

Tree Structure and Algorithms for Physical Design

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Outline:

- Introduction
- Ancestor Trees
- Column Generation
- Alphabetical Trees
- Conclusion

Introduction

Prof. T. C. Hu has made significant contributions to broad areas in computer science, including network flows, integer programming, shortest paths, binary trees, global routing, etc. since 1954 (Ph.D. Program, IBM Research Center).

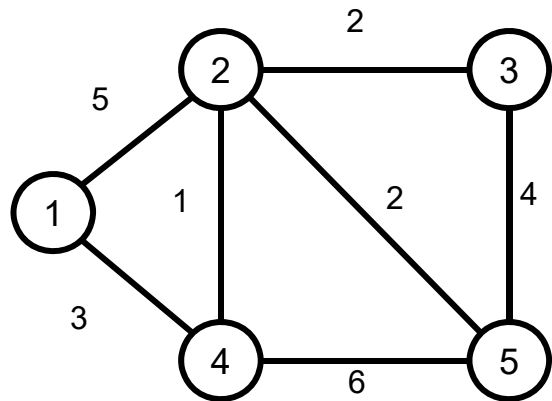
In this talk, we select and summarize three important and interesting tree related topics, in the highlights of Prof. T. C. Hu's contributions to physical design (1985+).

The Ancestor Tree

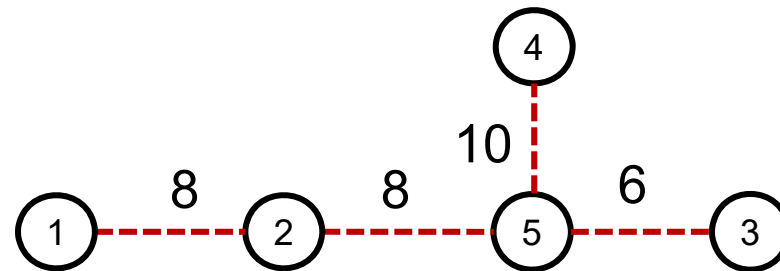
- Gomory and Hu's Cut Tree
 - Tree representation of all pairs of maximum flow minimum cuts
 - Journal of SIAM, 1961
- Ancestor Tree
 - Tree representation of all pairs of cuts (arbitrary objective function)
 - Annals of Operations Research ,1991

The Gomory-Hu Cut Tree

- Maximum flow minimum cut: Given a graph, and a pair of nodes s , and t , the maximum flow from s to t forms a minimum cut.
- # pair of nodes: Given an undirected graph with n nodes, we can choose $C(n, 2)$ pairs of nodes.
- Gomory and Hu: The $n - 1$ minimum cuts determine the maximum flow between all pairs of nodes.



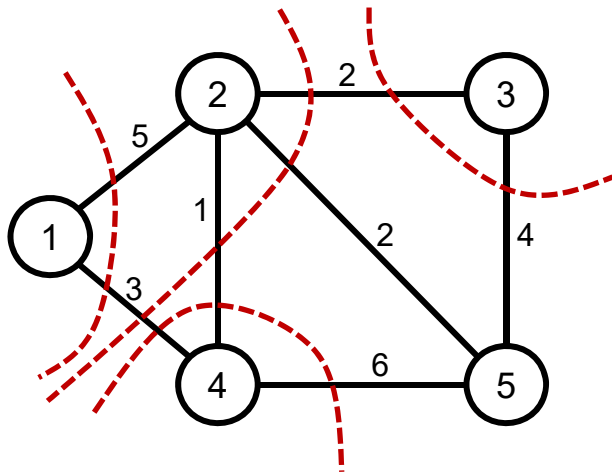
A Network Example



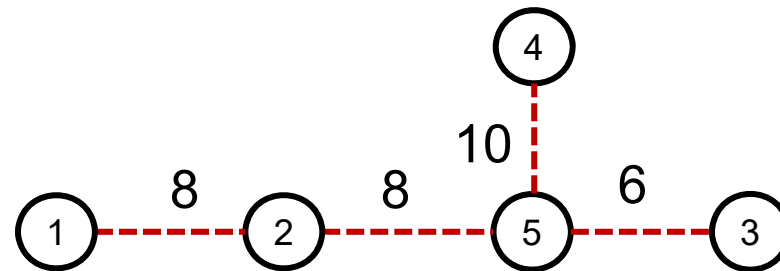
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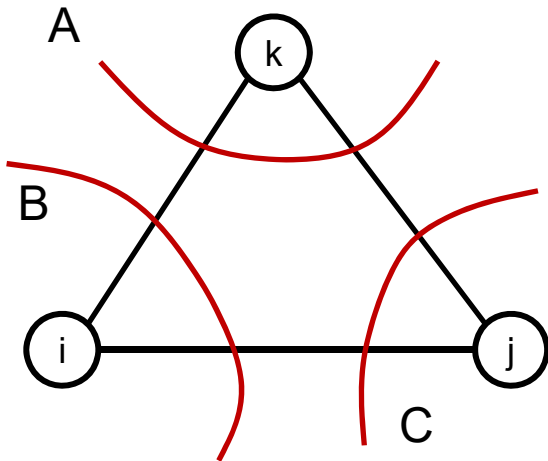


The Gomory-Hu Cut Tree

The Foundation of Cut Tree

Theorem: A necessary and sufficient condition for a set of non-negative numbers $f_{ij} = f_{ji}$ ($i, j = 1, \dots, n$) to be the minimum cut separating nodes i, j .

$$f_{ik} \geq \min(f_{ij}, f_{jk}), \forall i, j, k$$



Lemma: For any three nodes of the network, at least two of the cut costs between them must be equal.

