Efficient Generation of Short and Fast Repeater Tree Topologies

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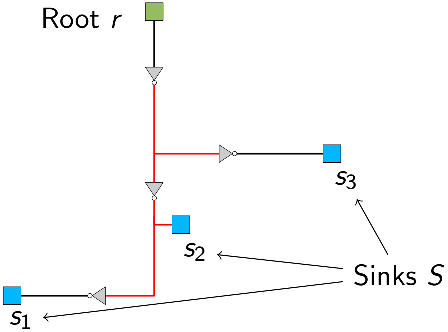
Outline

- Repeater Tree Problem
- Delay Model
- Topology Construction Algorithm

The Repeater Tree Problem



- A signal has to be distributed from a source to a set of sinks.
- The delay on a source-sink path increases
  - linearly in path length (assuming ideal repeater insertion),
  - with every bifurcation on the path.

The Repeater Tree Problem

Objectives

- Minimize power consumption
- Minimize wiring
- Maximize worst slack σ_r, where

$$\sigma_r := \min_{s \in S}\{RAT_s - \text{signal\_delay}(r, s)\}$$

The Repeater Tree Problem

Two-step Approach

First a repeater tree topology is constructed. Then repeaters are inserted in a second step (for example using a van Ginneken's style algorithm).

One-step Approach

Repeater insertion and topology generation are interleaved.

In this paper we focus on the topology generation in a two-step approach.

Previous Work

General Approaches to Topology Generation

- Minimum length rectilinear steiner tree
- Minimum spanning tree
- Shortest path trees

Problem-specific Approaches to Topology Generation

- C-Tree [Alpert et al., 2001]
- PRAB [Hu, Alpert, 2004]

Delay Estimation

- BELT [Alpert et al., 2004]

Our contribution

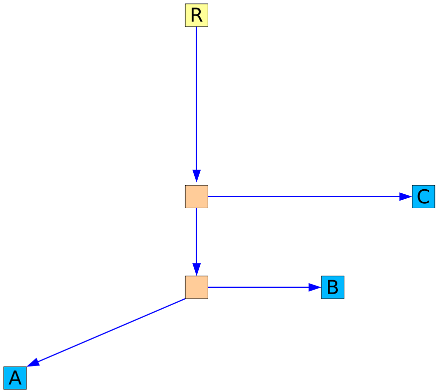
- A new delay-model for evaluating repeater tree topologies
- Theoretical bounds on the achievable slack
- A fast algorithm for topology construction considering our delay-model
- Optimality statements for our topology generation

Topology

A topology T is a directed tree rooted at r with $\delta^+(r) = 1$ and $\delta^+(u) = 2$ for all internal nodes u.

The set of leaves is a subset of S.

All internal nodes u are assigned placement coordinates $Pl(u)$.



Delay Model

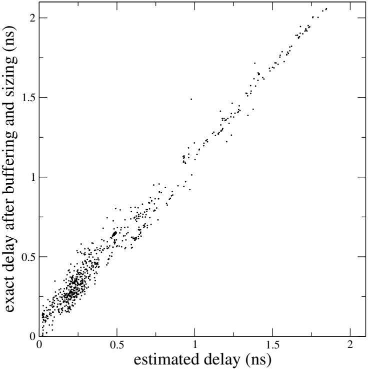
The delay from r to a sink s in a given topology is modeled as:

$$c_{node} \cdot (|E(T_{[r,s]})| - 1) + c_{wire} \sum_{(u,v) \in E(T_{[r,s]})} dist(Pl(u), Pl(v))$$

- c_{node}: Delay penalty for bifurcation
- c_{wire}: Delay per unit length
- Typical values are $c_{node} = 20$ ps and $c_{wire} = 220$ ps/mm.

Justification of Delay Model

Relation between critical path delays in our model (estimated delay) and with exact timing analysis after repeater insertion.



Bound on Wire Length

A lower bound on the wire length in our model is given by a minimum length rectilinear steiner tree (SMT).

Bound on Slack for Integer Values

Theorem 1

For $c_{wire} = 0$, $c_{node} = 1$ and integer values for AT_r and RAT_s for each $s \in S$ the maximum possible slack with respect to our delay model is:

$$-\left\lceil \log_2 \left(\sum_{s \in S} 2^{AT_r - RAT_s} \right) \right\rceil$$

Proof of Theorem 1

By Kraft's inequality there exists a rooted binary tree with n leaves at depth $l_1, l_2, \ldots, l_n$ if and only if

$$\sum_{i=1}^{n} 2^{-l_i} \leq 1$$

To realize a slack of at least σ we must find a topology in which $RAT_s - AT_r - d_s \geq \sigma$ holds for every sink s. The value d_s corresponds to the depth of sink s.
The maximum slack that can be realized is the largest integer σ_{max} that satisfies:

$$\sum_{s \in S} 2^{AT_r - RAT_s + \sigma_{\max}} \leq 1$$

Bound on Slack

Theorem 2

The maximum possible slack $\sigma_{\max}$ with respect to our delay model at root is at most:

$$-c_{node} \cdot \log_2 \left(\sum_{s \in S} 2^{-\left(\frac{RAT_s - c_{wire} dist(Pl(r), Pl(s))}{c_{node}} \right)} \right)$$

Sketch of Proof

Using Kraft's inequality and

$$RAT_s - AT_r - c_{wire} dist(Pl(r), Pl(s)) - c_{node} d_s \geq \sigma_{\max}$$

Improving the Upper Bound

The closed formula has two drawbacks:

- Integrality properties of the topology are neglected.
- Correct evaluation leads to numerical problems.

