Statistical Clock Tree Routing for Robustness to Process Variations

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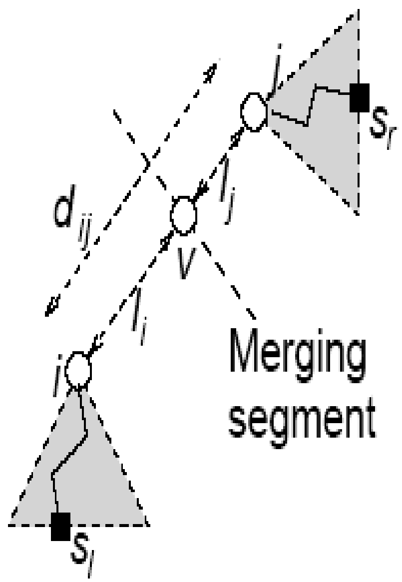
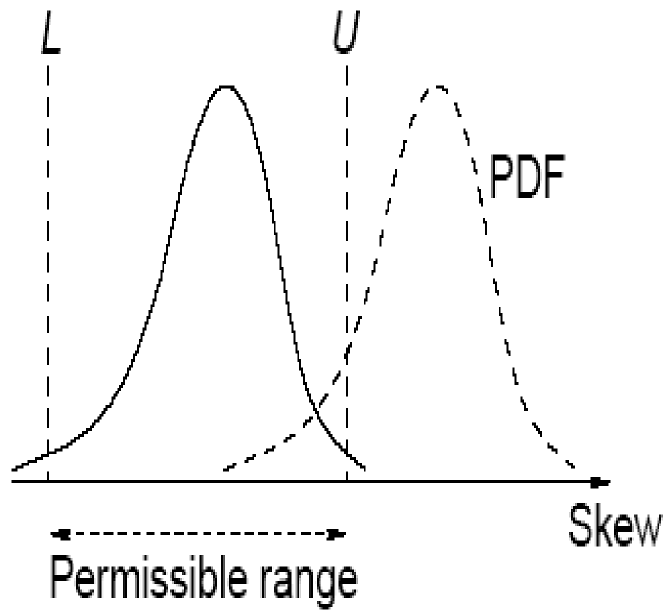
Outline

- Previous research
- Motivation of this project
- Variation aware delay model
- Statistical centering algorithm
- Examples
- Conclusion

Clock Tree Routing

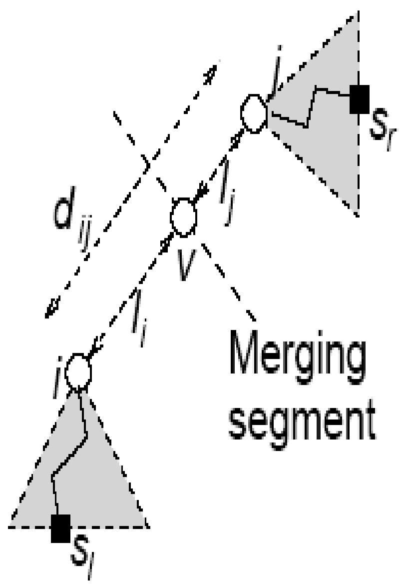


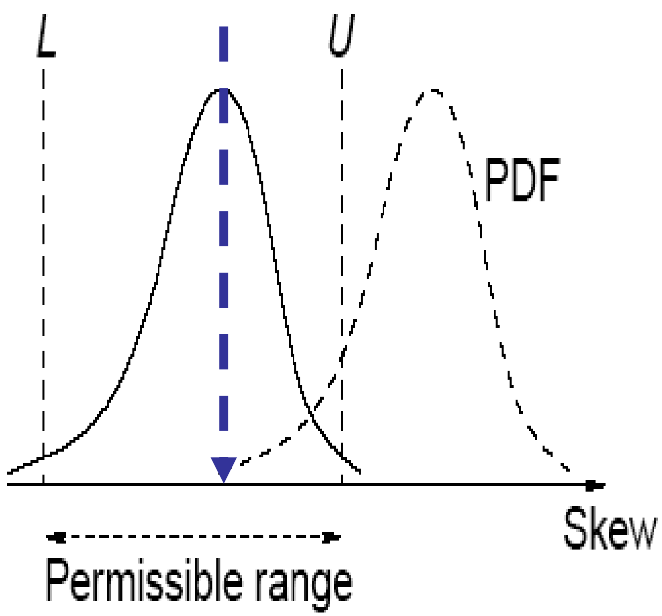


Previous Research

- Layout embedding method to achieve zero skew
- Deferred Merging Embedding technique (DME)
- Nearest neighbor based abstract tree method (NNA)
- Bounded skew tree (BST) algorithm
- Minimal Skew Violations (MinSV)