The Foundation of Cut Tree

By induction, we have

$$f_{ip} \geq \min (f_{ij}, f_{jk}, f_{kl}, \dots, f_{op})$$

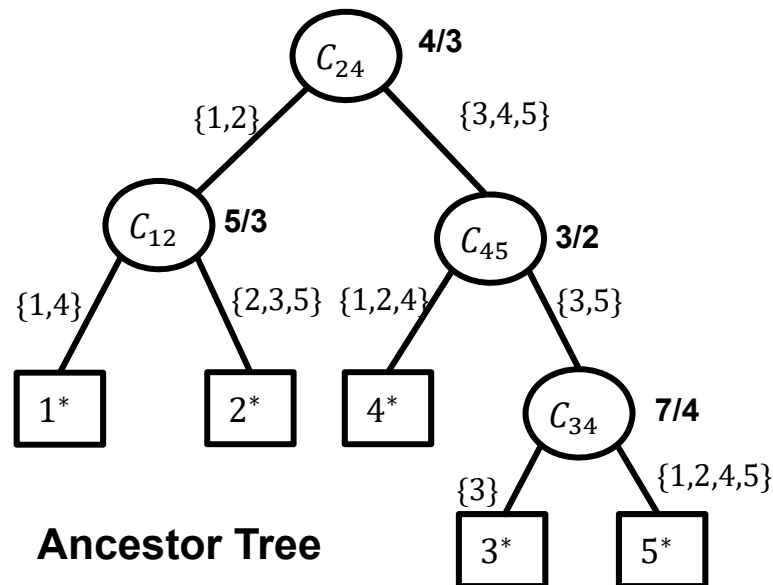
Where indices i, j, \dots, p represent an arbitrary **sequences of nodes in the network**

Lemma: There is no loop in the cut representation.

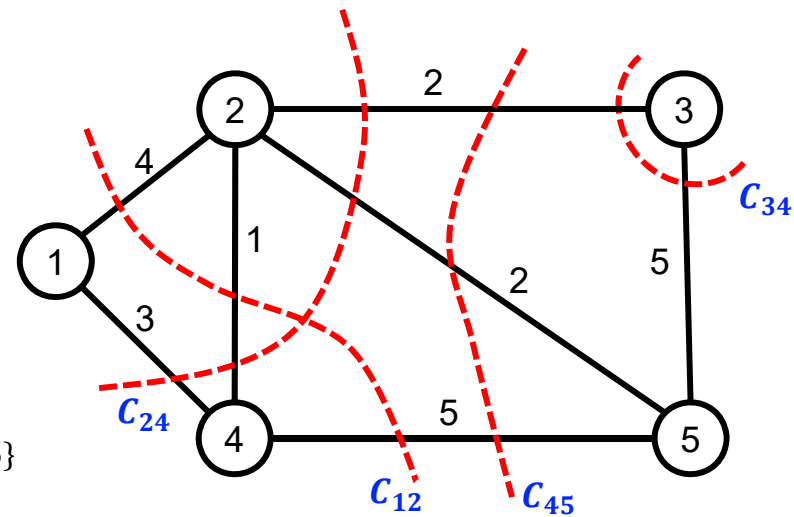
The Ancestor Tree

The minimum cut tree for an arbitrary cut cost.

Ratio Cut Example: $F_{ij} = \min \frac{C(X, \bar{X})}{|X| \cdot |\bar{X}|}$ with nodes $i \in X$ and $j \in \bar{X}$. Ratio cut is an NP-complete problem.

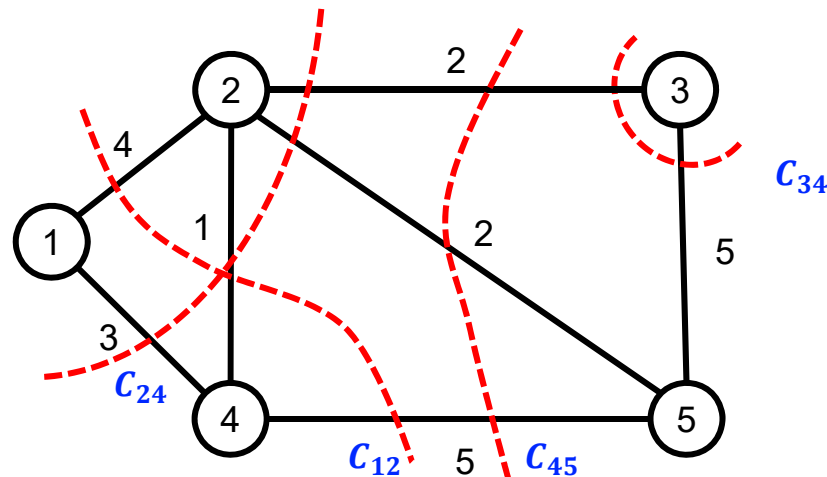


Ancestor Tree



The Properties of Ancestor Tree

- The ancestor tree algorithms derives the essential cut set with $n - 1$ minimum cut calls.
- The technique has been applied to solving complex multi-commodity network optimization problems as well as network partitioning problems
- This partitioning can be further applied to solve VLSI design problems for logic synthesis and physical layout.



