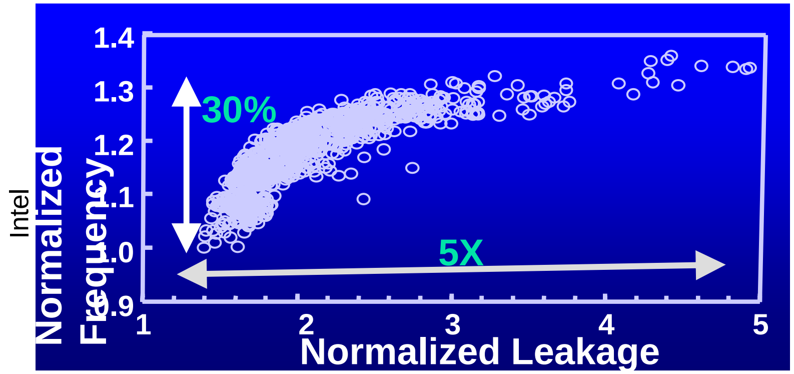


25-nm Transistors Produced with 248-nm Lithography

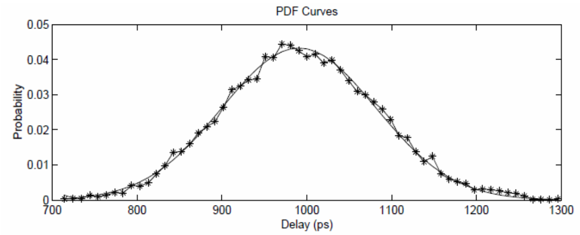


  - **Process: across-die/within-die**
  - **Environment: T, V_{dd}**

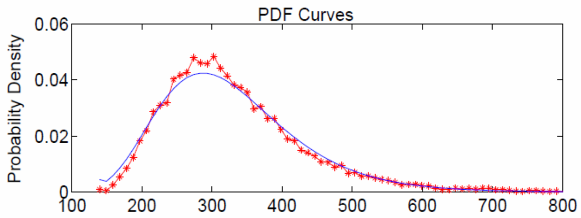
- **These lead to circuit performance deviations**



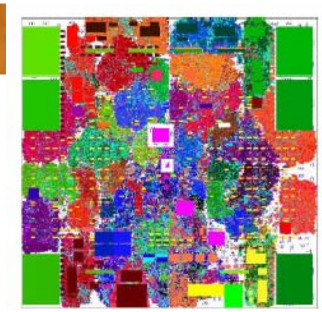
- **Timing PDF**



- **Power PDF**



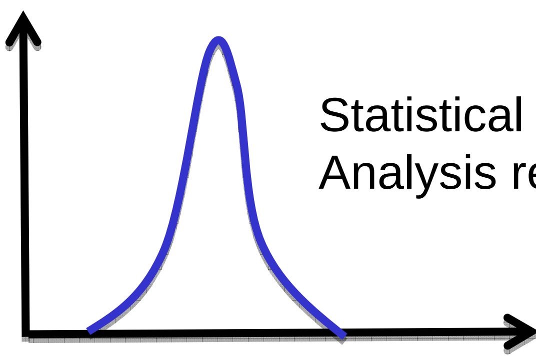
Design Phase: Pre-Silicon

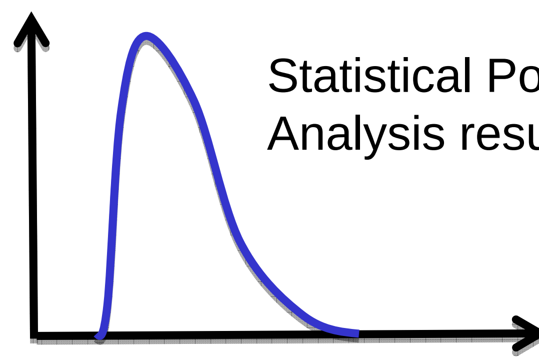


Pre-Silicon Analysis ⇄ Pre-Silicon Optimization

Deterministic/Statistical
Timing/Power Analysis
…

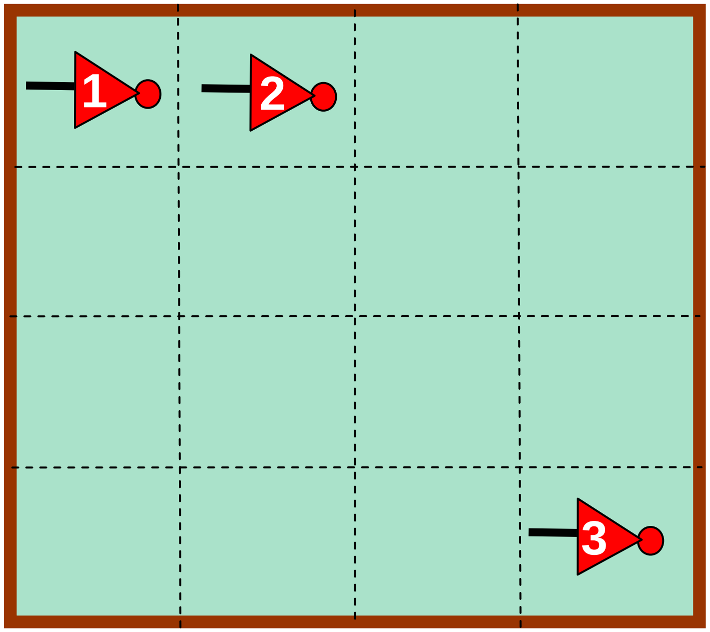
Synthesis
Gate Sizing
…





Statistical Static Timing Analysis (SSTA)

- **Spatial correlation**



- **Canonical form** $d_c = \mu_c + \mathbf{a}^T \mathbf{p} + R_c \qquad \mathbf{p} \sim N(\mathbf{0}, \mathbf{I})$