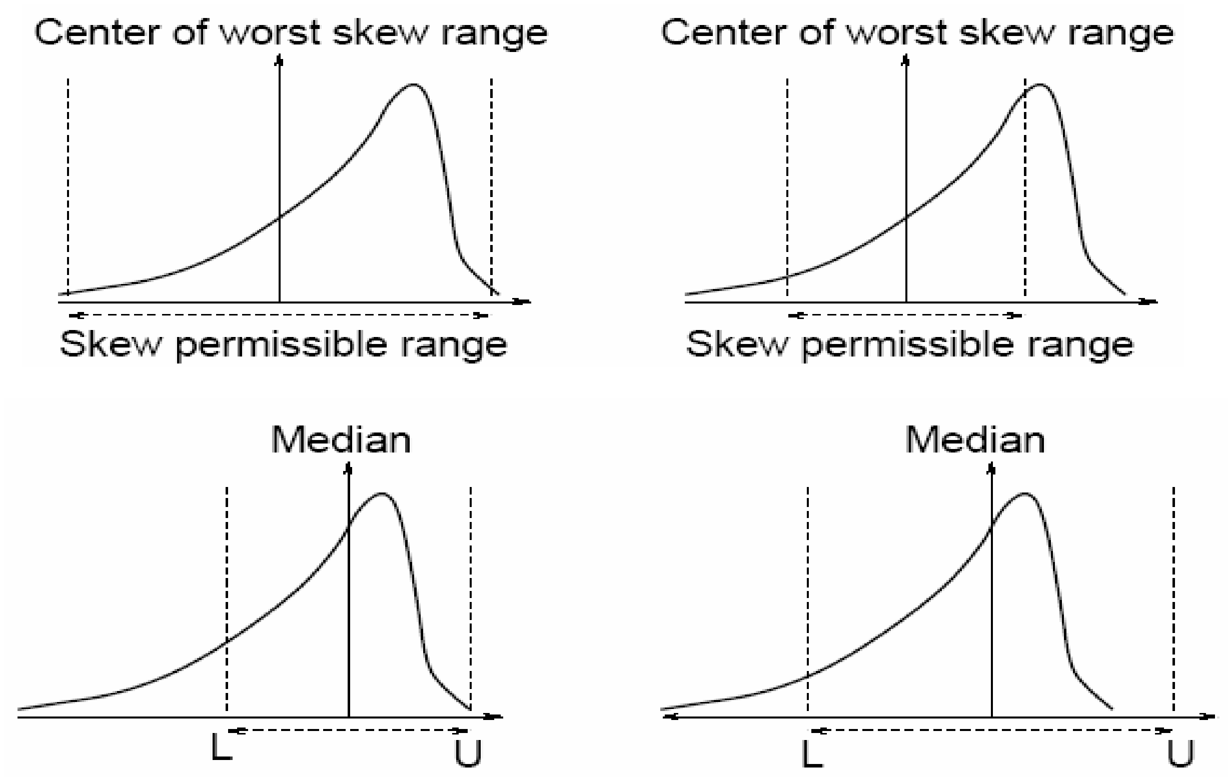
Permissible Range



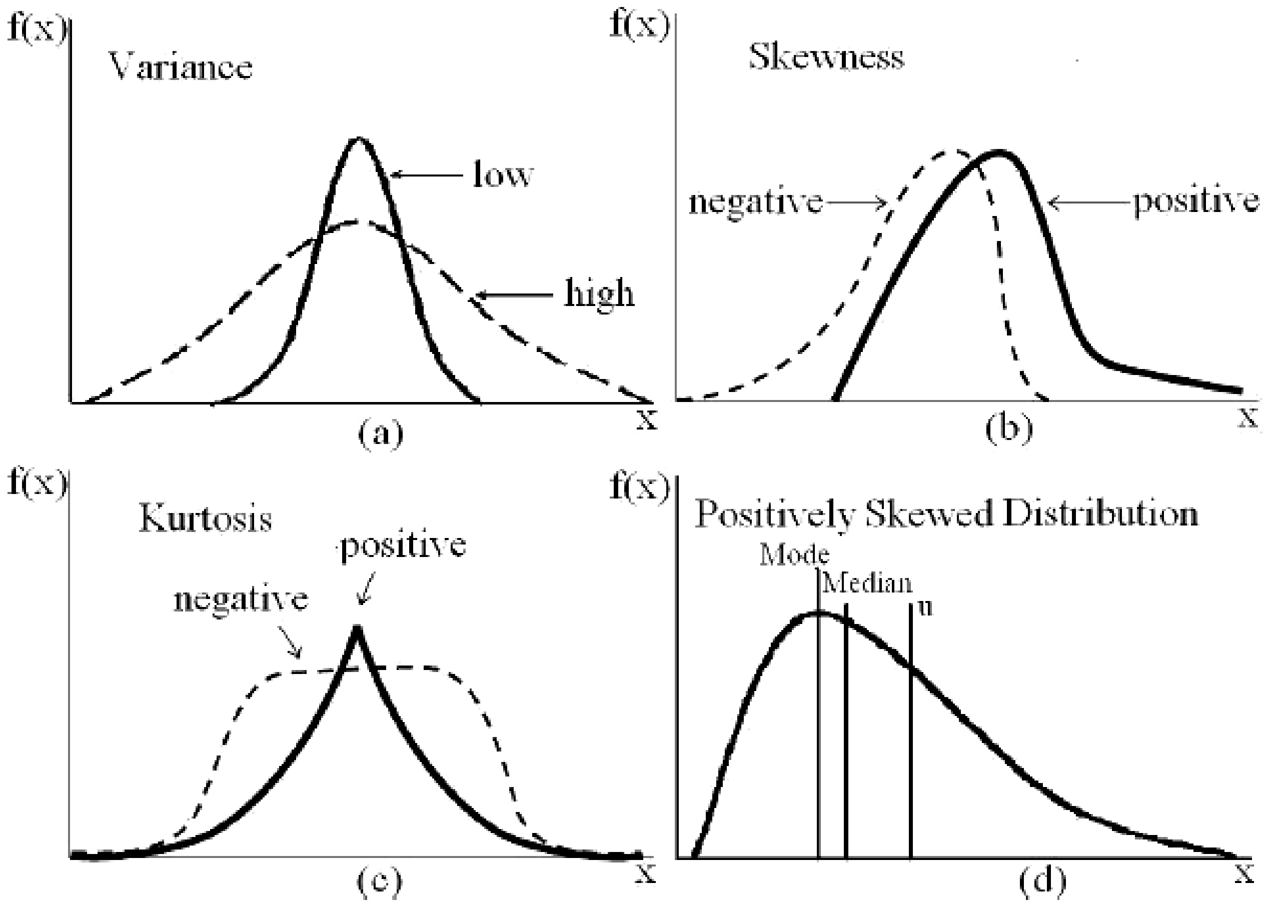


Align the mean to the center

Skew and Permissible Range



Basic Concepts



Existing Issues

- Most of the research works are for deterministic clock tree routing
- Some research works consider corner cases

not work well with statistical clock skew

Possible Strategies

- Skew can be non-Gaussian distributed: asymmetric
- Only mean and variance are not enough
- Our methodology:
  - include high order moments
  - center measures include: Mode, Median and Mean

A Task List

- Variation aware delay model
- Parameter reduction based on ANOVA and OPCA
- Choose among Mean, Median and Mode as Central Measures
- Select wire length based on Central Measures
- Variation aware abstract tree topology

Delay model

$$d_{FEDk} = Ar_d c_d \sum_{i \in T} l_i w_i + Br_d c_f \sum_{i \in T} l_i + Cr_d \sum_{j \in S} cL_j$$

$$+ D\rho c_a \sum_{i \in P_k} \frac{l_i}{w_i t_i} \left(\frac{l_i w_i}{2} + \sum_{j \in E_i} l_j w_j \right)$$

$$+ E\rho c_f \sum_{i \in P_k} \frac{l_i}{w_i t_i} \left(\frac{l_i}{2} + \sum_{j \in E_i} l_j \right)$$

$$+ F\rho \sum_{i \in P_k} \frac{l_i}{w_i t_i} \left(\sum_{j \in S_i} cL_j \right)$$

