Stochastic Analog Circuit Behavior Modeling by Point Estimation Method

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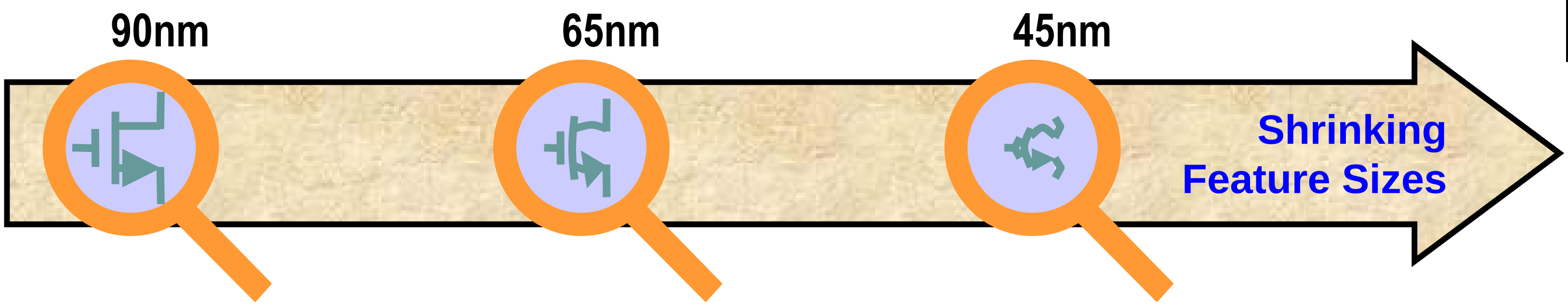
Outline

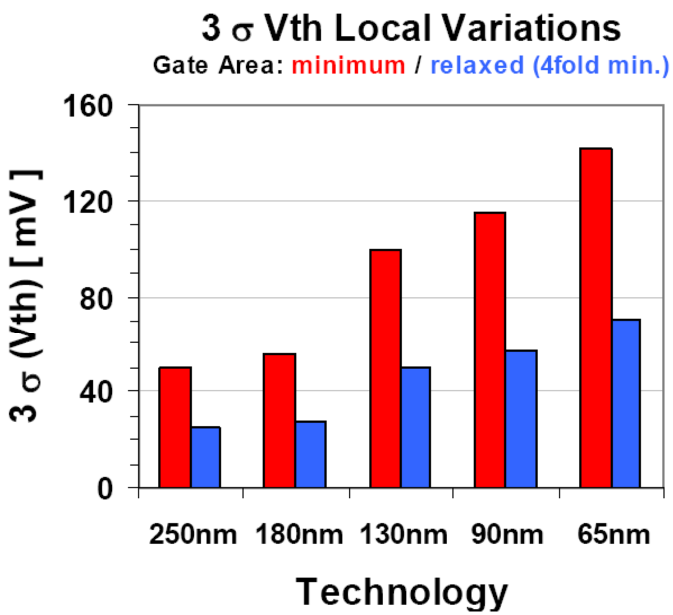
- Backgrounds
- Existing Methods and Limitations
- Proposed Algorithms
- Experimental Results
- Conclusions

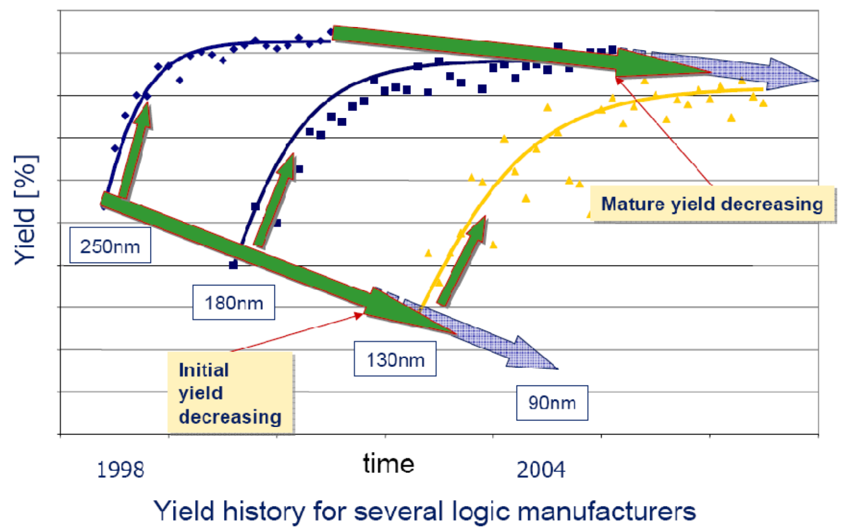
IC Technology Scaling

- Feature size keeps scaling down to 45nm and below



- Large process variation lead to ***circuit failures*** and ***yield problem***.

