SAMSON: A Generalized Second-order Arnoldi Method for Reducing Multiple Source Linear Network with Susceptance

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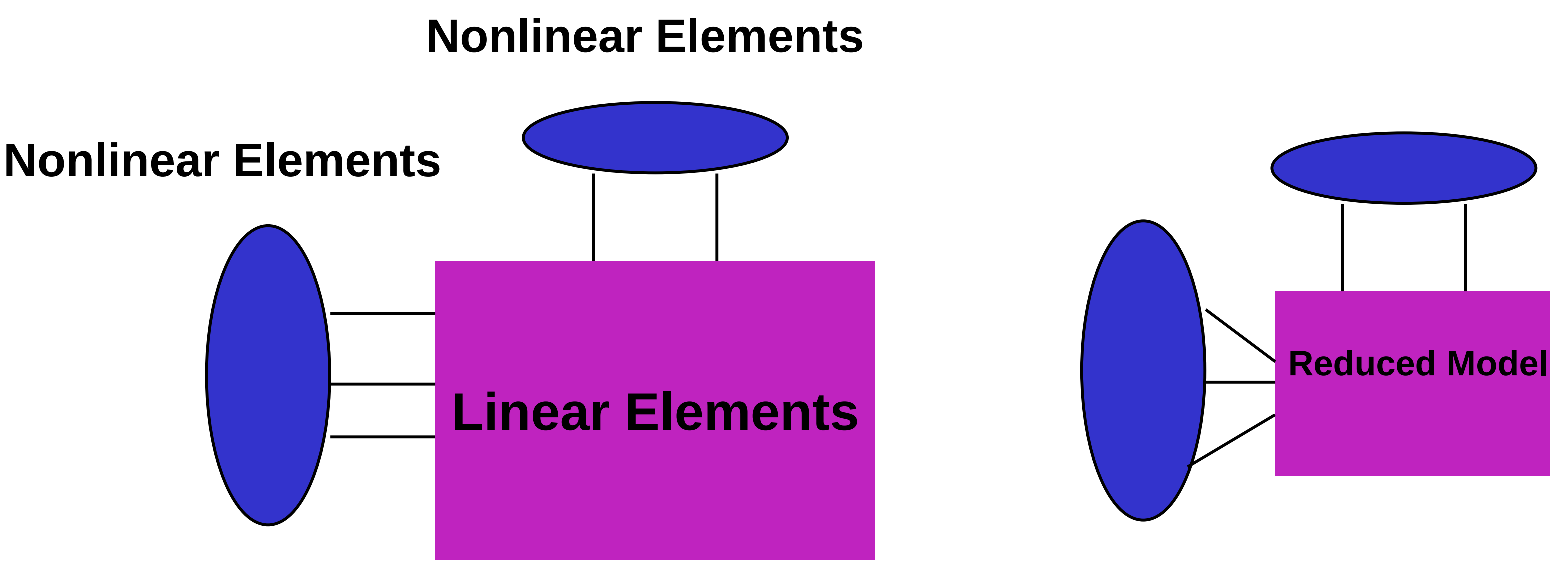
Outline

- Review and Motivation
- SAMSON Algorithm
- Experimental Results
- Conclusions

Motivation for Model Order Reduction

- Deep submicron design needs to consider a large number of linear elements
  - Interconnect, Substrate, P/G grid, and Package
- Accurate extraction leads to the explosion of data storage and runtime
- Need efficient macro-model



Non-RHS MOR vs. RHS MOR

- Non-RHS MOR reduces transfer function H(s)
  - First Order Method to handle L
    - PRIMA [Pileggi et al, TCAD'98]
  - Second Order Methods to handle S (susceptance)
    - ENOR [Sheehan, DAC'99]
    - SMOR [Pileggi et al, ICCAD'02]
    - SAPOR [Su et al, ICCAD'04] [Liu et al, ASPDAC'05]
- Main Limitation
  - Can only match up to ceil(n/n_p) moments
  - Accuracy is significantly limited when the port number n_p is large

- RHS MOR reduces output vector y(s)
  - [Chiprout, ICCAD'04]
    - explicit moment matching is used (lack in numerical stability)
  - EKS [J. M. Wang et al, DAC'00]
    - Implicit moment matching based on Incremental orthonormalization
    - frequency domain shifting to deal with 1/s and $1/s^2$ terms
  - IEKS [Y. Lee et al, TCAD'05]
    - Based on the observation that there are no 1/s and $1/s^2$ terms for PWL sources in finite time