Delay Mean and variance

First Order Estimates

$$\mu_T = d_{FED}\,|_{w=\mu_w,\,t=\mu_t,\,r_d=\mu_{rd},\,c_L=\mu_{cL}} +$$

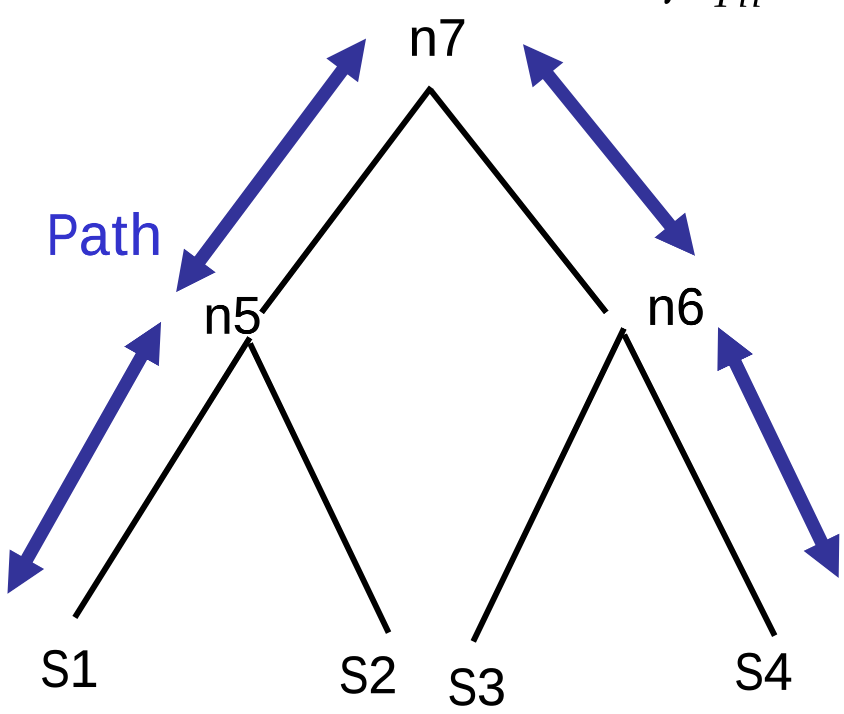
$$+\frac{1}{2}[(E\frac{\rho c_f l^2}{t} + 2F\frac{\rho l c_L}{t})\frac{1}{\mu_w^3}$$

$$+(D\rho c_a l^2 + E\frac{\rho c_f l^2}{w} + 2F\frac{\rho l c_L}{w})\frac{1}{\mu_t^3}]$$

$$\sigma_T^2 \approx (\frac{\partial d_{FED}}{\partial r_d})^2\sigma_{rd}^2 + (\frac{\partial d_{FED}}{\partial c_L})^2\sigma_{cL}^2$$

$$+(\frac{\partial d_{FED}}{\partial w})^2\sigma_w^2 + (\frac{\partial d_{FED}}{\partial t})^2\sigma_t^2$$

Delay Mean and Variance



$$\mu_{Til} = \sum_{k \in P_{il}} \mu_{Tk} \qquad \sigma^2_{Til} = \sum_{k \in P_{il}} \sigma^2_{Tk}$$

$$\mu_{Slr} = \mu_{Til} - \mu_{Tjr}$$

$$\sigma^2_{Slr} = \sigma^2_{Til} + \sigma^2_{Tjr}$$

Delay Median

Edge Delay Median:

$$M_{\mathbf{T}} = D\frac{\rho c_a l^2}{2M_t} + \frac{E\rho c_f l^2}{2M_t M_w} + \frac{F\rho M_{c_L}}{M_t M_w}$$

Path Delay Median:

$$M_{\mathbf{T}il} = \sum_{k \in P_{il}} M_{\mathbf{T}k}$$

Skew Median:

$$M_{\mathbf{S}lr} = M_{\mathbf{T}il} - M_{\mathbf{T}jr}$$

Delay Mode

Matching the delay mode by Weibul function:

$$f_{WB}(x) = \alpha\beta^{-\alpha}x^{\alpha-1}e^{-(x/\beta)^{\alpha}}$$

$$\mu_{WB} = \beta\Gamma(1+\theta)$$

$$\sigma^2_{WB} = \beta^2\left(\Gamma(1+2\theta) - \Gamma^2(1+\theta)\right)$$

$$\frac{\Gamma(1+2\theta)}{\Gamma^2(1+\theta)} = \frac{\sigma^2_{Slr}+\mu^2_{Slr}}{\mu^2_{Slr}} \qquad \beta = -m_1/\Gamma(1+\theta).$$