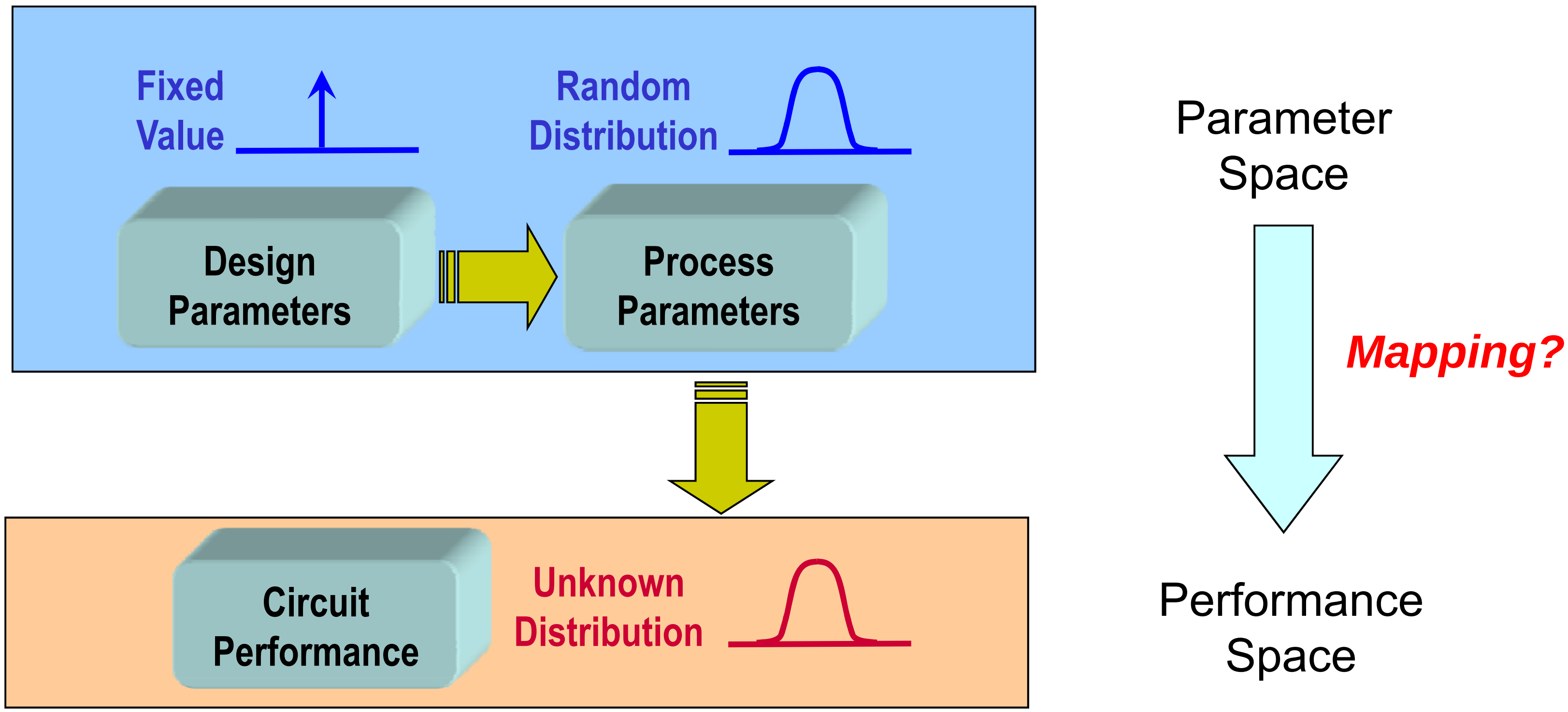




* Data Source: Dr. Ralf Sommer, DATE 2006, COM BTS DAT DF AMF;

Statistical Problems in IC Technology

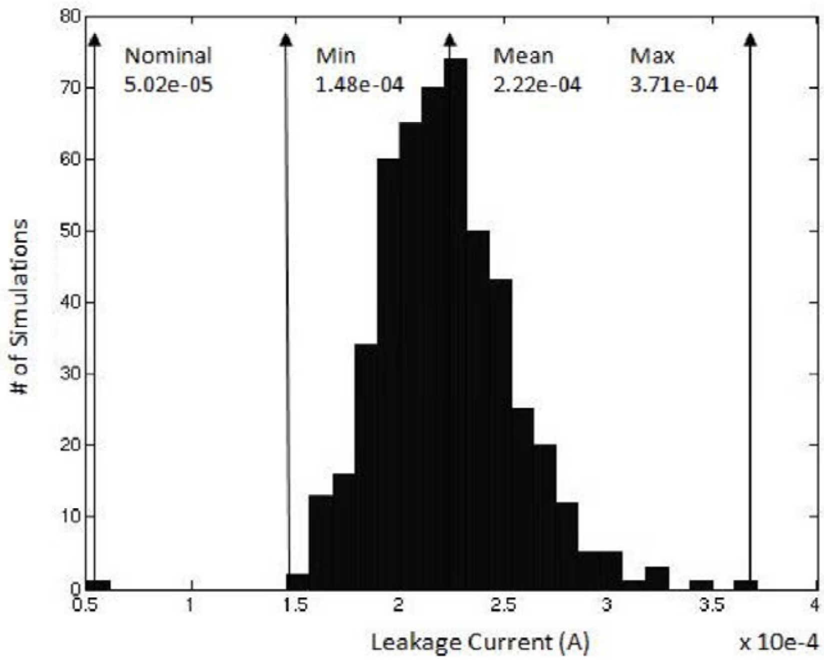
- Statistical methods were proposed to address variation problems
- Focus on performance probability distribution extraction in this work



How to model the stochastic circuit behavior (performance)?

Leakage Power Distribution

- An example ISCAS-85 benchmark circuit:
  - all threshold voltages (Vth) of MOSFETs have variations that follow ***Normal distribution***.
- The leakage power distribution follow ***lognormal*** distribution.