Design Phase: Post-Silicon

Post-Silicon Analysis

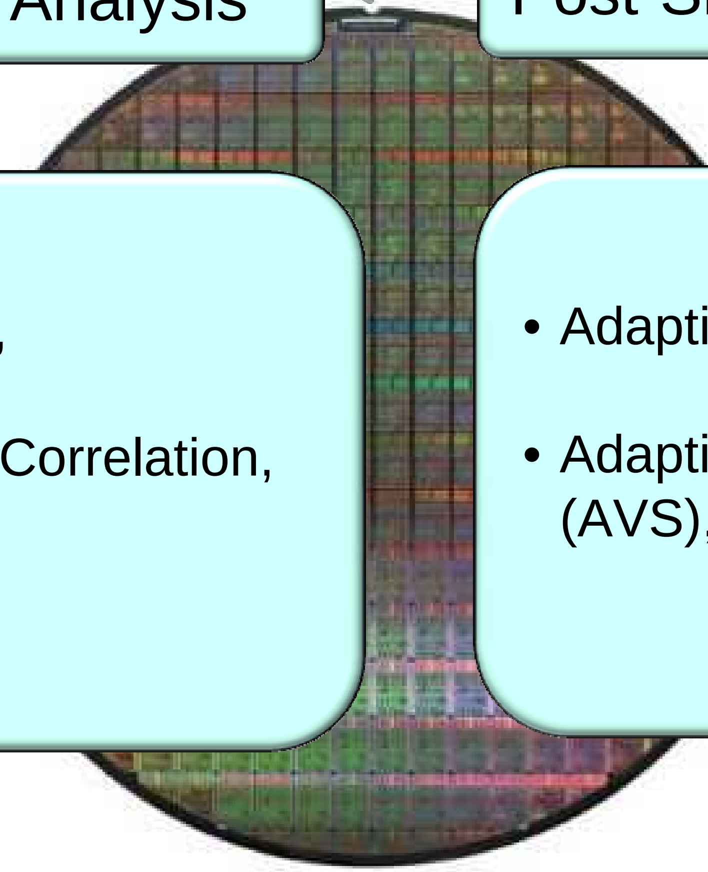
- Delay Analysis,
- Design-Silicon Correlation,

…

Post-Silicon Optimization

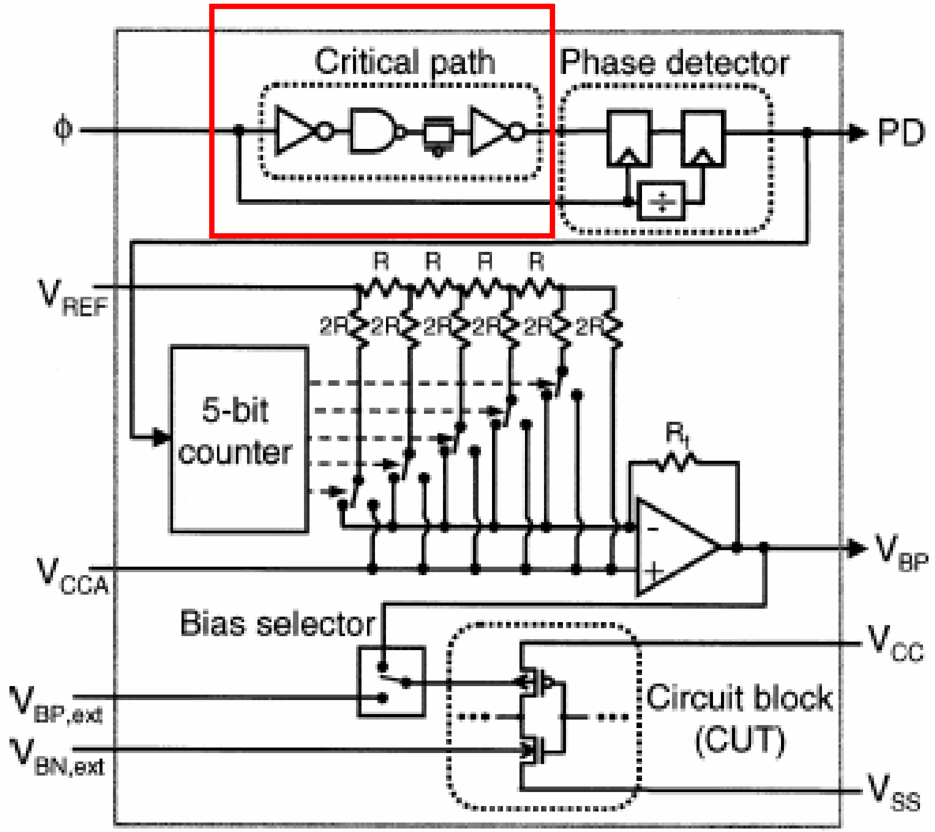
- Adaptive Body Bias(ABB),
- Adaptive Voltage Scaling (AVS),

…



[amd.com]

Adaptive Body Bias (ABB)



[Tschanz *et al.*, JSSC02]

Limitations of Critical Path Replica (CPR)

- Only the **nominal** critical path is replicated
- Numerous near-critical paths in modern VLSI circuits
- Nominal critical path not necessarily critical in the manufactured die (process variations)

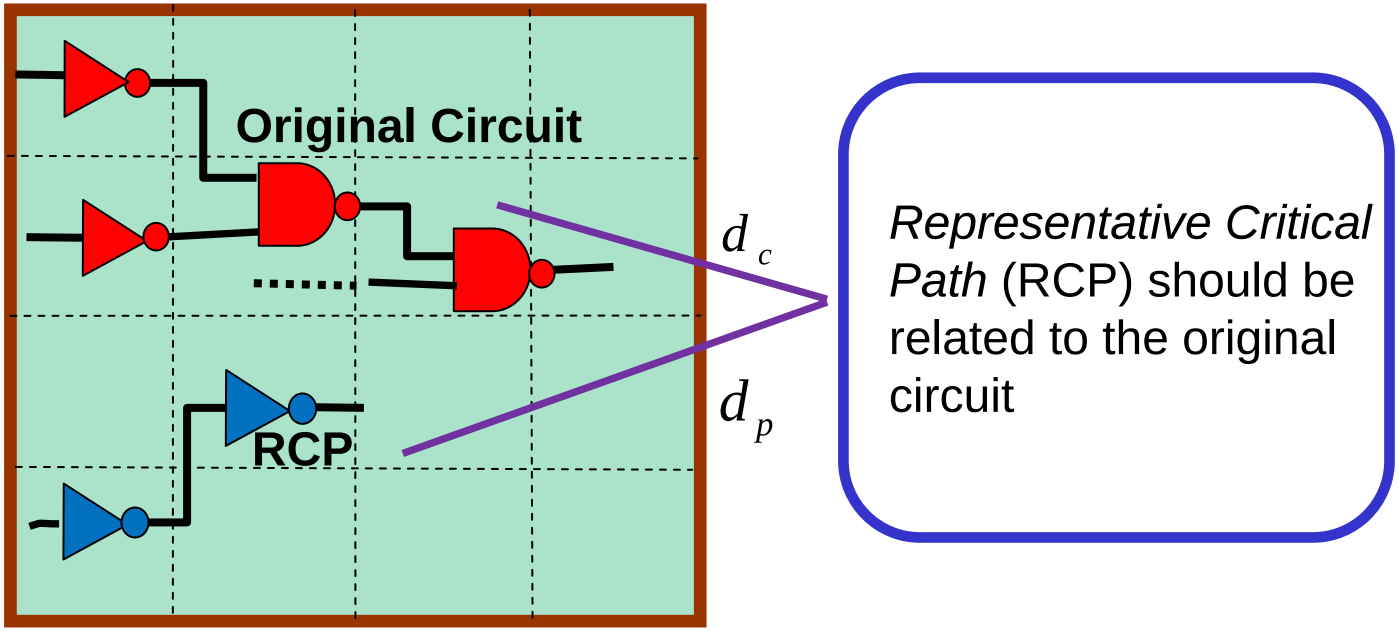
- Representative Critical Path (RCP)
    - Always predicts the worst case delay