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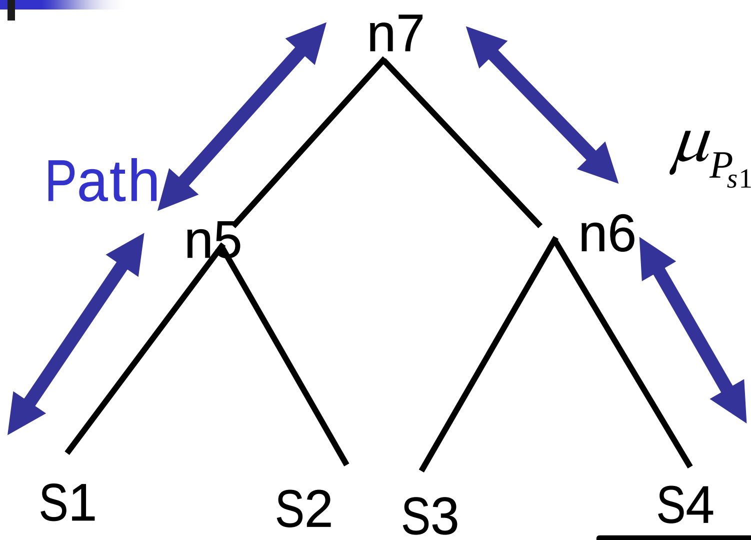
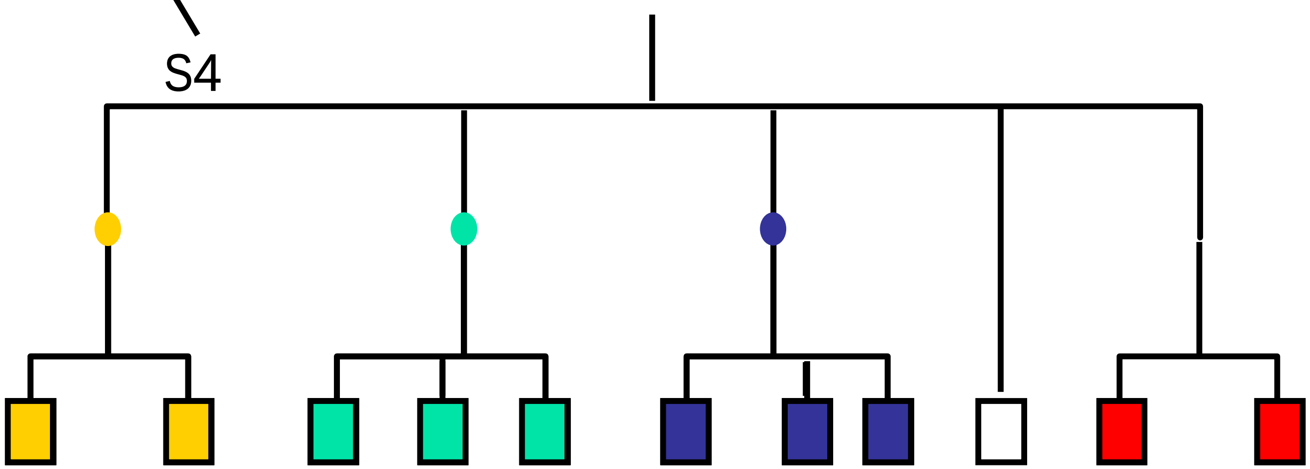
$$Mode = \sqrt[\alpha]{\frac{\alpha-1}{\beta^{-\alpha}\alpha}}$$

ANOVA



$$\mu_{P_{s1,n7}}(w_1,t_1,w_2,t_2)=\mu_{s_1,n_5}(w_1,t_1)+\mu_{n_5,n_7}(w_2,t_2)$$