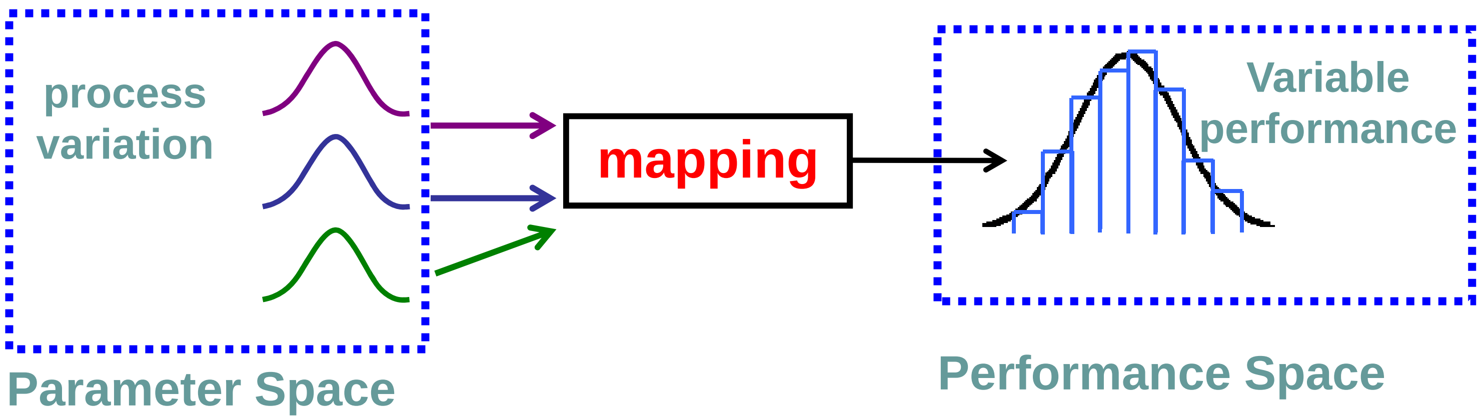
*Courtesy by Fernandes, R.; Vemuri, R.; , *ICCD 2009.* pp.451-458, 4-7 Oct. 2009

- It is desired to extract the arbitrary (usually non-normal) distribution of performance exactly.

Problem Formulation

- ***Given***: random variables in **parameter space**
  - a set of (normal) random variables $\{\varepsilon_1, \varepsilon_2, \varepsilon_3, ...\}$ to model process variation sources.

- ***Goal***: extract the arbitrary probability distribution of performance $f(\varepsilon_1, \varepsilon_2, \varepsilon_3, ...)$ in **performance space**.



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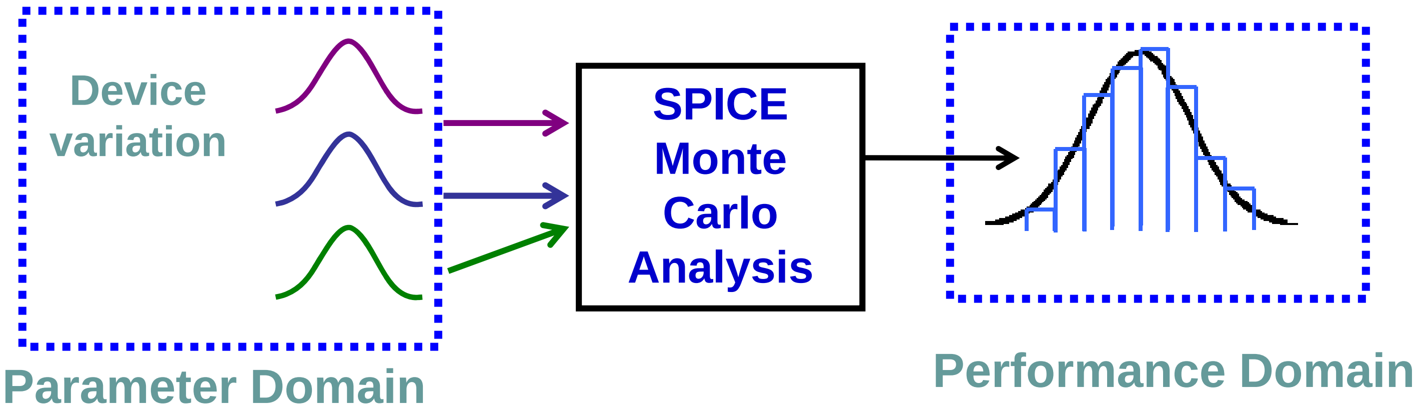
Monte Carlo simulation

- Monte Carlo simulation is the most straight-forward method.

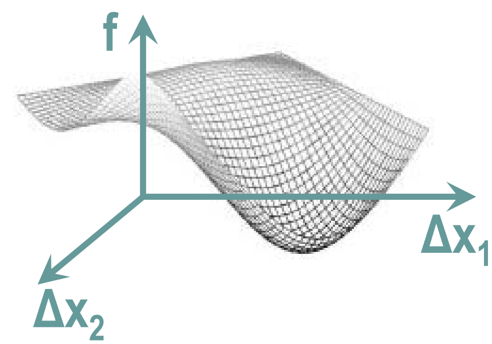


- However, it is highly time-consuming!

Response Surface Model (RSM)

- Approximate circuit performance (e.g. delay) as an analytical function of all process variations (e.g. ΔV_{TH}, etc)
  - Synthesize analytical function of performance as random variations.
  - Results in a multi-dimensional model fitting problem.