Outline

- Problem Formulation
- Representative Critical Path Synthesis
- Experimental Results

Problem Formulation

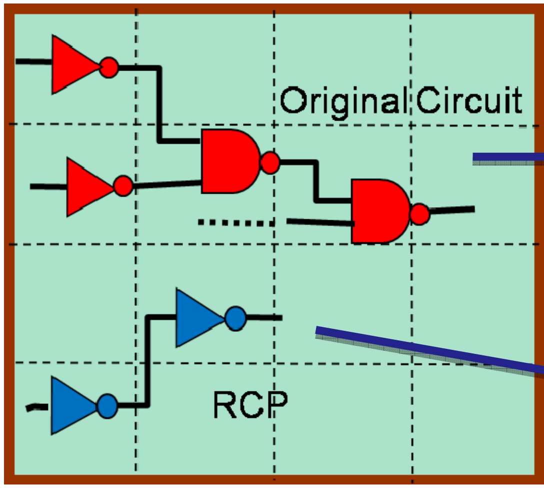
Circuit with Gaussian process parameter variations



Build the RCP to reveal most information about the original circuit.

Mathematical Formulation

From SSTA (Pre-Silicon)



$$d_c = \mu_c + \mathbf{a}^T\mathbf{p} + R_c$$
$$\sigma_c^2 = \mathbf{a}^T\mathbf{a} + \sigma_{R_c}^2$$

$$d_p = \mu_p + \mathbf{b}^T\mathbf{p} + R_p$$
$$\sigma_p^2 = \mathbf{b}^T\mathbf{b} + \sigma_{R_p}^2$$

$$\begin{bmatrix} d_c \\ d_p \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_c \\ \mu_p \end{bmatrix}, \begin{bmatrix} \sigma_c^2 & \mathbf{a}^T\mathbf{b} \\ \mathbf{a}^T\mathbf{b} & \sigma_p^2 \end{bmatrix}\right)$$

Our goal: d_c

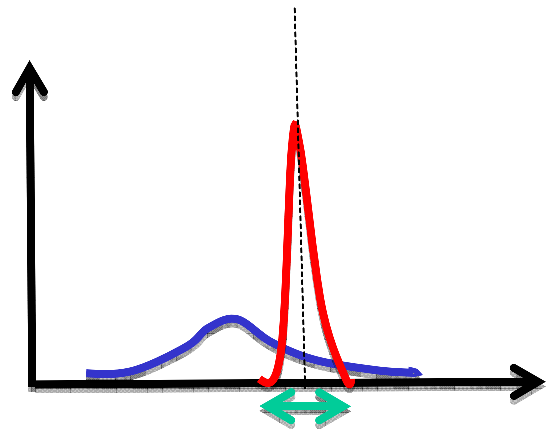
From measurement data (Post-Silicon) $d_p = d_{pr}$

Conditional PDF

$$d_c \mid (d_p = d_{pr}) \sim N(\bar{\mu}, \bar{\sigma}^2)$$

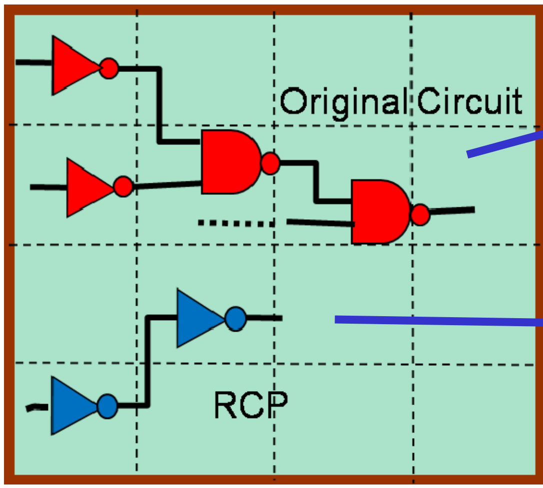
$$\bar{\mu} = \mu_c + \frac{\mathbf{a}^T\mathbf{b}}{\sigma_p}(d_{pr} - \mu_p)$$
$$\bar{\sigma}^2 = \sigma_c^2\left(1 - \frac{\mathbf{a}^T\mathbf{b}}{\sigma_c\sigma_p}\right)$$

ρ Maximum



Minimum

Full Correlated Case



$$d_c = \mu_c + \mathbf{a}^T\mathbf{p} + R_c$$

$$d_p = \mu_p + \mathbf{b}^T\mathbf{p} + R_p$$

Fully Correlated

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \Lambda = \frac{a_n}{b_n} = k$$