$$\mu_{P_{s1,n7}}(w_1,t_1,\mu_{n_5,n_7})=\mu_{s_1,n_5}(w_1,t_1)+\mu_{n_5,n_7}$$

Statistical Centering Based Layout Embedding

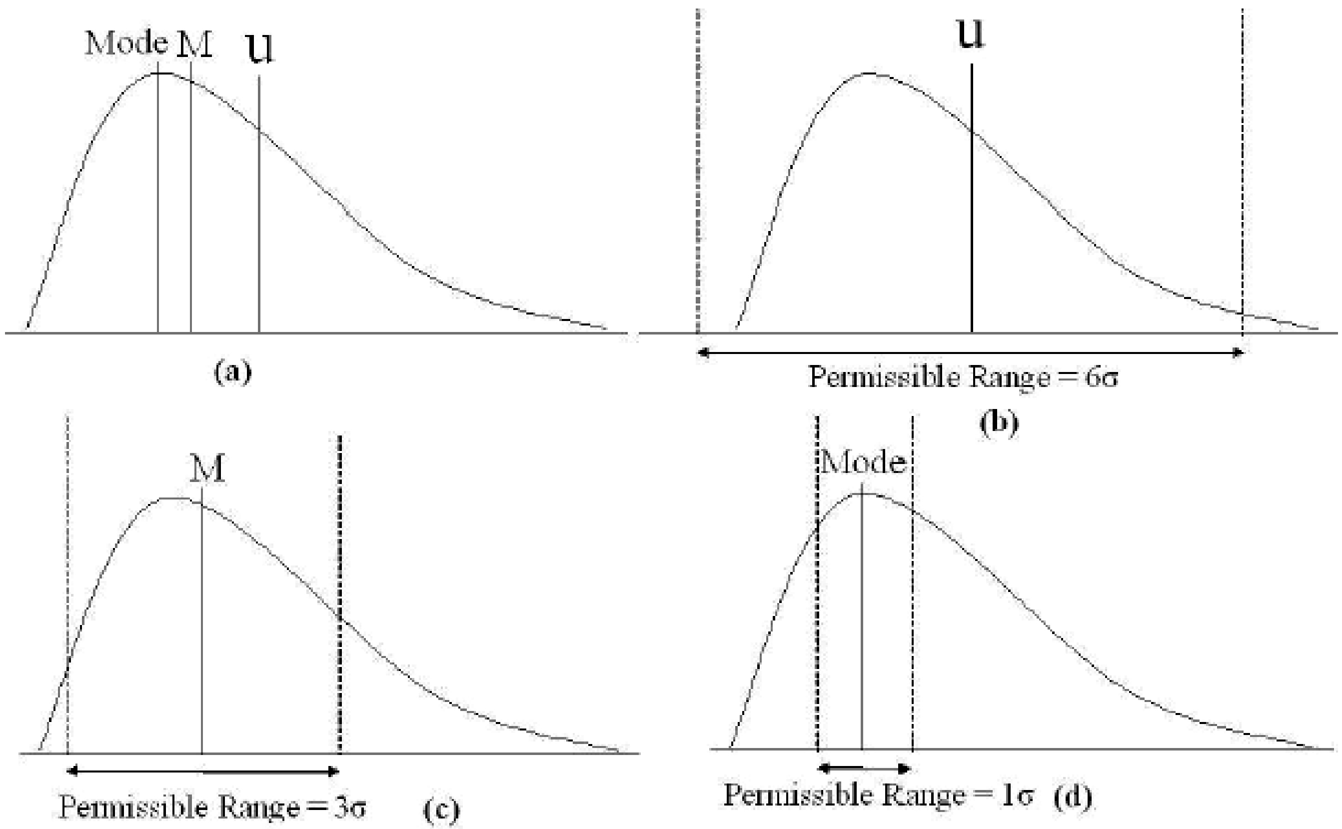
CASE I. $|\varsigma| \leq \varepsilon, \forall |PR_{lr}|$. The distribution is symmetric or almost symmetric. Therefore, $CM_{\mathrm{S}lr} = \mu_{\mathrm{S}lr}$.

CASE II. $|\varsigma| > \varepsilon$, $|PR_{lr}| > 5\sigma$. The distribution can be easily fit within the permissible range by aligning the mean. Therefore, $CM_{\mathrm{S}lr} = \mu_{\mathrm{S}lr}$. Choosing the median or mode might lead to an excessive part of the distribution lying outside the permissible range.

CASE III. $|\varsigma| > \varepsilon$, $2\sigma < |PR_{lr}| < 5\sigma$. The permissible range is not large enough to fit the entire distribution and is not extremely narrow either. As shown in Figure 6(c), $CM_{\mathrm{S}lr} = M_{\mathrm{S}lr}$ is the best choice in this case.

CASE IV. $|\varsigma| > \varepsilon$, $|PR_{lr}| < 2\sigma$. The permissible range represents a very stringent constraint. Based on the previous discussion, we choose $CM_{\mathrm{S}lr} = Mode_{\mathrm{S}lr}$ to maximize the area within bounds.

Statistical Centering Based Layout Embedding



Mean, Median and Mode based design

$$q_{ij} = A_s {l_i}^2 + B_s l_i - C_s$$

where

$$A_s = \frac{\rho}{2}[c_a(\frac{D_i}{t_i} - \frac{D_j}{t_j}) + c_f(\frac{E_i}{w_i t_i} - \frac{E_j}{w_j t_j})]l$$

$$B_s = \rho[d_{ij}(\frac{D_j c_a}{t_j} + \frac{E_j c_f}{w_j t_j}) + \frac{F_i c_{Li}}{w_i t_i} + \frac{F_j c_{Lj}}{w_j t_j}]$$