Problems Still Remain Unsolved

- None of the existing RHS MOR methods can deal with RCS circuits with susceptance elements
  - Both EKS and IEKS are first order methods
  - Directly applying them to RCS circuits cannot guarantee passivity
- There is still much room to improve accuracy
  - Incremental Orthonormalization causes error to accumulate
  - When matching high order moments, it becomes inaccurate
- None of the existing MOR methods can handle arbitrary independent inputs
  - Especially when they contain $1/s^i$ terms (i>0)
    - frequency domain shifting (inaccurate)
  - In the existence of infinite PWL sources
    - EKS and IEKS cannot consider s=0 (cannot perform DC analysis)

Major Contributions of SAMSON

- It is an RHS MOR method
  - Compare with SAPOR and other non-RHS methods, it is more accurate
  - Can handle a large number of ports
- It can deal with all kinds of input sources accurately without frequency domain shifting or incremental orthonormalization
  - Numerically more stable, more efficient and more accurate in the whole frequency domain, especially at DC (s=0)
- It is based upon generalized second order Arnoldi method
  - Can handle RCS circuits with passivity

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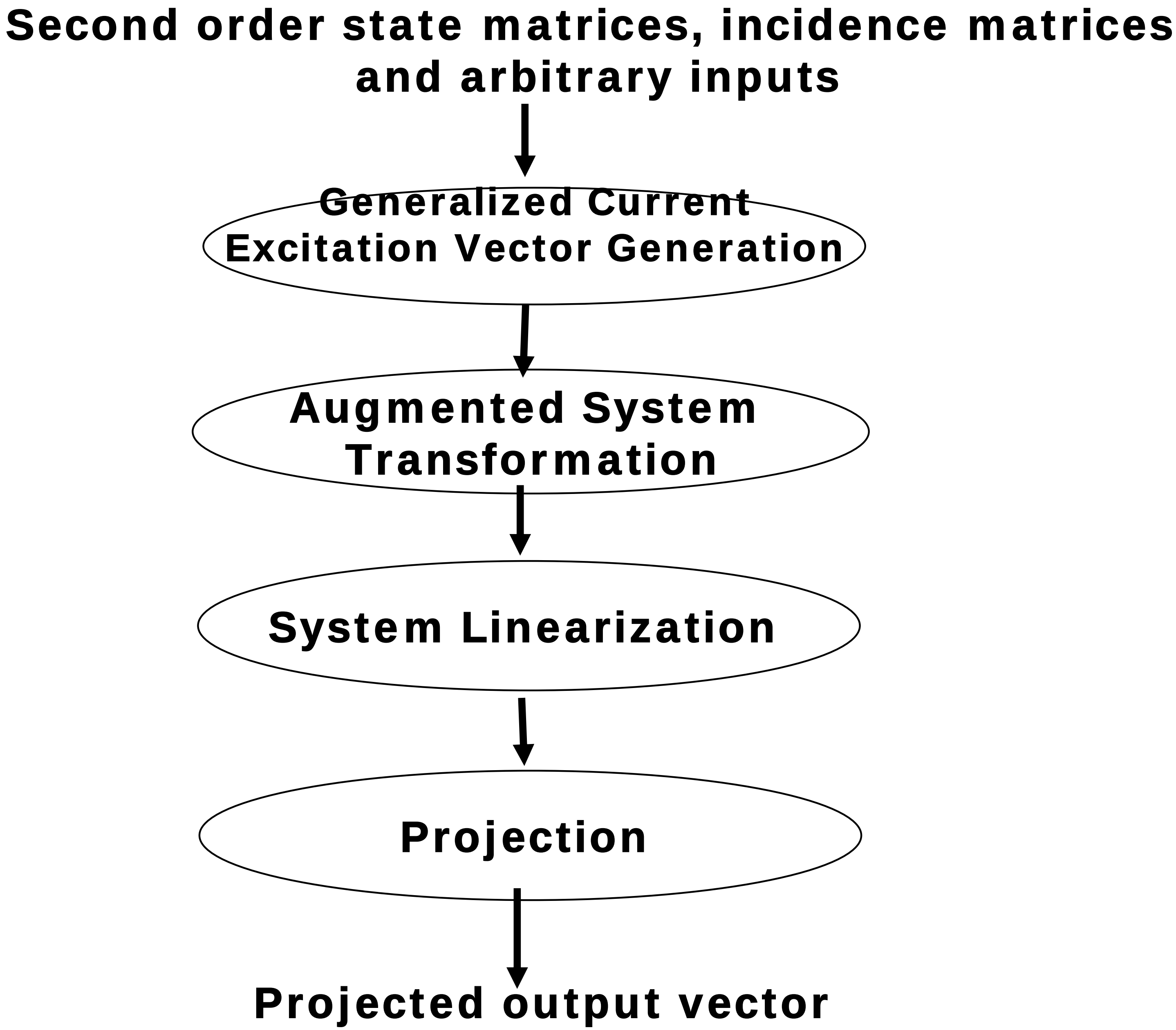
SAMSON Algorithm: Overview