- Response surface model can be used to
  - Estimate performance variability
  - Identify critical variation sources
  - Extract worst-case performance corner
  - Etc.



$$f(\varepsilon) = p_0 + \alpha_1\varepsilon_1 + \cdots + \alpha_N\varepsilon_N$$

Flow Chart of APEX*

Synthesize analytical function of performance using RSM

$$f(\varepsilon) = p_0 + \alpha_1\varepsilon_1 + \cdots + \alpha_N\varepsilon_N$$

Calculate moments

$$\widehat{m}_f^k = \frac{(-1)^k}{k!} \cdot \int_{-\infty}^{+\infty} f^k \cdot pdf(f)\,df$$

Calculate the probability distribution function (PDF) of performance based on RSM

$$\widehat{m}_t^k = \frac{(-1)^k}{k!} \cdot \int_{-\infty}^{+\infty} t^k \cdot h(t)\,dt$$

$$\widehat{m}_f^k = \widehat{m}_t^k = -\sum_{r=1}^{M} \frac{a_r}{b_r^{k+1}}$$

h(t) can be used to estimate ***pdf(f)***

$$h(t) = \begin{cases} \sum_{r=1}^{M} a_r \cdot e^{b_r^{k+1} \cdot t} & (t \geq 0) \\ 0 & (t < 0) \end{cases}$$

*Xin Li, Jiayong Le, Padmini Gopalakrishnan and Lawrence Pileggi, "*Asymptotic probability extraction for non-Normal distributions of circuit performance*," *IEEE/ACM International Conference on Computer-Aided Design* (ICCAD), pp. 2-9, 2004.

Limitation of APEX

- RSM based method is time-consuming to get the analytical function of performance.
  - It has exponential complexity with the number of variable parameters n and order of polynomial function q.

$$f(x_1, x_2, \cdots, x_n) = (\alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n)^q$$

  - e.g., for 10,000 variables, APEX requires 10,000 simulations for linear function, and 100 millions simulations for quadratic function.

- RSM based high-order moments calculation has high complexity
  - the number of terms in f^k increases exponentially with the order of moments.

$$E(f^p) = \int_{-\infty}^{+\infty} (f^p \cdot pdf(f))df$$

$$f^k(x_1, x_2, \cdots, x_n) = (\alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n)^{k \times q}$$

Contribution of Our Work

Step 1: Calculate High Order Moments of Performance

APEX

Find analytical function of performance using RSM

$$f(\varepsilon) = p_0 + \alpha_1 \varepsilon_1 + \cdots + \alpha_N \varepsilon_N$$

↓

Calculate high order moments

$$m_f^k = \int_{-\infty}^{+\infty} (f^k \cdot pdf(f)) df$$

Proposed Method

A few samplings at selected points.

↓

Calculate moments by Point Estimation Method

↓

Step 2: Extract the PDF of performance

$$\widehat{m}_f^k = \frac{(-1)^k}{k!} \cdot \int_{-\infty}^{+\infty} (f^k \cdot pdf(f)) df = \widehat{m}_t^k = \frac{(-1)^k}{k!} \cdot \int_{-\infty}^{+\infty} (t^k \cdot h(t)) dt = -\sum_{r=1}^{M} \frac{a_r}{b_r^{k+1}} \Rightarrow h(t) = \sum_{r=1}^{M} a_r \cdot e^{b_r^{k+1} \cdot t} \approx pdf(f)$$

- Our contribution:
  - We do NOT need to use analytical formula in RSM;
  - Calculate high-order moments efficiently using Point Estimation Method;

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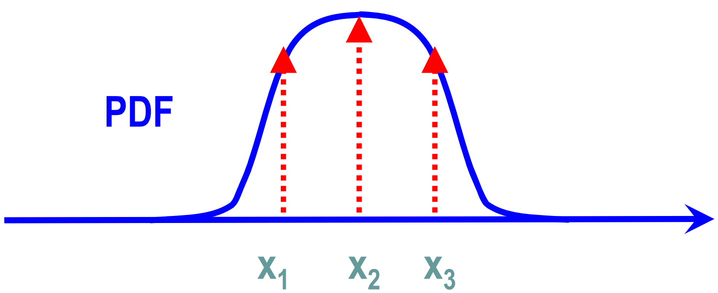
Moments via Point Estimation

- Point Estimation: approximate high order moments with a *weighted sum of sampling values of f(x).*
  - $x_j \ (j = 1, \cdots, p)$ are estimating points of random variable.
  - P_j are corresponding weights.
  - k-th moment of $f(x)$ can be estimated with

$$m_f^k = \int_{-\infty}^{+\infty} f^k \cdot pdf(f)df \approx \sum_{j=1}^{p} P_j \cdot f(x_j)^k.$$



- Existing work in mechanical area* only provide empirical analytical formulae for x_j and P_j for *first four moments*.

Question – how can we accurately and efficiently calculate the higher order moments of $f(x)$?

* Y.-G. Zhao and T. Ono, "New point estimation for probability moments," Journal of Engineering Mechanics, vol. 126, no. 4, pp. 433-436, 2000.

Calculate moments of performance

- Theorem in Probability: assume x and $f(x)$ are both continuous random variables, then:

$$E(f^k(x)) = \int f^k(x) \cdot pdf(f) df = \int f^k(x) \cdot pdf(x) dx$$

- Flow Chart to calculate high order moments of performance:

pdf(x) of parameters is known

↓

Step 1: calculate moments of parameters

$$m_x^k = \int_{-\infty}^{+\infty} (x^k \cdot pdf(x)) dx \approx \sum_{j=1}^{m} P_j \cdot (x_j)^k$$

↓

Step 2: calculate the estimating points x_j and weights P_j

→

Step 3: run simulation at estimating points x_j and get performance samplings $f(x_j)$

↑

Step 4: calculate moments of performance

$$m_f^k = \int_{-\infty}^{+\infty} (f^k \cdot pdf(x)) dx \approx \sum_{j=1}^{m} P_j \cdot (f(x_j))^k$$

↑

Step 5: extract performance distribution ***pdf(f)***

Step 2 is the most important step in this process.