A better upper bound can be obtained algorithmically by using Huffman coding:

- No closed formula.
- Slightly better bounds.
- Numerical stable and loglinear runtime.

Using Huffman Coding

1. Set $\sigma_s = RAT_s - AT_r - c_{wire} dist(Pl(r), Pl(s))$ for all $s \in S$.
2. Order these values

$$\sigma_{s_1} \leq \sigma_{s_2} \leq \ldots \leq \sigma_{s_n}$$

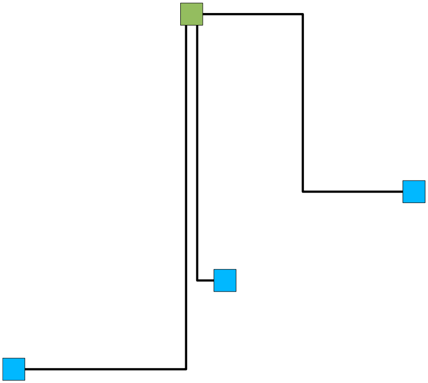
3. Replace the largest two $\sigma_{s_{n-1}}$ and σ_{s_n} by

$$-c_{node} + min\{\sigma_{s_{n-1}}, \sigma_{s_n}\} = -c_{node} + \sigma_{s_{n-1}}$$

4. Go to 2.

Realization of the Maximum Slack

The maximum possible slack can be obtained by a shortest path tree:

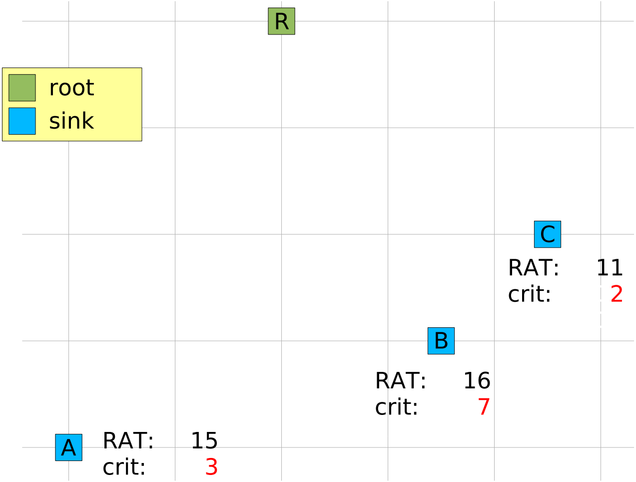


All distance delays are minimum: For each sink s, the distance part of the modeled delay attains the minimum possible value.

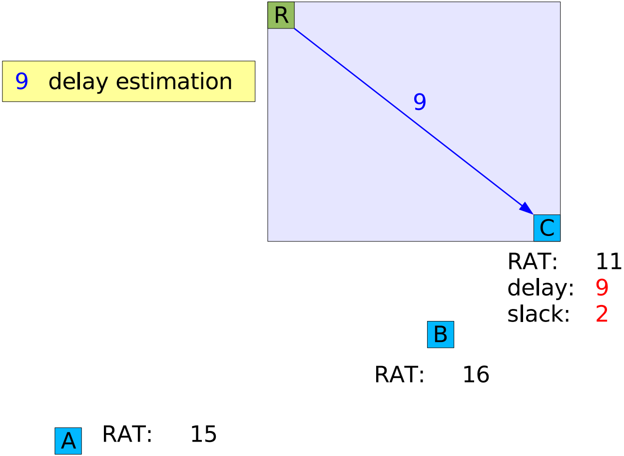
Topology Construction Algorithm

1. Sort sinks according to criticality (worst to best).
2. Start with a tree consisting of r and the first sink.
3. For each sink s, connect s to an edge of the tree, minimizing the cost function.