$$d_c - \mu_c = k\left(d_p - \mu_p\right)$$

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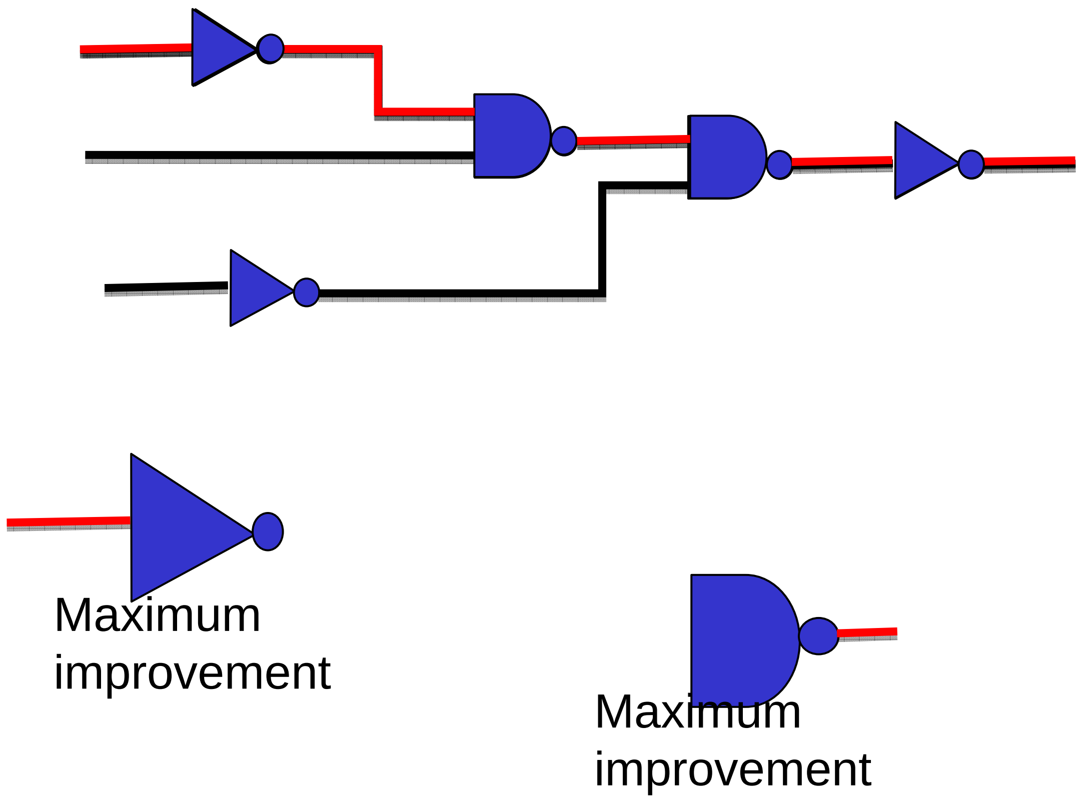
- Experimental Results

Representative Critical Path (RCP) Synthesis (Method I)



$$d_c = \mu_c + \mathbf{a}^T \mathbf{p} + R_c$$

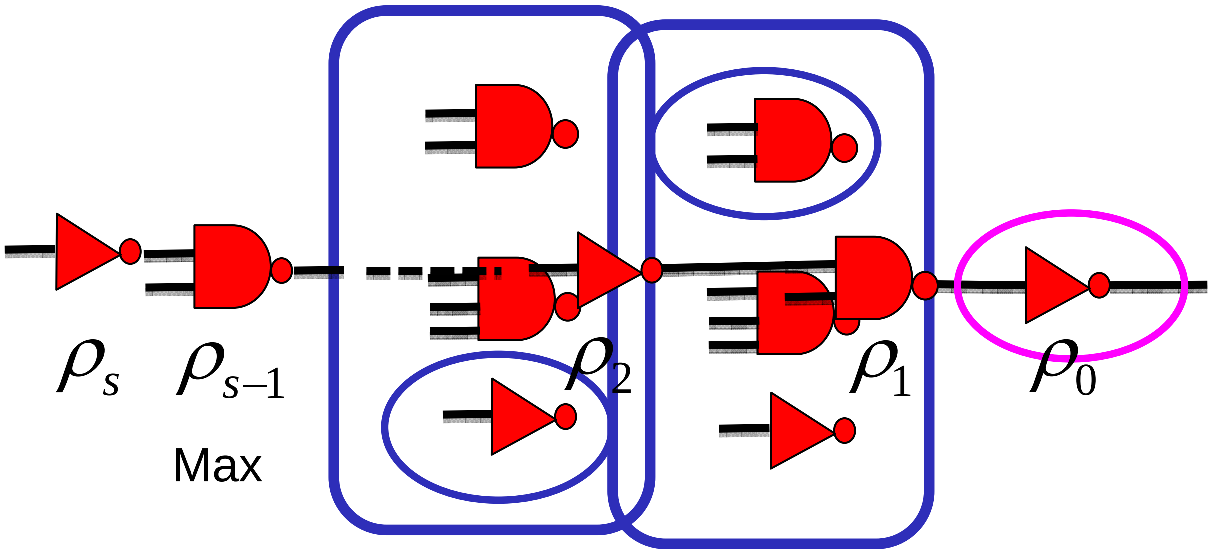
$$d_p = \mu_p + \mathbf{b}_2^T \mathbf{p} + R_p$$

$$\rho = \frac{\mathbf{a}^T \mathbf{b}_2}{\sigma_c \sqrt{\mathbf{b}_2^T \mathbf{b}_2 + \sigma_{R_p}^2}}$$

Comments on Method I

- **Advantage**
  - **Guaranteed to do no worse than CPR**
  - **Exact solution when there is clearly one dominating path**
- **Drawback**
  - **Flexibility of the solution is limited**
- **Runtime: O(Ks)**
  - **Saved by only updating the SSTA results of stages adjacent to the one sized up**
  - **K: number of iterations**
  - **s:number of stages**

RCP Generation (Method II)



$$d_c = \mu_c + \mathbf{a}^T \mathbf{p} + R_c$$

$$d_p = \mu_p + \mathbf{b}_1^T \mathbf{p} + R_p$$

$$\rho = \frac{\mathbf{a}^T \mathbf{b}_1}{\sigma_c \sqrt{\mathbf{b}_1^T \mathbf{b}_1 + \sigma_{R_p}^2}}$$

Comments on Method II

- **Advantage**
  - **More flexibility, not tied to a specific path**
- **Drawbacks**
  - **No exact solution when there is only one dominating path**
  - **Not guaranteed to be always better than CPR**
- **Runtime: O(kcs)**
  - **k: number of starting locations**
  - **c: number of choices for each iteration**
  - **s: maximum number of stages**