Generalized Current Excitation Vector

$$(G + sC + \frac{\Gamma}{s})V(s) = BJ_e$$

Original system equation

Superposed SIMO system equation

$$\sum_{i=0}^{m+1} \gamma_i s^i V(s) = \sum_{i=0}^{n} \Theta_i s^i$$

- Define the generalized current excitation vector J_{ex} as

$$J_{ex} = BJ_e$$

- J_{ex} can be divided into two categories
  - Rational

$$J_{ex}(s) = \frac{a_0 + a_1 s + \ldots + a_n s^n}{b_0 + b_1 s + \ldots + b_m s^m}$$

  - Irrational
    - Can be expanded into Taylor series and take dominant terms. Then it becomes a special case of the Rational category

$$J_{ex}(s) = \sum_{i=0}^{n} J_i s^{i-m}$$

- Multiply both sides by

$$b_0 + b_1 s + \ldots + b_n s^m$$

SAMSON Algorithm: Overview

Second order state matrices, incidence matrices and arbitrary inputs

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Generalized Current Excitation Vector Generation

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Augmented System Transformation

↓

System Linearization

↓

Projection

↓

Projected output vector

Augmented System Transformation

$$\sum_{i=0}^{m+1} \gamma_i s^i V(s) = \sum_{i=0}^{n} \Theta_i s^i$$

Superposed SIMO system equation

Augmented system equation

$$\sum_{i=0}^{n+1} \Psi_i s^i U = \begin{bmatrix} \sum_{i=0}^{n} \Theta_i s^i \\ 0 \end{bmatrix}$$

$$U = [V, V_1, V_2, \ldots, V_{n-m}]^T$$

- To linearize, we have to make LHS exactly one order higher than RHS by raising the order of LHS by n-m .
  - Introduce auxiliary variables V_1, V_2, …, V_n satisfying