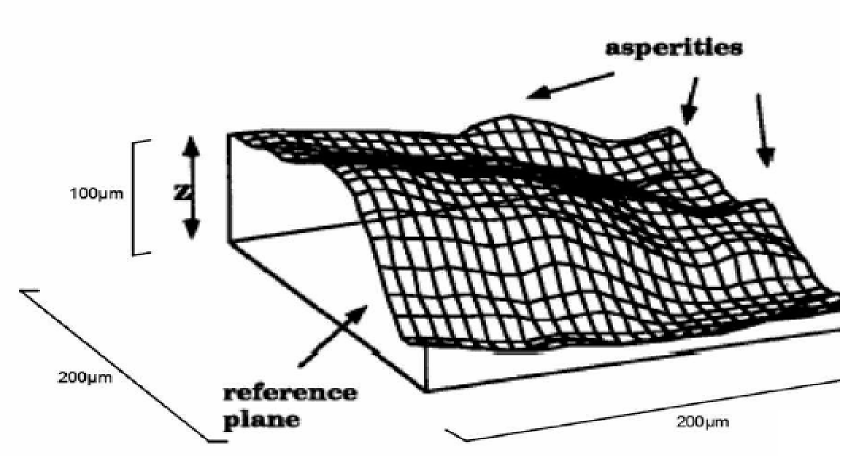
$$C_s = F_j \rho c_{Lj} d_{ij} \frac{1}{w_j t_j}$$

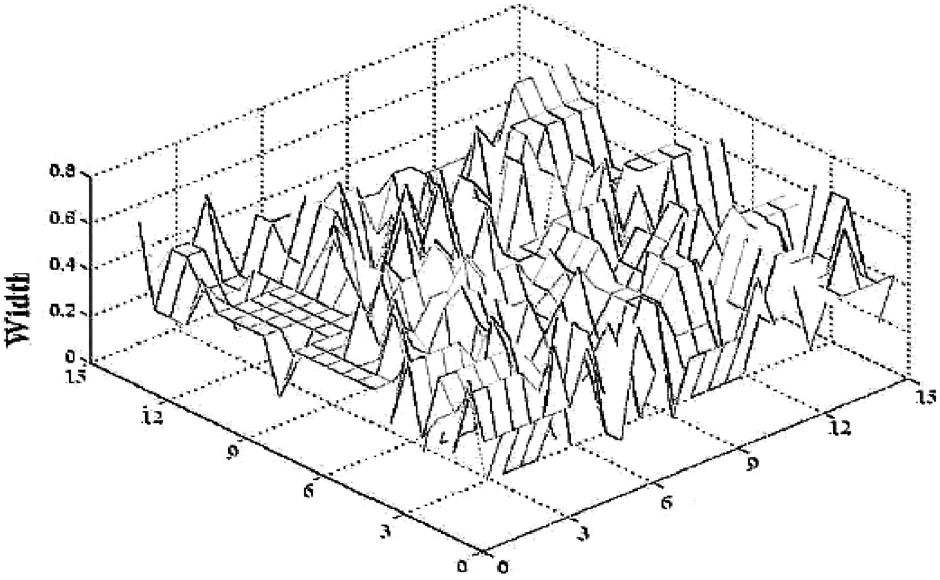
Variation Aware Abstract Tree Topology

- Extend the Nearest Neighbor Algorithm (NNA)
- Length is positive dependent on the distance between nodes $\partial l = f(d_{ij})\partial d_{ij}$
- The minimal composite distance

$$d'_{ij} = d_{ij}^2 \left(\lambda + \left| \frac{1}{w_i} - \frac{1}{w_j} \right| \right)$$

