Signal Speed Optimization in Routing

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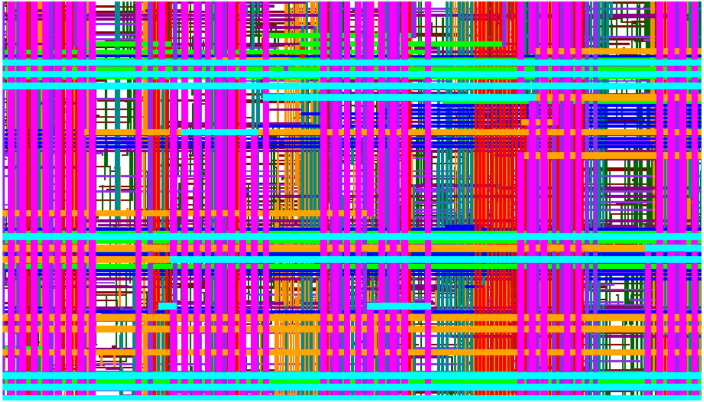

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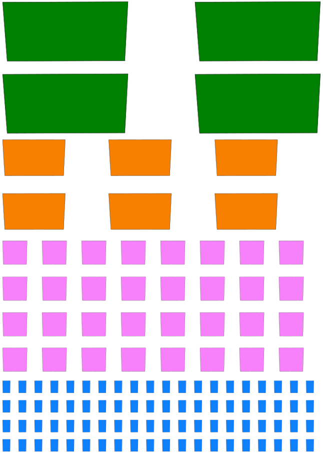
Speed impact of heterogeneous metal stacks

Wires on higher layers have less resistance and are much faster.

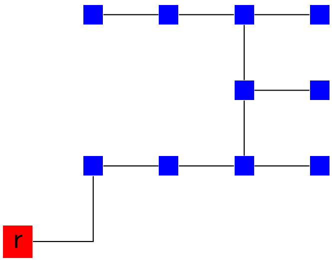


Cross-section of wires

Speed impact of Steiner trees

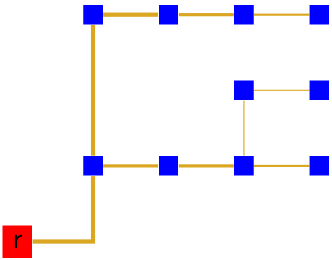
timing-unaware

- minimum total length (causing source-sink detours)
- minimum width



timing-aware

- short path lengths
- fast wires/high layers at root



Timing-constrained global routing: approaches

Based on resource sharing /multicommodity flow

- Net delay bounds (Huang et al. 1993)
- Path delay bounds (Hong et al. 1997)
- FPTAS for net delay bounds and two-terminal nets (Albrecht et al. 2002)
- Net delay or path delay resources (Vygen 2004)

Further approaches

- Congestion aware embedding of fast Steiner topologies (HS 2002, YLC 2006, . . .).
- Timing-aware layer assignment (BLCP 2015, LLCY 2016, LZGH 2022,. . .)

Summary

Models are either over constrained or have exponential size.

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Global routing: min-max resource sharing (Müller, Radke & Vygen '11)

generalizing multi-commodity flow routing (Shragowitz & Keel '87)

Global routing:

- $\mathcal{N}$:= nets
- $\mathcal{R}$:= set of resources = edges in global routing graph
- $\mathrm{usg}_r(Y) := \frac{\text{width+spacing of } Y \text{ on } r}{\text{capacity}(r)}$ = relative usage of $r \in R$ by Steiner tree Y.

Goal: Compute a steiner trees $(Y_N)_{N \in \mathcal{N}}$, such that the maximum relative usage

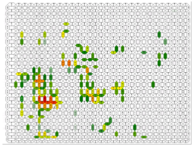
$$\max_{r \in \mathcal{R}} \sum_{N \in \mathcal{N}} \mathrm{usg}_r(Y_N)$$

is minimized.