Estimating Points x_j and Weights P_j

- With moment matching method, x_j and P_j can be calculated by

$$\sum_{j=1}^{m} P_j = 1 = m_x^k$$
$$\sum_{j=1}^{m} P_j \cdot x_j = E(x) = m_x^1$$
$$\sum_{j=1}^{m} P_j \cdot x_j^2 = E(x^2) = m_x^2$$
$$\cdots$$
$$\sum_{j=1}^{m} P_j \cdot x_j^{2m-1} = E(x^{2m-1}) = m_x^{2m-1}$$

  - $m_x^k (k = 0, \ldots, 2m-1)$ can be calculated exactly with ***pdf(x).***

- Assume residues $a_j = P_j$ and poles $b_j = 1/x_j$

$$\begin{bmatrix} a_1 + a_2 + \cdots a_m \\ \frac{a_1}{b_1} + \frac{a_2}{b_2} + \cdots \frac{a_m}{b_m} \\ \frac{a_1}{b_1^2} + \frac{a_2}{b_2^2} + \cdots \frac{a_m}{b_m^2} \\ \vdots \\ \frac{a_1}{b_1^{2m-1}} + \frac{a_2}{b_2^{2m-1}} + \cdots \frac{a_m}{b_m^{2m-1}} \end{bmatrix} = \begin{bmatrix} m_x^0 \\ m_x^1 \\ m_x^2 \\ \vdots \\ m_x^{2m-1} \end{bmatrix} \Rightarrow$$

$$\widehat{m}_f^k = \frac{(-1)^k}{k!} \cdot \int_{-\infty}^{+\infty} f^k \cdot pdf(f) df \approx \frac{(-1)^k}{k!} \cdot \sum_{j=1}^{m} P_j \cdot f(x_j)^k$$

  - system matrix is well-structured (Vandermonde matrix);
  - nonlinear system can solved with deterministic method.

Extension to Multiple Parameters

- Model moments with multiple parameters as a linear combination of moments with single parameter.

$$m^k_{f(x_1,x_2,\cdots,x_n)} = \sum_{i=1}^{n} g_i m^k_{f(x_i)}$$

$$g_i = c \cdot \frac{\partial\left(f(x_i)\right)}{\partial x_i}$$

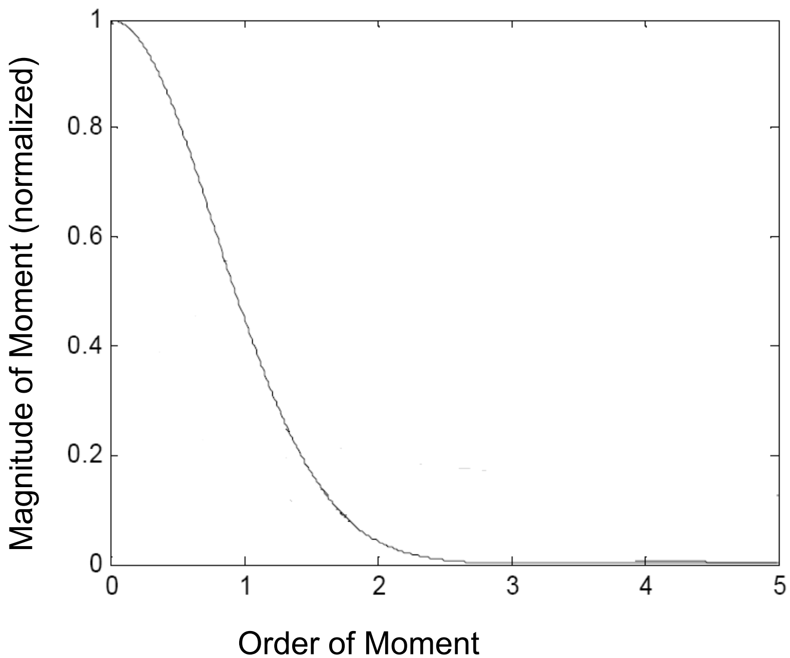
$$c = 1 \Bigg/ \sum_{i=1}^{n} \frac{\partial\left(f(x_i)\right)}{\partial x_i}$$

  - $f(x_1,x_2,\ldots,x_n)$ is the function with multiple parameters.
  - $f(x_i)$ is the function where xi is the single parameter.
  - g_i is the weight for moments of $f(x_i)$
  - c is a scaling constant.

Error Estimation

- We use approximation with ***q+1*** moments as the exact value, when investigating PDF extracted with ***q*** moments.

- When moments decrease progressively $|m_f^p| \geq |m_f^{q+1}| \; (\; p \leq q+1)$



$$m_f^k = \int_{-\infty}^{+\infty} (f^k \cdot pdf(f))df$$

$0 < f < 1$

$$Error \leq \left| \frac{(-j\omega)^{q+1}}{(q+1)!} \cdot \left(\sum_{p=0}^{q+1} \frac{(-j\omega)^p}{p!} \right)^{-1} \right|.$$

- Other cases can be handled after ***shift (f<0), reciprocal (f>1) or scaling*** operations of performance merits.