Outline

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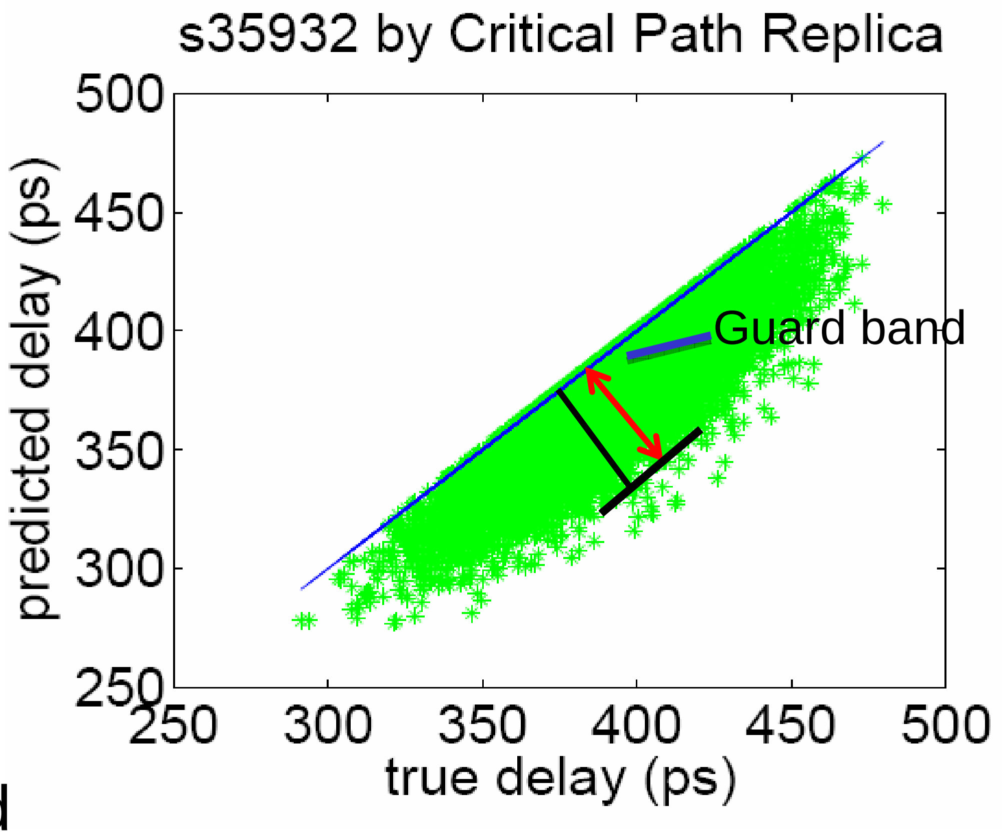
Comparison Metric

- Average error, maximum error w.s.t. Monte-Carlo analysis

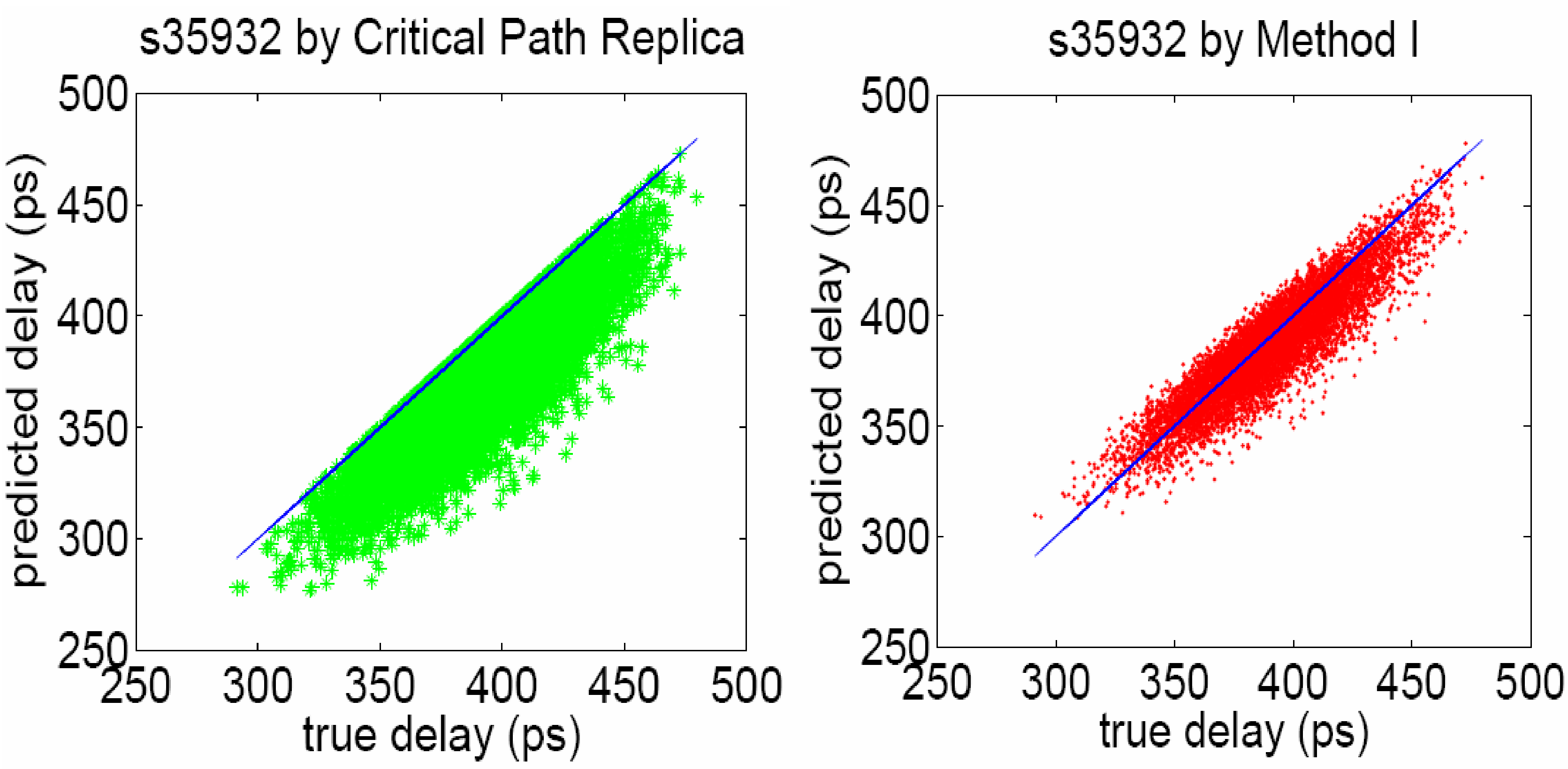


- Guard band

Experimental Results (Method I)

Circuit	Average Error		Maximum Error		Guard Band (ps)	
	Method I	CPR	Method I	CPR	Method I	CPR
s9234	1.59%	2.84%	10.51%	15.20%	28.5	44.1
s13207	0.59%	1.07%	6.30%	7.33%	20.3	28.6
s15850	1.16%	2.13%	8.99%	11.52%	39.0	56.9
s35932	2.35%	5.77%	13.72%	20.83%	33.7	59.1
s38584	1.98%	3.26%	14.70%	17.66%	48.9	74.2
s38417	2.80%	5.24%	15.80%	21.32%	53.2	84.1