Applications of Ancestor Tree

- S.-J. Chen and C. K. Cheng, “Tutorial on VLSI Partitioning”, *VLSI Design* 11(3) (2000), pp. 175-218.
- M. E. Kuo and C. K. Cheng, “A Network Flow Approach for Hierarchical Tree Partitioning”, *Proc. DAC*, 1997, pp. 512-517.
- Network Flows, Prentice Hall, R.K. Ahuja, T.L. Magnanti, J.B. Orlin, 1993
- V. Gabrel, A. Knippel and M. Minoux, “Exact Solution of Multicommodity Network Optimization Problems with General Step Cost Functions”, *Operations Research Letters* 25(1) (1999), pp. 15-23.
- D. Jungnickel, *Graphs, networks and algorithms (Vol. 5)*. Berlin: Springer 1999.
- M. Queyranne, “Minimizing symmetric submodular functions”, *Mathematical Programming*, 82(1-2) 1998, pp. 3-12.

Column Generation

Linear programming problem with a large number (exponential) of variables.

- Primal dual formulation
- Shadow price
- Column generation when possible columns of the primal problem is huge

$$\begin{array}{ll} \text{Maximize} & z = \mathbf{c}\mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{array}$$

$$\begin{array}{ll} \text{Minimize} & w = \mathbf{y}\mathbf{b} \\ \text{subject to} & \mathbf{y}\mathbf{A} \geq \mathbf{c} \\ & \mathbf{y} \geq \mathbf{0} \end{array}$$

Primal and Dual Formulation for Routing

- Linear Programming formulation

$$\begin{aligned} & \text{minimize } \sum_{r=1}^m s_r \\ & \text{subject to} \end{aligned}$$

$$\begin{aligned} & \sum_{t \in T_n} f(n, t) \geq b(n), \quad \forall t \in T_n, \forall n \in N \\ & \sum_{n \in N} \sum_{t \in T_n} f(n, t) \cdot \delta(n, t, e) \leq s_r, \quad \forall e \in E_{Hr}, \\ & \sum_{n \in N} \sum_{t \in T_n} f(n, t) \cdot \delta(n, t, e) \leq c(e), \quad \forall e \in E_V, \\ & f(n, t) \geq 0, \quad \forall t \in T_n, \forall n \in N, \\ & s_r \geq 0, \quad \forall r \in \{1, 2, \dots, m\}. \end{aligned}$$

#tree route in T_n is exponential

- Dual Linear Programming Formulation

$$\text{maximize } \sum_n \lambda(n)b(n) - \sum_{e \in E_V} d(e)c(e)$$

subject to

$$\sum_{e \in E_{Hr}} d(e) \leq 1, \quad \forall r \in \{1, 2, \dots, m\},$$

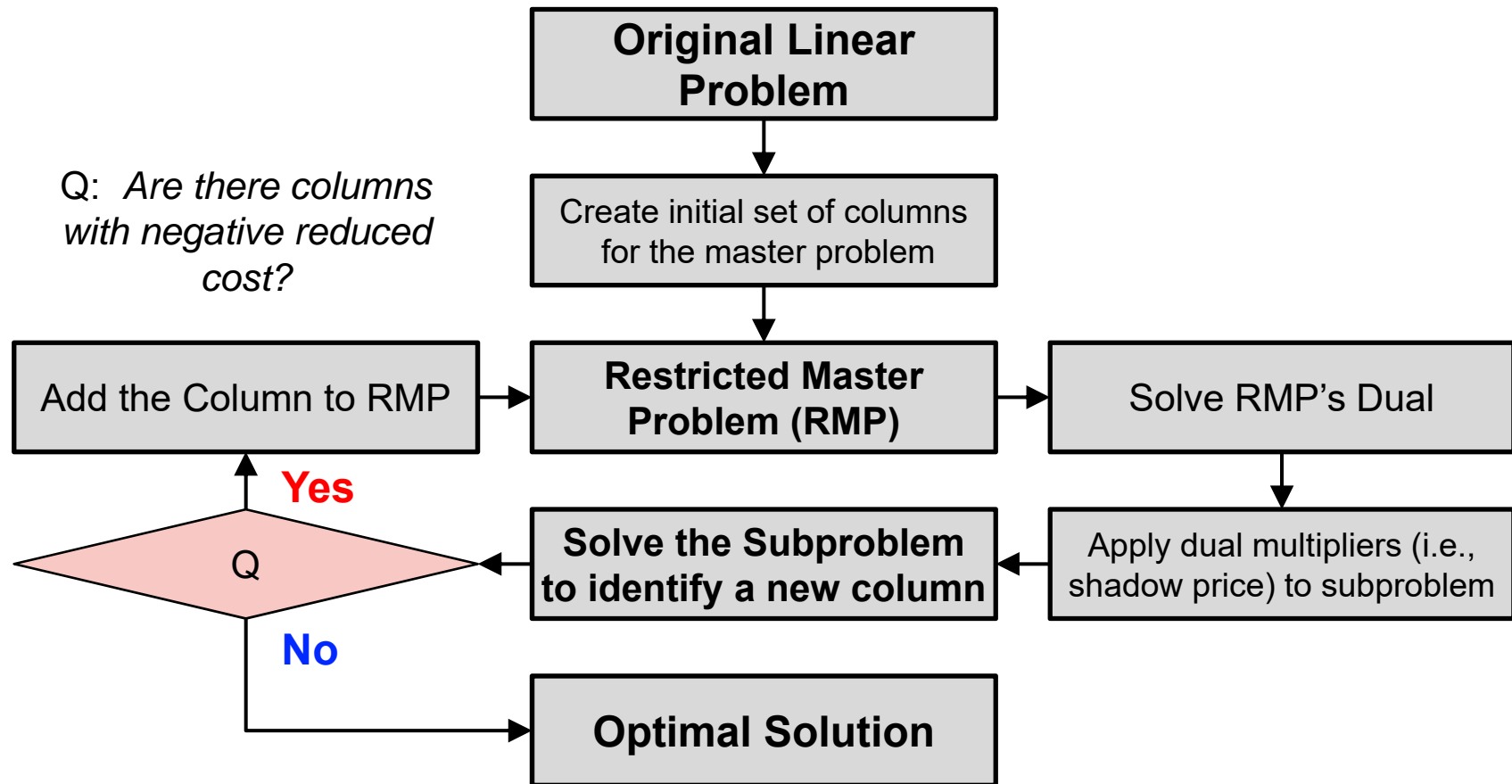
$$\sum_{e \in T} d(e) \geq \lambda(n), \quad \forall t \in T_n, \forall n \in N,$$