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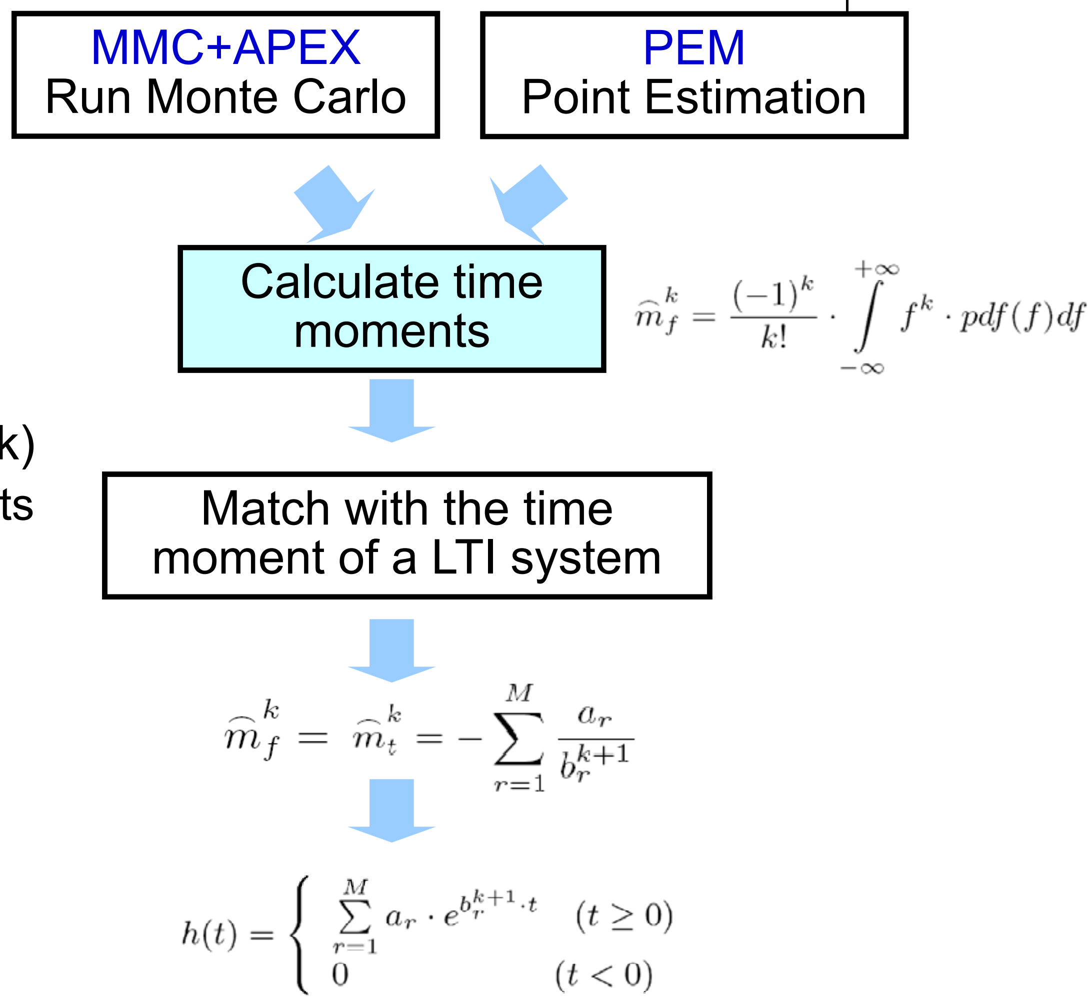
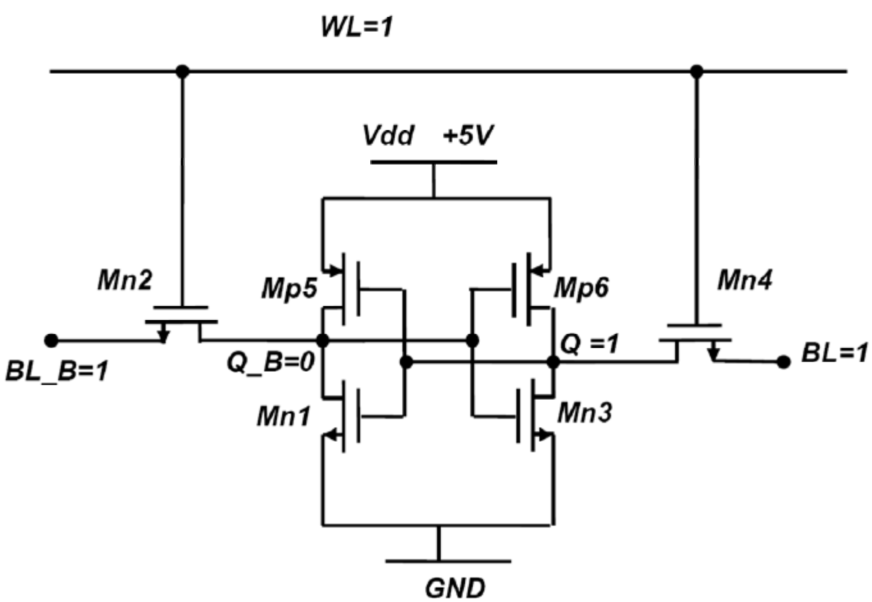
(1) Validate Accuracy: Settings