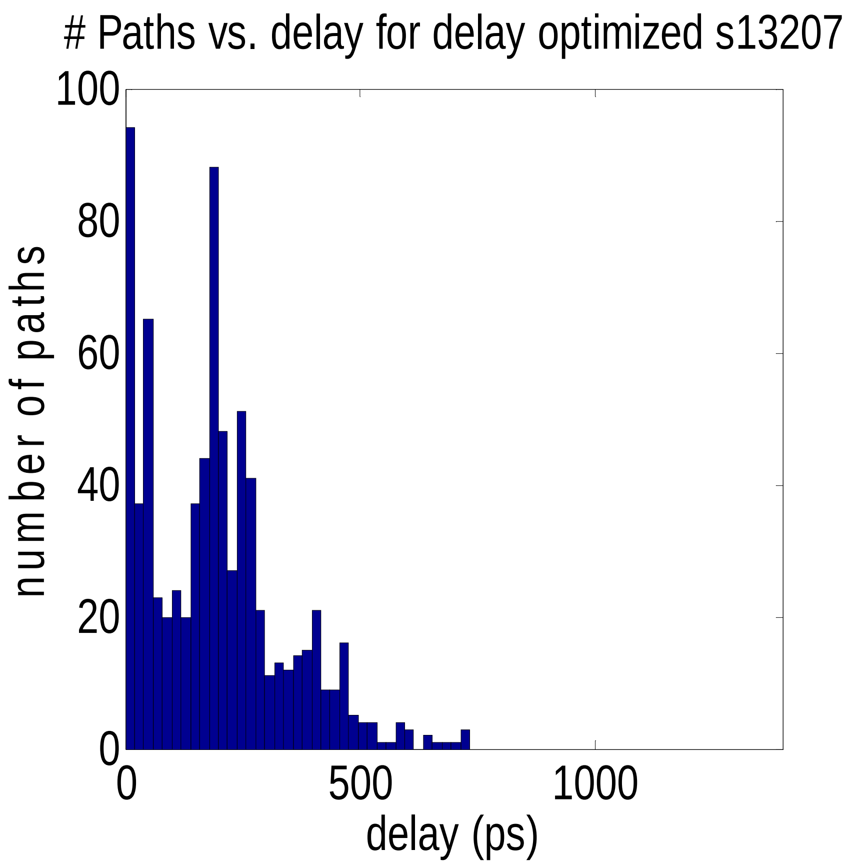
Scatter Plots (Method I)

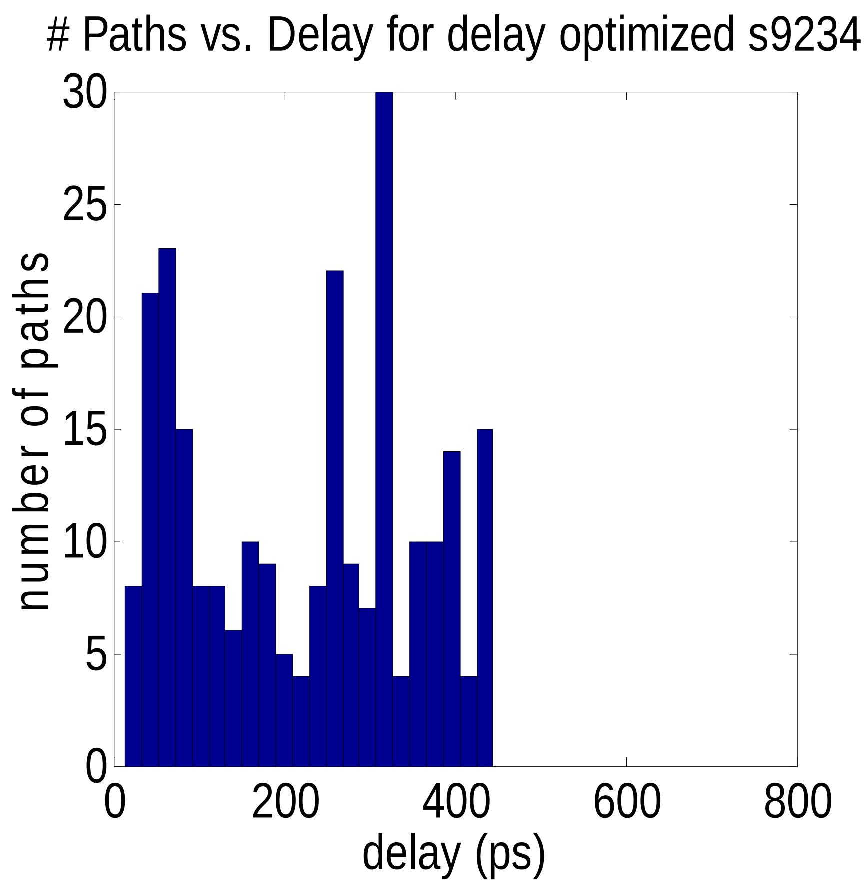


Experimental Results (Method II)

Circuit	Average Error			Maximum Error			Guard Band (ps)		
	Method I	Method II	CPR	Method I	Method II	CPR	Method I	Method II	CPR
s9234	1.59%	1.98%	2.84%	10.51%	10.57%	15.20%	28.5	31.4	44.1
s13207	0.59%	1.51%	1.07%	6.30%	8.51%	7.33%	20.3	35.3	28.6
s15850	1.16%	1.73%	2.13%	8.99%	9.22%	11.52%	39.0	45.4	56.9
s35932	2.35%	2.27%	5.77%	13.72%	13.91%	20.83%	33.7	32.3	59.1
s38584	1.98%	2.11%	3.26%	14.70%	10.89%	17.66%	48.9	43.0	74.2
s38417	2.80%	2.28%	5.24%	15.80%	12.01%	21.32%	53.2	42.4	84.1