$$\lambda(n) \geq 0, \quad \forall n \in N,$$

$$d(e) \geq 0, \quad \forall e \in E.$$

$\lambda(n)$ shadow price of net n
 $\lambda(e)$ shadow price of edge e

Flow Chart for Column Generation



Primal iteration finds a routing solution according to a given shadow price.

Dual iteration updates the shadow price.

The iterations converge to an optimal solution.

Applications of Column Generation

Physical Design

- *R. C. Carden, J. Li and C. K. Cheng, "A Global Router with a Theoretical Bound on the Optimal Solution", IEEE Trans. on CAD, 15(2) (1996), pp. 208-216.*
- *J. Hu and S. S. Sapatnekar, "A Survey on Multi-Net Global Routing for Integrated Circuits", Integration, the VLSI Journal, 31(1) (2001), pp. 1-49.*

Routing + timing cost

- *J. Huang, X. L. Hong, C. K. Cheng and E. S. Kuh, "An Efficient Timing-Driven Global Routing Algorithm", Proc. DAC, 1993, pp. 596-600.*

Routing + wirelength cost

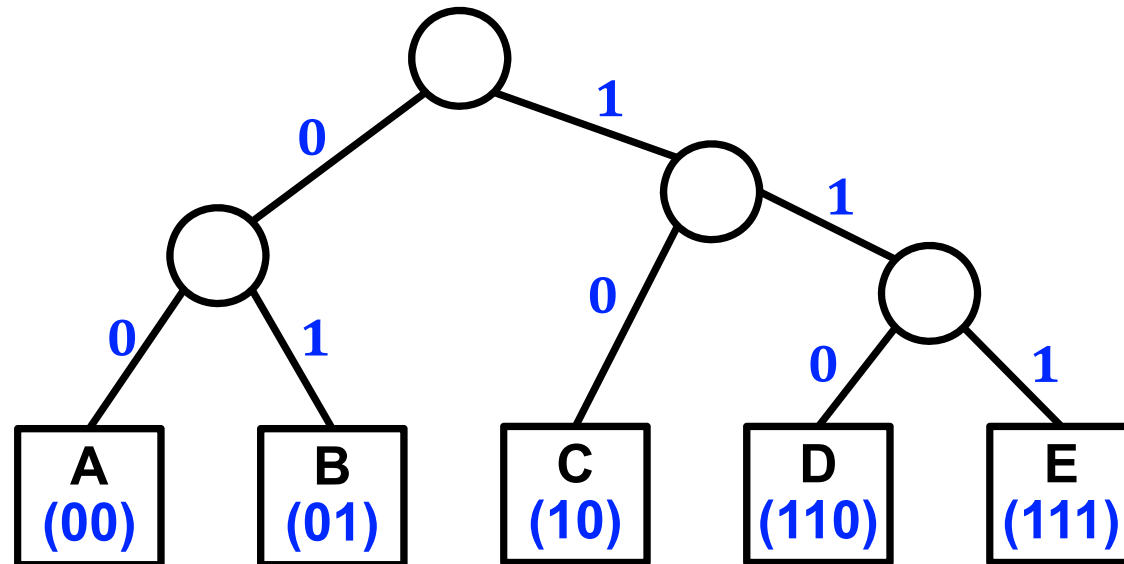
- *C. Albrecht, "Provably Good Global Routing by A New Approximation Algorithm for Multicommodity Flow", Proc. ISPD, 2000, pp. 19-25.*

Routing tree candidates + Integer programming

- *T. H. Wu, A. Davoodi and J. T. Linderoth, "GRIP: Global Routing via Integer Programming", IEEE Trans. on CAD, 30(1) (2011), pp. 72-84.*