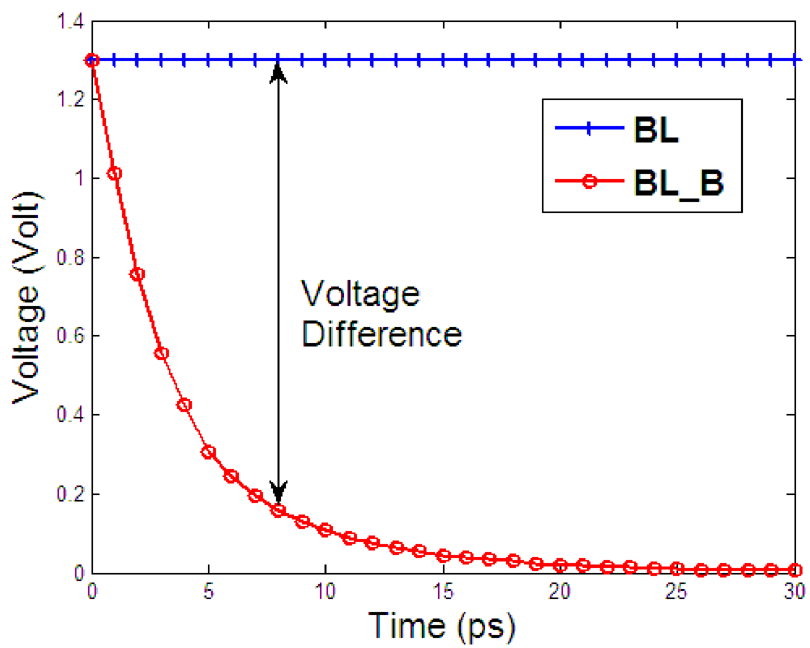
- To validate accuracy, we compare following methods:
  - Monte Carlo simulation.
    - run tons of SPICE simulations to get performance distribution.
  - PEM: point estimation based method (proposed in this work)
    - calculate high order moments with point estimation.
  - MMC+APEX:
    - obtain the high order moments from Monte Carlo simulation.
    - perform APEX analysis flow with these high-order moments.



6-T SRAM Cell

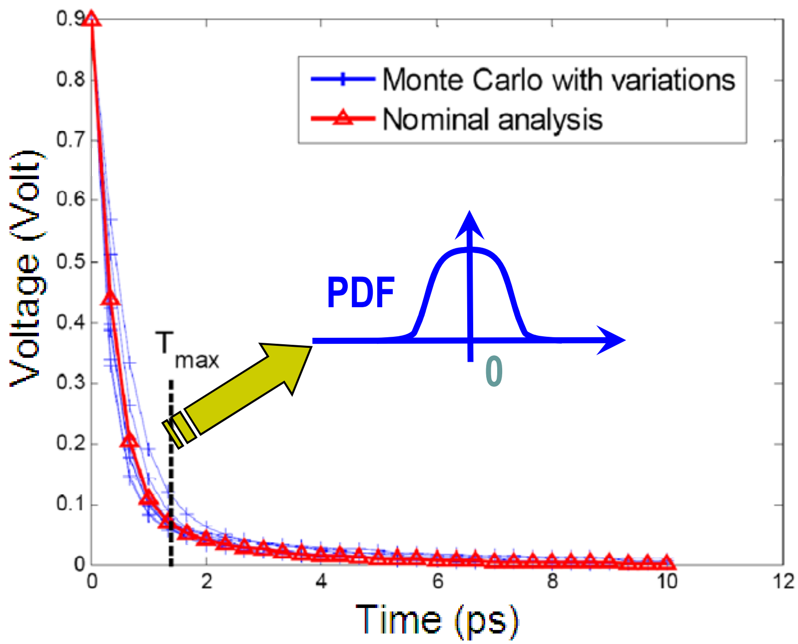
- Study the discharge behavior in *BL_B* node during reading operation.
- Consider threshold voltage of all MOSFETs as independent Gaussian variables with 30% perturbation from nominal values.
- Performance merit is the voltage difference between *BL* and *BL_B* nodes.

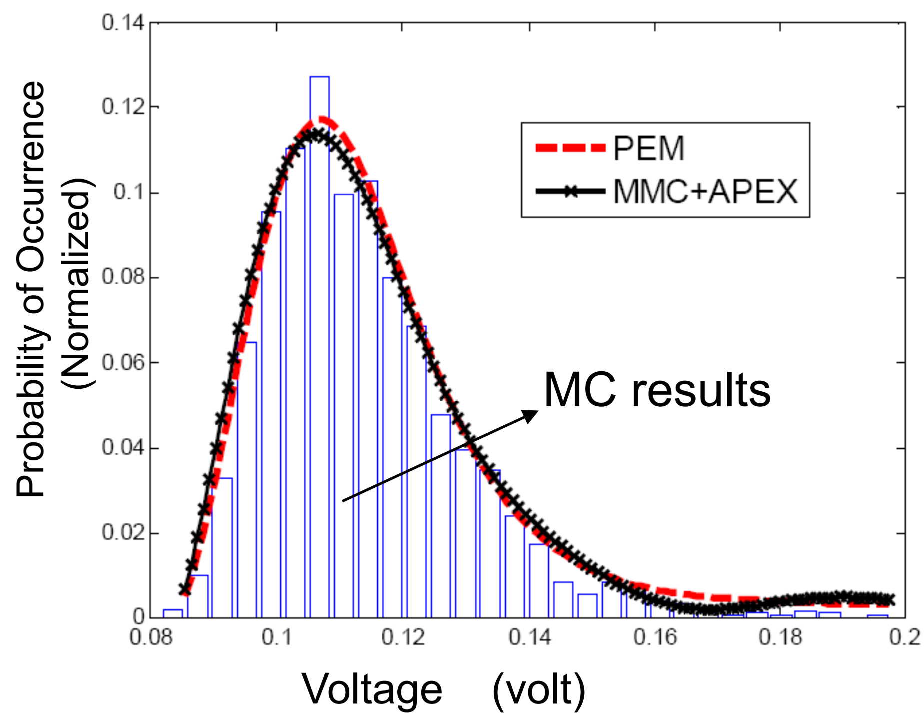




Accuracy Comparison

- Variations in threshold voltage lead to deviations on discharge behavior
  - Investigate distribution of node voltage at certain time-step.
- Monte Carlo simulation is used as baseline.
- Both APEX and PEM can provide high accuracy when compared with MC simulation.