Number of Paths vs. Delay





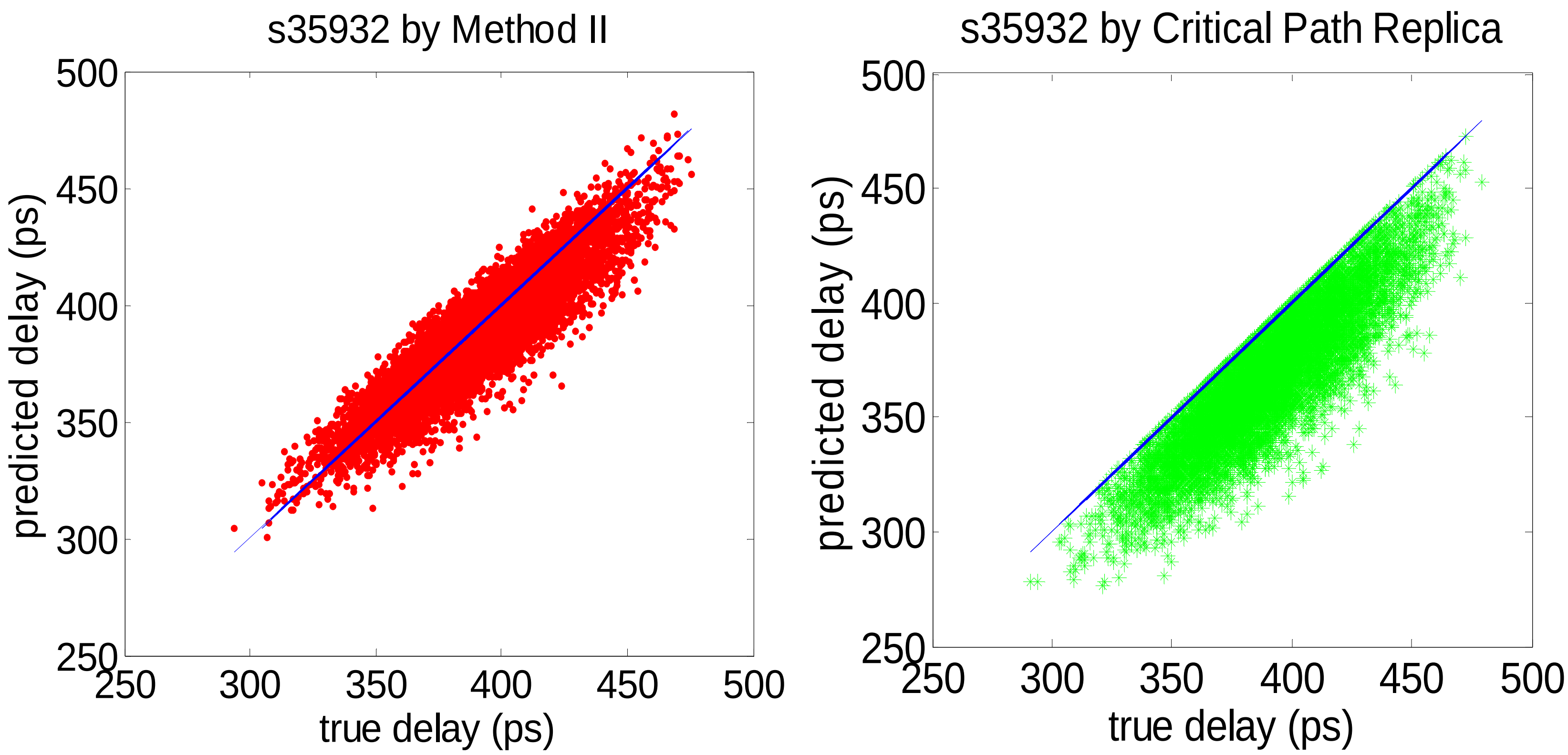
Conclusion and Future Work

- **Two novel methods for synthesizing a representative critical path under process variations are presented**
- **Average prediction error: below 2.8%**
- **To ensure 99% of the predictions pessimistic, requiring guard band 30% smaller than CPR**
- **Future work**
  - **Test on real silicon**

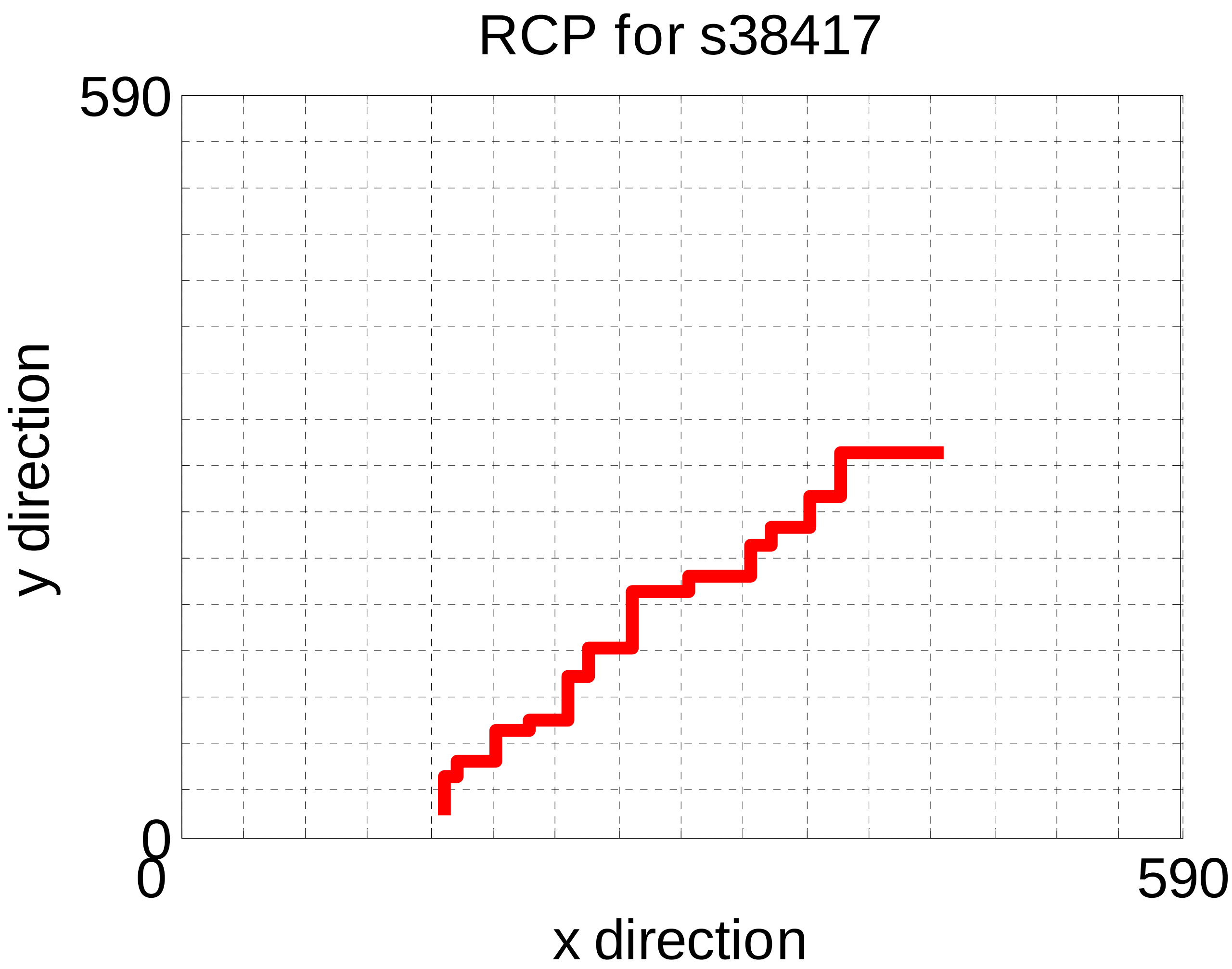
Thank you!