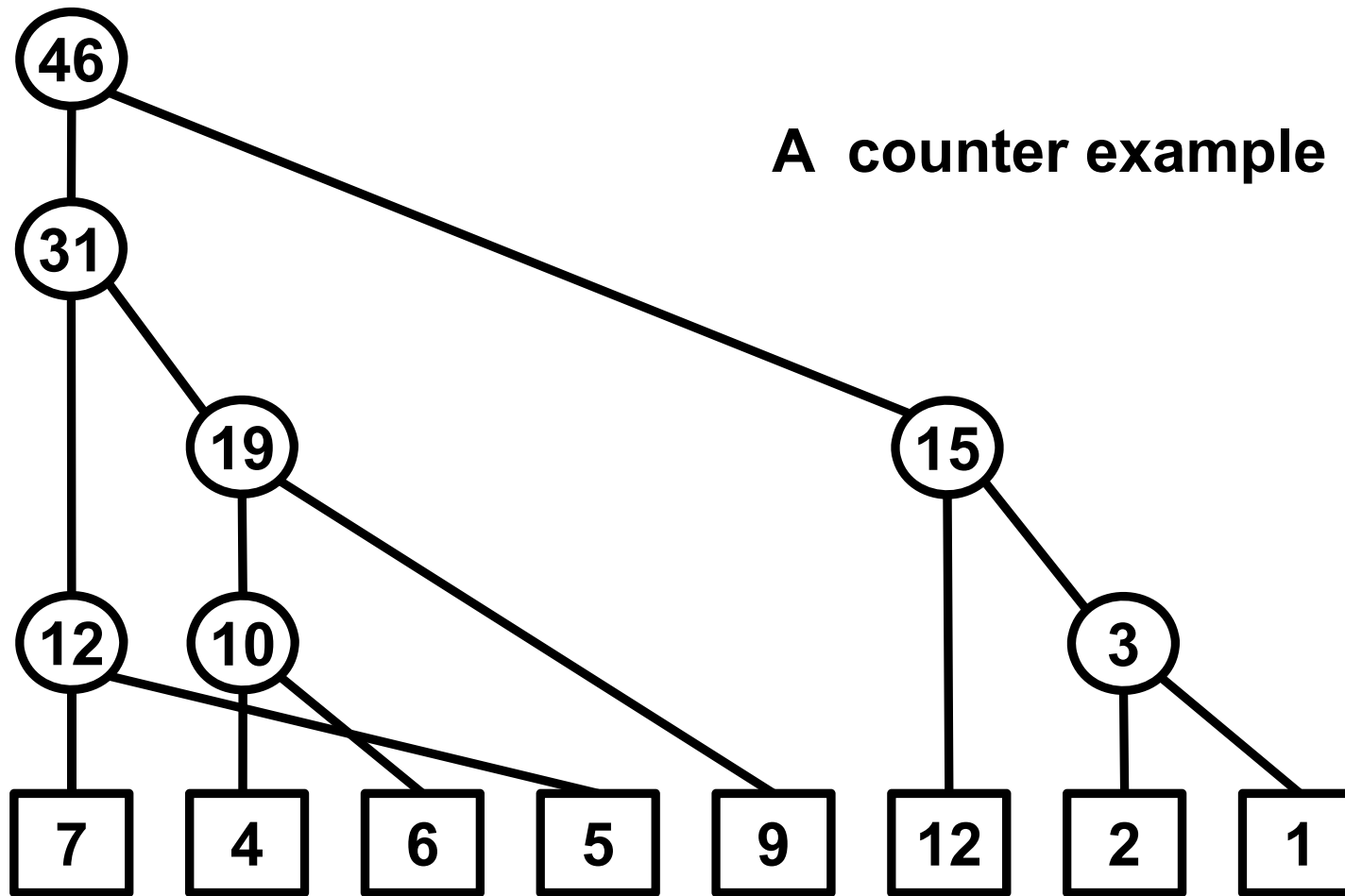
An Alphabetical Tree

A tree that preserves the sequence of the leaves, i.e. no two edges cross each other in the layout.