(2)Validate Efficiency: PEM vs. MC

- For 6-T SRAM Cell, Monte Carlo methods requires 3000 times simulations to achieve an accuracy of 0.1%.

- Point Estimation based Method (PEM) needs only 25 times simulations, and achieve up to 119X speedup over MC with the similar accuracy.

Method	Time (second)	Speedup
Monte Carlo (3×10^3)	7644	1x
PEM (5 point)	64.12	119.2x

Compare Efficiency: PEM vs. APEX

- To compare with APEX:
  - One Operational Amplifier under a commercial 65nm CMOS process.
  - Each transistor needs 10 independent variables to model the random variation*.

Circuit Name	Transistor #	Mismatch Variable #
SRAM Cell	~ 6	~ 60
Operational Amplifier	~ 50	~ 500
ADC	~ 2K	~ 20K
SRAM Critical Path	~ 20K	~ 200K

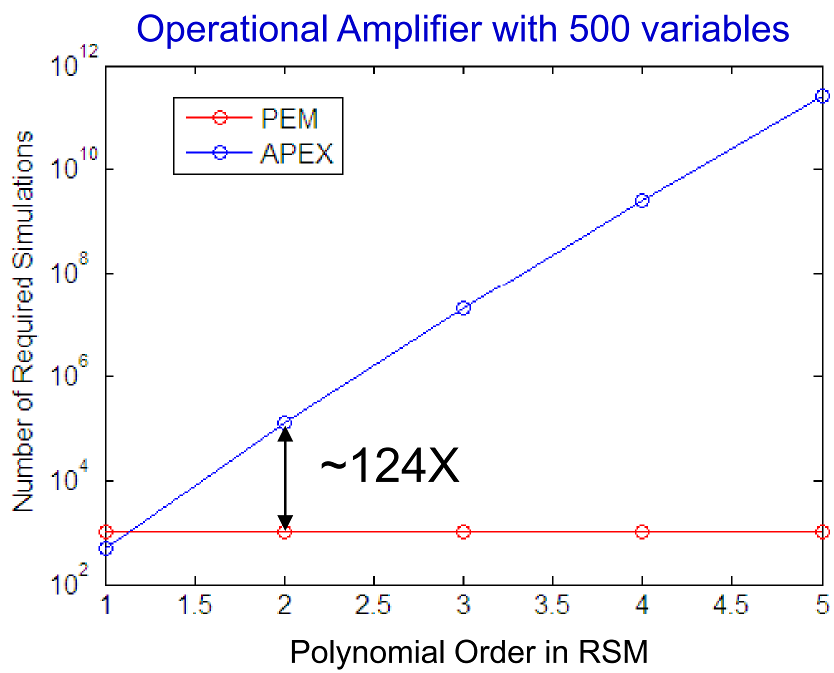
- We compare the efficiency between PEM and APEX by the *required number of simulations*.

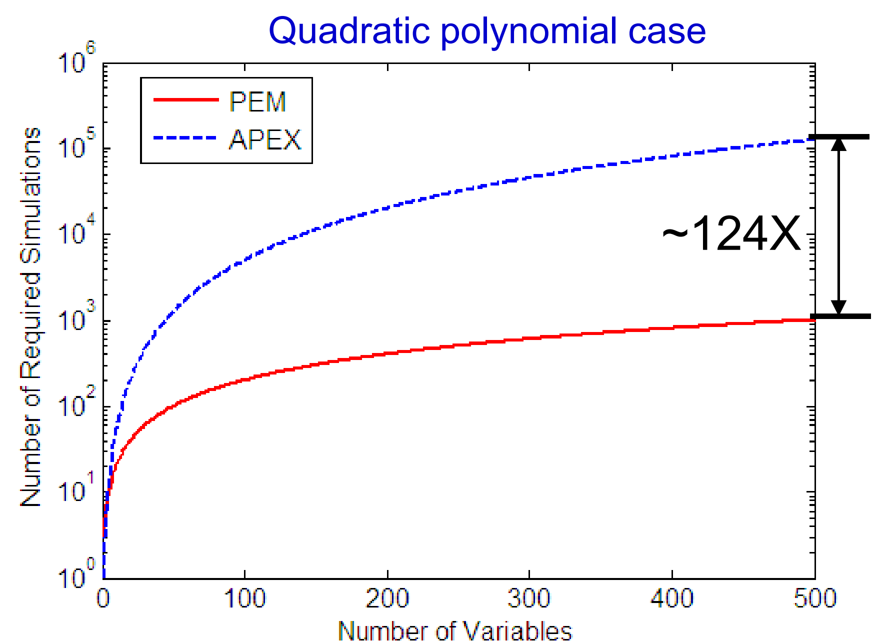
- **Linear vs. Exponential Complexity**:
  - PEM: a linear function of number of sampling point and random variables.
  - APEX: an exponential function of polynomial order and number of variables.

* X. Li and H. Liu, "Statistical regression for efficient high-dimensional modeling of analog and mixed-signal performance variations," in *Proc. ACM/IEEE Design Automation Conf. (DAC)*, pp. 38-43, 2008.

Operational Amplifier

- A two-stage operational amplifier
  - complexity in APEX increases exponentially with polynomial orders and number of variables.
  - PEM has linear complexity with the number of variables.





The Y-axis in both figures has **log scale!**

Conclusion

- Studied stochastic analog circuit behavior modeling under process variations
- Leverage the Point Estimation Method (PEM) to estimate the high order moments of circuit behavior systematically and efficiently.
- Compared with exponential complexity in APEX, proposed method can achieve linear complexity of random variables.

Thank you!

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Fang Gong, Hao Yu and Lei He