Process variation model





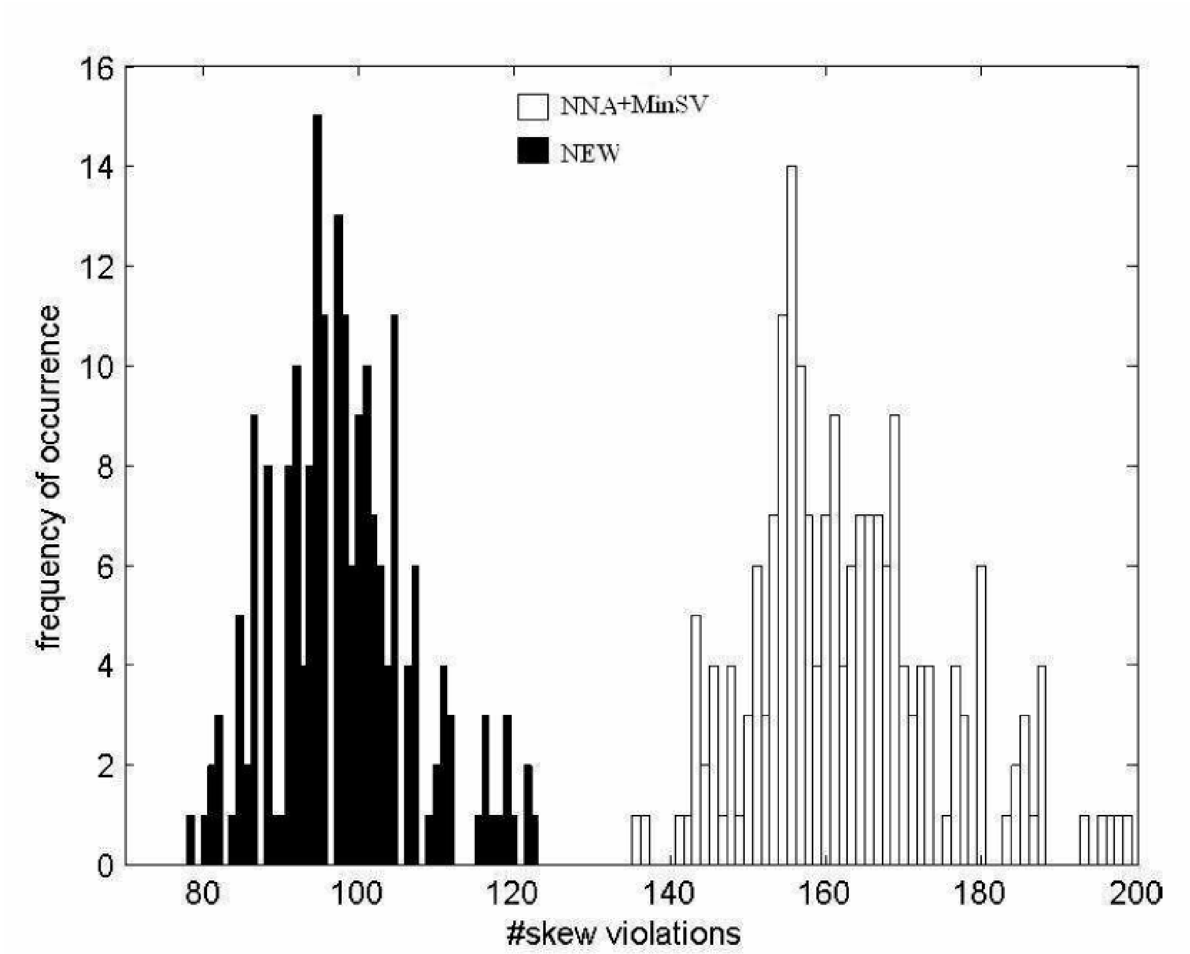
Experimental Results

	Profile1					
	wirelength(μm)			#skew violations		
	NNA+MinSV	NEW	Imprv	NNA+MinSV	NEW	Imprv
r1	137848	127364	8%	74	61	17%
r2	292006	257338	12%	218	140	36%
r3	339958	322035	5%	246	194	21%
r4	695116	657460	5%	358	287	20%
r5	1034488	983639	5%	501	363	28%
Prim1	134045	131013	2%	42	37	12%
Prim2	352054	308380	12%	155	97	37%
s1423	110625	104935	5%	26	20	23%
s5378	221017	169052	23%	75	56	25%
s15850	446656	431137	3%	226	199	12%

Experimental Results

Profile2					
wirelength(μm)			#skew violations		
NNA+MinSV	NEW	Imprv	NNA+MinSV	NEW	Imprv
135012	131155	3%	53	42	21%
286685	262892	8%	163	102	37%
347469	329361	5%	173	138	20%
691496	664143	4%	240	196	18%
1036414	986358	5%	351	272	22%
135352	132238	2%	32	27	16%
328610	312532	5%	118	94	20%
109588	108727	0%	19	16	16%
174745	170404	2%	50	44	12%
445104	437260	2%	174	145	17%

Experimental Results



Run Time Comparison

	Sinks	CPU(s)	
		NNA+MinSV	NEW
r1	267	1.5	1.5
r2	598	2.3	2.3
r3	862	3.3	3.4
r4	1903	10.1	10.8
r5	3101	24.5	26.1
Prim1	269	1.5	1.5
Prim2	603	2.2	2.3
s1423	74	1.3	1.3
s5378	179	1.4	1.4
s15850	597	2.3	2.3

Conclusions

- A Statistical centering approach
- A Fitted Elmore delay model with analysis of variance and principle component analysis
- A topology generation algorithm which takes the width and thickness into consideration

Thank you