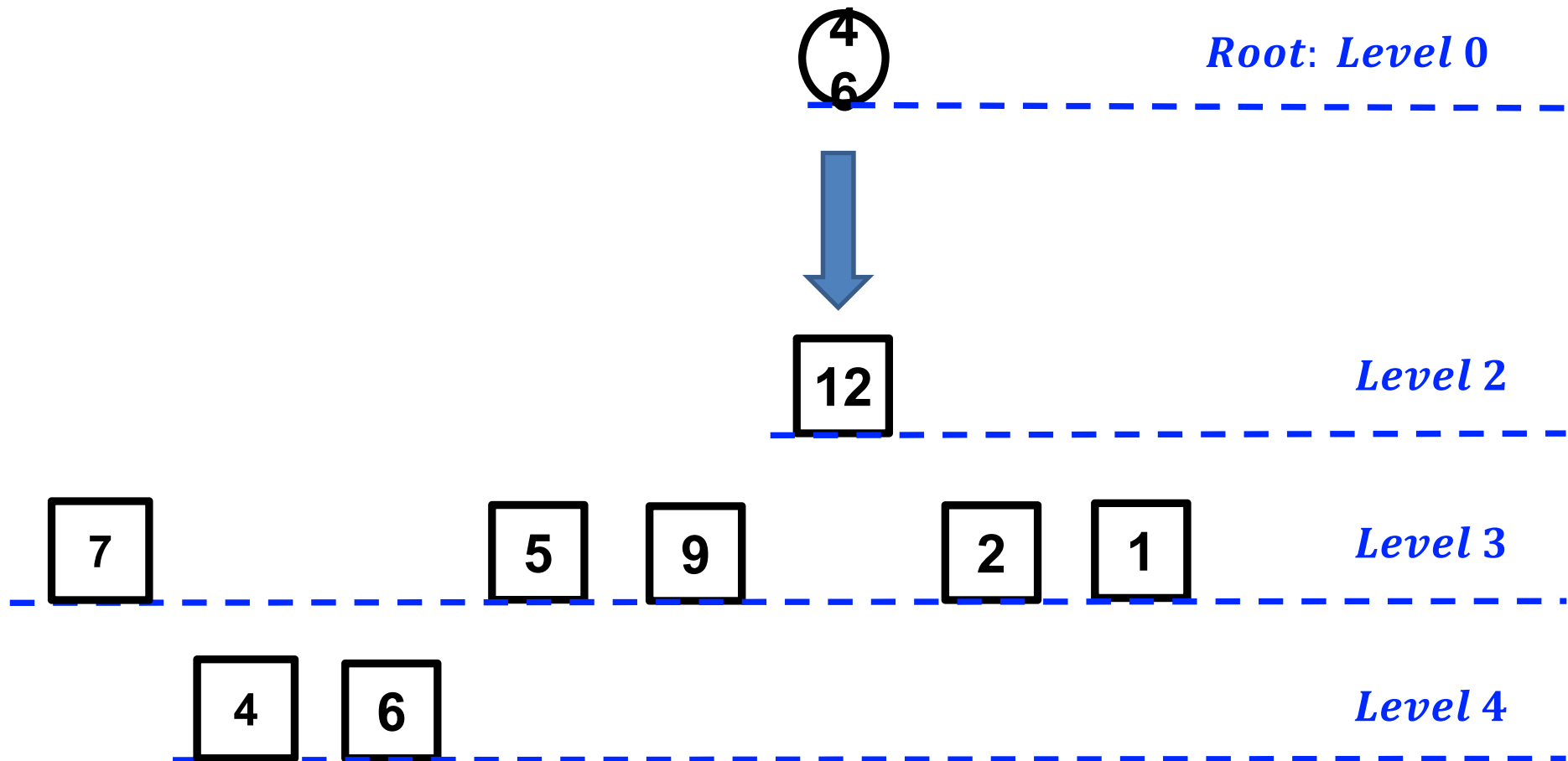
The constraint fits physical layout.
The formulation reduces the solution space.

The Alphabetical Tree (Hu-Tucker Algorithm)



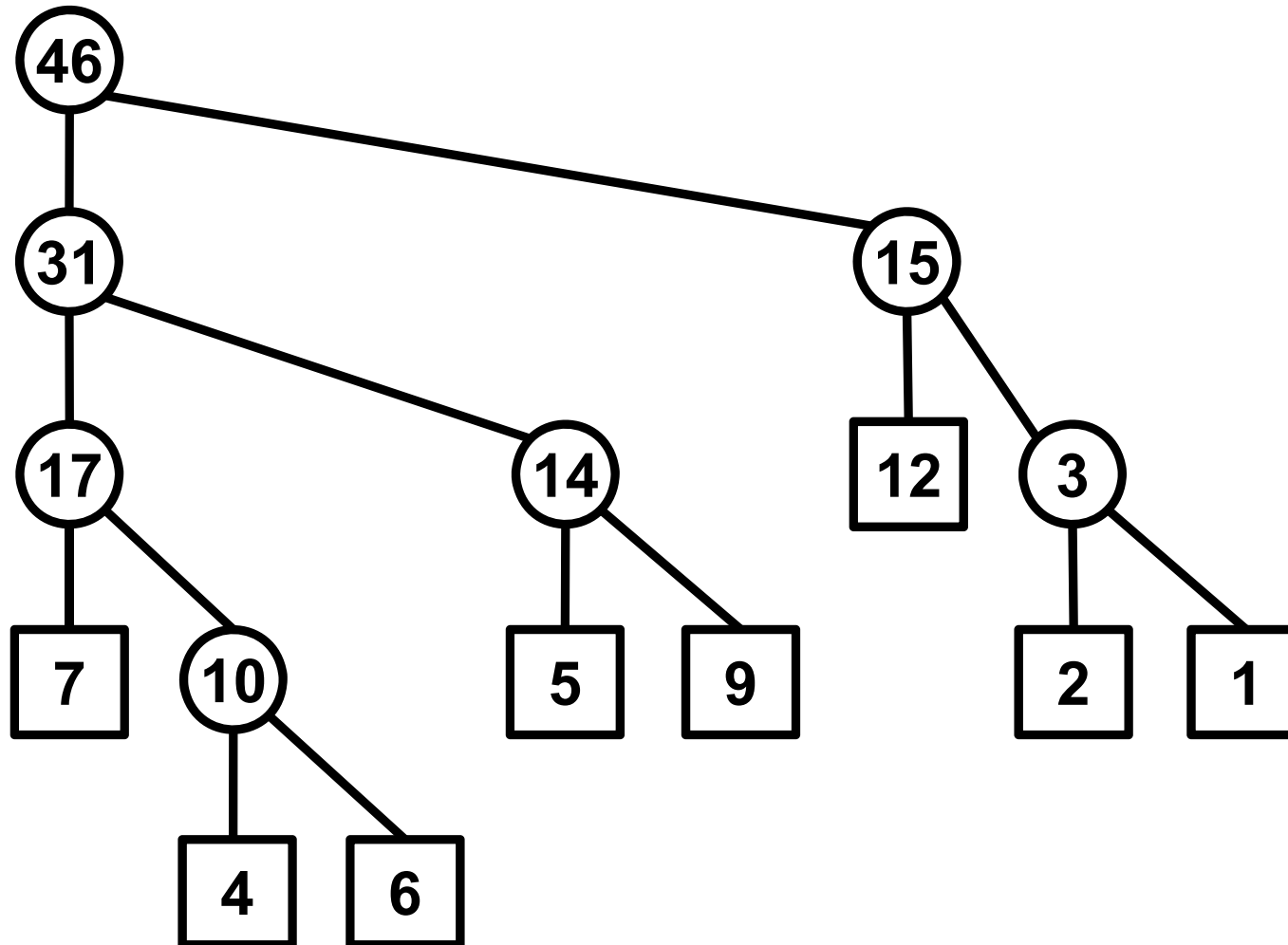
The Hu-Tucker Algorithm – Level Assignment