Extra Slides

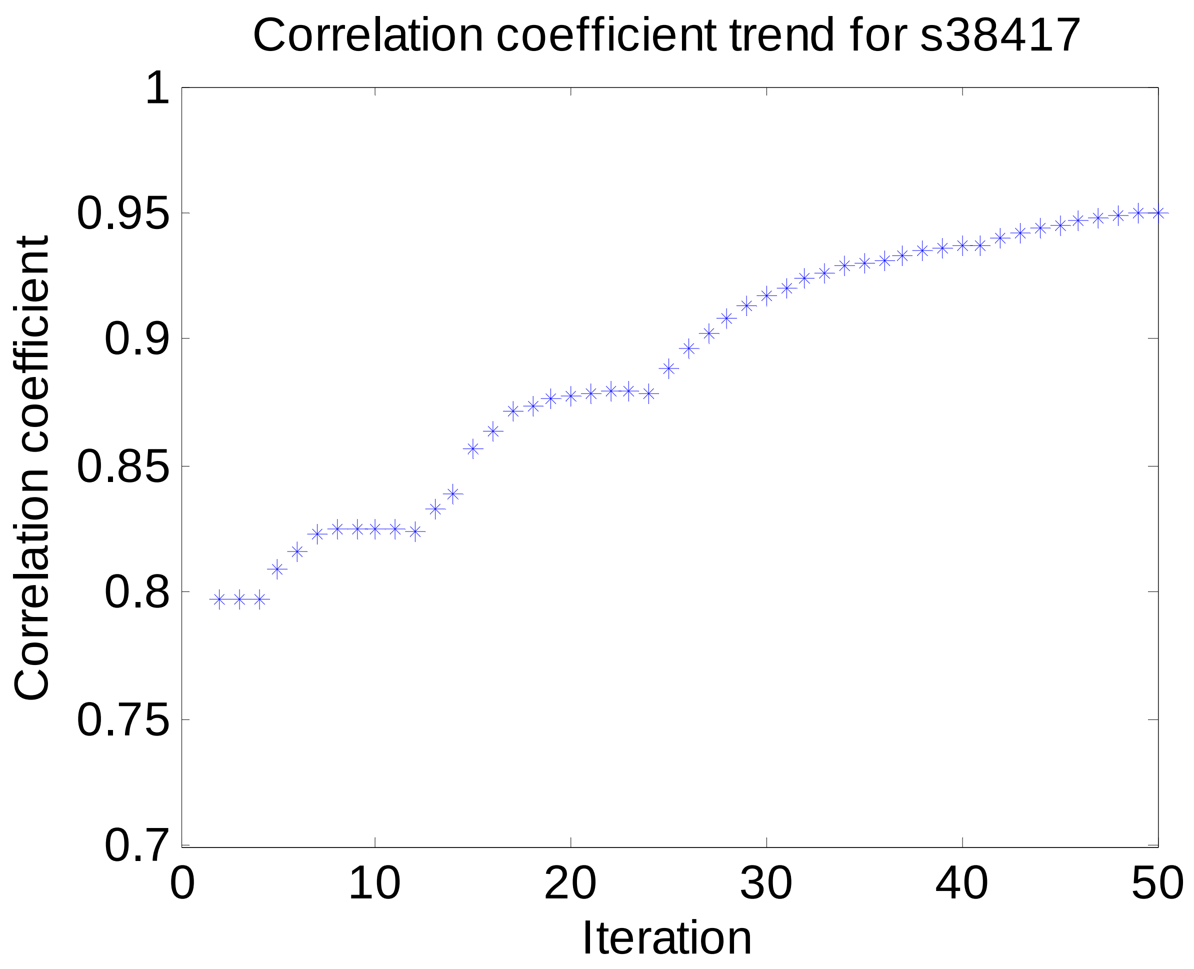
Scatter Plots (Method II)



An Example RCP (Method II)



ρ vs. iteration number (Method II)



Equations for Editing

$$\begin{bmatrix} d_c \\ d_p \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_c \\ \mu_p \end{bmatrix}, \begin{bmatrix} \sigma_c^2 & \mathbf{a}^T\mathbf{b} \\ \mathbf{a}^T\mathbf{b} & \sigma_p^2 \end{bmatrix} \right)$$

$$d_c = \mu_c + \mathbf{a}^T\mathbf{p} + R_c$$

$$\sigma_c^2 = \mathbf{a}^T\mathbf{a} + \sigma_{R_c}^2$$

$$d_p = \mu_p + \mathbf{b}^T\mathbf{p} + R_p$$

$$\sigma_p^2 = \mathbf{b}^T\mathbf{b} + \sigma_{R_p}^2$$

$$d_c$$

$$d_p = d_{pr}$$

$$d_c \mid \left(d_p = d_{pr}\right) \sim N\left(\overline{\mu}, \overline{\sigma}^2\right)$$

$$\overline{\mu} = \mu_c + \frac{\mathbf{a}^T\mathbf{b}}{\sigma_p}\left(d_{pr} - \mu_p\right)$$

$$\overline{\sigma}^2 = \sigma_c^2\left(1 - \frac{\mathbf{a}^T\mathbf{b}}{\sigma_c\sigma_p}\right)$$

$$d_p = \mu_p + \mathbf{b}_3^T\mathbf{p} + R_p$$

$$\rho = \frac{\mathbf{a}^T\mathbf{b}_1}{\sigma_c\sqrt{\mathbf{b}_1^2\mathbf{b}_1 + \sigma_{R_p}^2}}$$

Parameter Variations

Parameter Variations

	L	W	W_{int}	T_{int}	H_{ILD}
Mean	60.0	150.0	150.0	500.0	300.0
3σ	12.0	22.5	30.0	75.0	45.0

Use the Grid-Based Correlation Model ([Chang, ICCAD03])