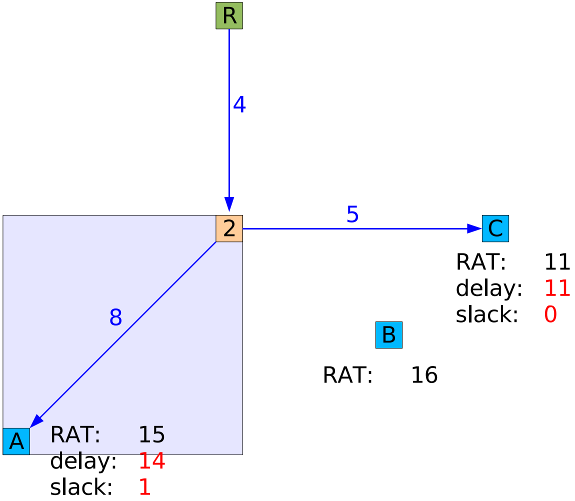
Example Problem Instance



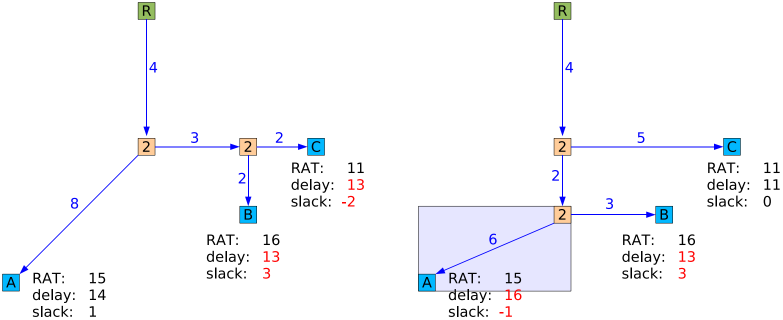
Connect first sink



Connect second sink



Connect third sink



Prim-Heuristic for Steiner Trees

Wire Length Minimization:

- Instead of choosing next critical sink:
- Choose sink, which is closest to the preliminary topology T'.
- Well known heuristic existing in many variants.

Hwang $\Longrightarrow$ $\frac{3}{2}$-approximation algorithm for SMT.

Theorem 3

For $c_{wire} = 0$, $c_{node} = 1$ and integer values for RAT_s, $s \in S$, the algorithm generates a topology that realizes the maximum possible slack.

Proof.

Assume the sinks in $S' \subset S$ are already connected optimally in T'. Let $s' \in S \setminus S'$.

- If all $s \in S'$ have the same slack $\sigma_{S'}$ in T'.
  - They are connected at maximum possible slack.
  - The best possible slack for the set $S' \cup s'$ equals $\sigma_{S'} + 1$.
  - s' can be connected to any existing edge in T' such that its slack is $\leq \sigma_{S'} + 1$.
- Otherwise s' can be connected to any non-critical edge.

Running Time

The running time is $O(|S|^2 \cdot \Psi)$, where Ψ is the running time of the cost function.

Handling Large Instances

- Pre-clustering if $|S| > 10\,000$
- Facility location approximation [Massberg, Vygen 2005]
- Runtime: $O(|S| \log |S|)$

Parameter Generation

Delay per nanometer

Insert repeaters in a 5 m long two-point net such that delay is minimized.

Delay per bifurcation

Insert a medium-sized repeater half-way between two repeaters of such a net.

Experimental Results

- 2.3 million instances with up to 10 000 sinks were taken from current 90nm designs.
- The slack minimizing cost function is compared against the slack bound (Huffman Coding).
- A length minimizing cost function is compared against a length bound.
- The topologies were computed in ≤ 50 seconds on a 2.6 GHz Opteron.

Results

# Sinks	# Instances	Wirelength Deviation (%) avg.	Wirelength Deviation (%) worst	Slack Deviation (ps) avg.	Slack Deviation (ps) worst	Wirelength Deviation (%) avg.	Wirelength Deviation (%) worst	Slack Deviation (ps) avg.	Slack Deviation (ps) worst
1	1547517	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	319759	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	165448	0.00	0.00	13.89	82.72	12.19	99.60	0.12	20.00
4	86377	0.16	19.65	23.72	312.98	10.93	190.27	0.27	40.00
5	44301	0.16	21.51	33.40	174.51	14.01	188.15	0.34	52.45
6	27854	0.28	23.84	41.92	118.27	14.38	268.06	1.04	52.93
7	20523	0.45	22.24	52.19	285.43	22.26	248.77	0.42	52.51
8	19300	0.44	30.73	64.01	332.29	19.39	268.49	2.08	69.13
9	11085	0.81	26.26	71.11	465.77	29.58	250.04	3.36	60.00
10	11942	0.74	28.68	76.46	367.39	23.61	296.47	1.45	54.87
11-20	38184	1.60	28.00	101.16	427.25	32.57	426.68	1.73	76.80
21-30	11104	3.20	30.80	144.27	520.00	35.86	805.45	2.51	84.18
31-50	8647	2.99	33.16	226.05	793.70	70.29	1091.17	6.55	161.81
51-100	6621	4.06	26.34	344.88	1486.06	105.90	1782.56	12.23	203.48
101-200	1863	5.82	16.91	606.26	2019.90	135.84	1498.34	19.78	351.25
201-500	824	6.22	24.00	920.37	3711.47	209.77	2127.34	26.91	304.92
501-1000	205	7.62	19.40	1686.15	3563.61	569.58	2242.49	48.57	257.65
$>$ 1000	31	6.99	14.74	2929.08	7872.96	211.40	1124.99	17.78	89.88
Total	2321585	**0.66**	33.16	**9.92**	7872.96	**19.35**	2242.49	**0.21**	351.25
$>$ 2 sinks	774068	**1.31**	33.16	**50.69**	7872.96	**38.34**	2242.49	**1.08**	351.25

Table: Deviation from known bounds, 90 nm

Thank you