(b) Level Assignment: the path-length from the root node



The Hu-Tucker Algorithm - Reconstruction

(c) Reconstruction: T'_N (Alphabetical Tree)



Applications of Alphabetical Tree

Alphabetical tree + prefix adder

- *J. Liu, S. Zhou, H. Zhu and C. K. Cheng, "An Algorithmic Approach for Generic Parallel Adders", Proc. ICCAD}, 2003, pp. 734-740.*
- *Y. Zhu, J. Liu, H. Zhu and C. K. Cheng, "Timing-Power Optimization for Mixed-Radix Ling Adders by Integer Linear Programming", Proc. ASP-DAC, 2008,*

Alphabetical tree + timing

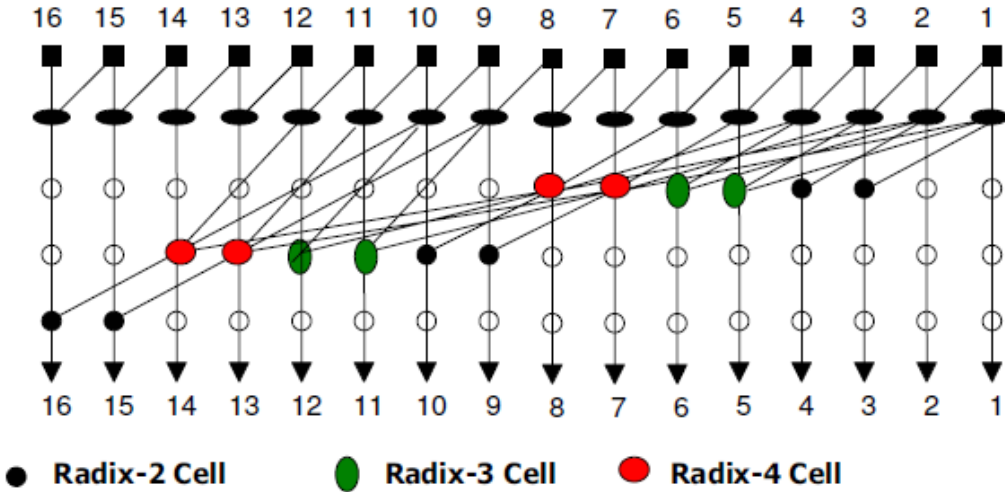
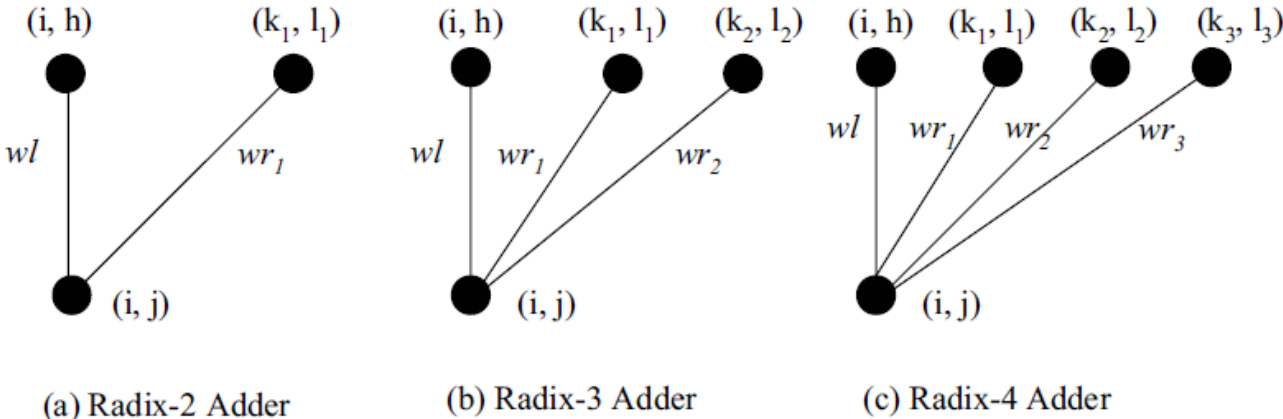
- *A. Vittal and M. Marek-Sadowska, "Minimal Delay Interconnect Design Using Alphabetic Trees", Proc. DAC, 1994, pp. 392-396.*