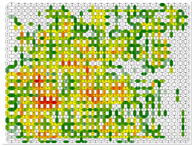
Multiplicative-weight updates (simplified):

$\mathrm{price}_r := 1 \ (\forall r \in \mathcal{R})$
Repeat:
  For each net $N \in \mathcal{N}$:
    Compute Steiner tree Y_N with minimum price
    $\mathrm{price}_r := \mathrm{price}_r \cdot e^{\mathrm{usg}_r(Y_N)} \ \forall r \in \mathcal{R}$

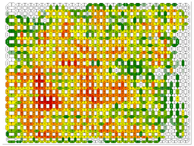
Price development after phase 1



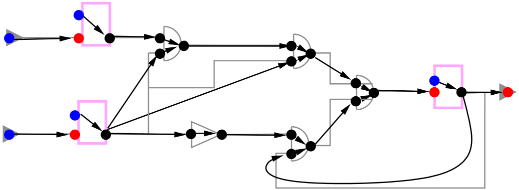
after phase 2



after phase 3



Adding timing-constraints as resources



For each path P in the timing graph:

$$\sum_{e \in E(P)} \mathrm{delay}(e) \leq T \quad \Longleftrightarrow \frac{\sum_{e \in E(P)} \mathrm{delay}(e)}{T} \leq 1$$

For each path/resource P traversing an edge $e \subseteq N \in \mathcal{N}$, the Steiner tree Y_N has a relative delay usage

$$\mathrm{usg}_P(Y_N) := \frac{\mathrm{delay}_{Y_N}(e)}{T}.$$

The prices can be computed implicitly in linear time! (Daboul, Hähnle, H., Schorr [2018])

Alt.: timing graph edges as resources (H., Müller, Rotter, Scheifele, Traub, Vygen [2018])

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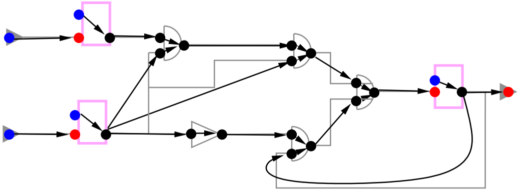

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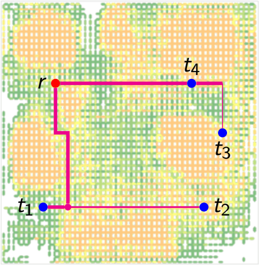
Cost-distance Steiner tree problem



Compute a Steiner tree Y minimizing

$$\sum_{f \in E(Y)} \mathrm{price}(f) + \sum_{t \text{ sink}} \mathrm{price}(r,t) \cdot \mathrm{delay}_Y(r,t).$$

↑ congestion cost ↑ delay cost

Timing-Constrained Global Routing

Theorem (H., Müller, Rotter, Traub, Scheifele, Vygen [2018])

Let $\epsilon > 0$.
Given a σ-approximation algorithm for the cost-distance Steiner tree problem, we can compute with $\tilde{O}(\epsilon^{-2}(|\mathcal{N}| + |\mathcal{R}|))$ oracle calls a randomized solution $(Y_N)_{N \in \mathcal{N}}$, satisfying

$$\max_{r \in \mathcal{R}} \mathbb{E}\left[\sum_{N \in \mathcal{N}} \mathrm{usg}_r(Y_N)\right] \leq \sigma(1+\epsilon) \cdot \mathrm{OPT}.$$

Unfortunately: $\sigma \geq o(\log \log |N|)$
(Chuzhoy et al. '05, slightly stronger assumption than $P \neq NP$).

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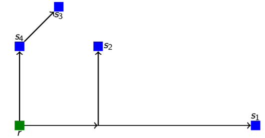
Heuristic oracle for global interconnect optimization

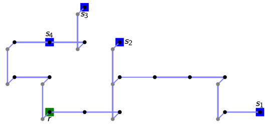
H., Müller, Rotter, Traub, Scheifele, Vygen [2018], Daboul, H., Natura, Rotter [2019]

Additional placement bin resources for repeater space.

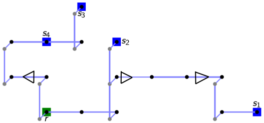
$$\sum_{e \text{ edge}} \text{price}(e) \cdot \text{width}(e) + \sum_{b \text{ repeater}} \text{price}(\text{tile}(b)) \cdot \text{size}(b) + \sum_{t \text{ sink}} \text{price}(t) \cdot \text{delay}(\text{root}, t)$$



Step 1: Uniform cost-dist Steiner tree in $(\mathbb{R}^2, l_1)$



Step 2: Embedding into GR graph (Dijkstra)



Step 3: Buffering

Elmore delay oracle: H., Müller, Rotter, Traub, Scheifele, Vygen [2018]

Summary & Major challenge

Consistent model representing global routing and global timing constraints

Practical impact: Timing-constrained (buffered) global routing is now default mode of global routing at IBM.

Major challenge: Better understanding of the cost-distance Steiner tree problem

$$\min_{Y}\left(\sum_{f\in E(Y)}\mathrm{price}(f)+\sum_{t\text{ sink}}\mathrm{price}(r,t)\cdot\mathrm{delay}_{Y}(r,t)\right).$$