Alphabetical tree + Logic synthesis (Fanout and tech decomposition)

- *M. Pedram and H. Vaishnav, "Technology Decomposition Using Optimal Alphabetic Trees", Proc. ECDA, 1993, pp. 573-577.*
- *H. Vaishnav and M. Pedram, "Alphabetic Trees - Theory and Applications in Layout-Driven Logic Synthesis", IEEE Trans. on CAD, 42(2) (2002), pp. 219-223.*

Alphabetical Applications (1)

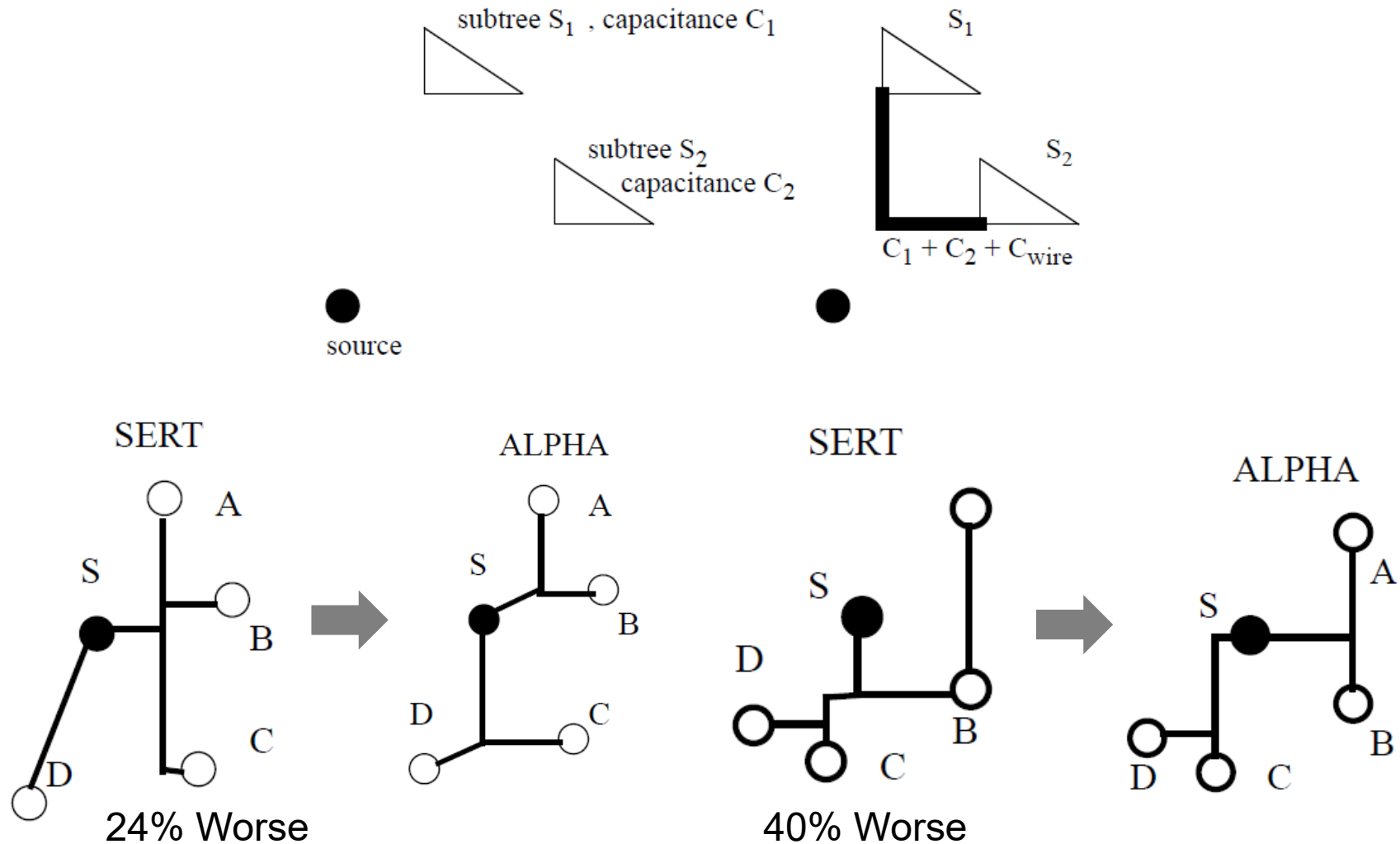
- Parallel adders based on prefix computation



Minimum Power Mixed-Radix 16-bit Ling Adders

Alphabetical Applications (2)

- Interconnect Model for minimal delay



Conclusion:

- Tree Structures
 - Ancestor Trees: Crossing Cuts
 - Column Generation: Shadow Price
 - Alphabetical Trees: Layout Sequence
- NP-Complete Problems
- Solid Theoretical Foundation
- Systematic Approaches with Elegant Processes
- Solutions for Physical Design and Other Fields

Thank You!