$$V = sV_1, \quad V_1 = sV_2,$$

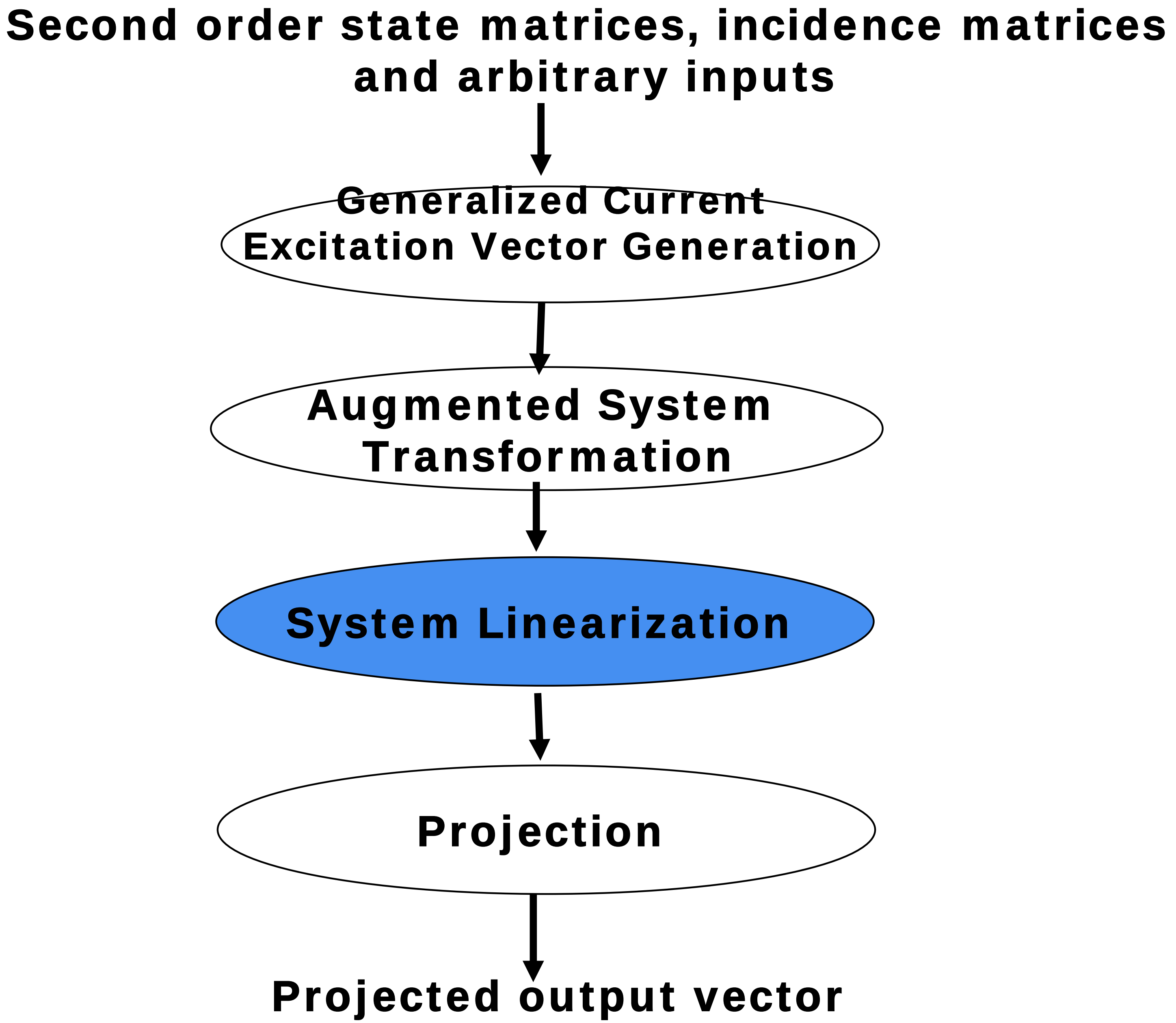
$$\ldots, \quad V_{n-m-1} = sV_{n-m}$$

Here we assume m<n. The transformation for the n<m case can be derived similarly

  - Insert into superposed SIMO system equation =>

$$\sum_{i=0}^{m+1} \gamma_i s^{i+n-m} V_{n-m}(s) = \sum_{i=0}^{n} \Theta_i s^i$$

SAMSON Algorithm: Overview



System Linearization

$$\sum_{i=0}^{n+1} \Psi_i s^i U = \begin{bmatrix} \sum_{i=0}^{n} \Theta_i s^i \\ 0 \end{bmatrix}$$

$$U = [V, V_1, V_2, \ldots, V_{n-m}]^T$$

Augmented system equation

e.g., $A_{n+1}\sigma U + Z_n = R_n$

$$\Rightarrow A_{n+1}\sigma^{n+1} U + Z_n \sigma^n = R_n \sigma^n$$

Insert it into

$$A_{n+1}\sigma^{n+1} U + \ldots = R_n \sigma^n + \ldots$$

and we get

$$(A_n U + Z_n)\sigma^n + \ldots = R_{n-1}\sigma^{n-1} + \ldots$$

- Expand the augmented system at a given frequency $s = s_0 + \sigma$

$$\Rightarrow \sum_{i=0}^{n+1} A_i \sigma^i U(\sigma) = \sum_{i=0}^{n} R_i \sigma^i$$

- Introduce auxiliary variables Z_1, Z_2, …Z_n, satisfying

$$A_{n+1} U \sigma + Z_n = R_n$$

$$(A_n U - Z_n)\sigma + Z_{n-1} = R_{n-1}$$

$$\ldots$$

$$(A_0 U - Z_1)\sigma + Z_1 = R_0$$

and we can obtain

$$(A_0 + A_1 \sigma) U - \sigma Z_1 = R_0$$

System Linearization

$$\sum_{i=0}^{n+1}\Psi_i s^i U = \begin{bmatrix}\sum_{i=0}^{n}\Theta_i s^i \\ 0\end{bmatrix}$$

$$U = [V, V_1, V_2, \ldots, V_{n-m}]^T$$

Augmented system equation

Moment space equation

$$(I-\sigma A)\begin{bmatrix}V\\ D\end{bmatrix} = \begin{bmatrix}q_0\\ p_0\end{bmatrix}$$

- Expand the augmented system at a given frequency $s = s_0 + \sigma$

$$\Rightarrow \sum_{i=0}^{n+1} A_i \sigma^i U(\sigma) = \sum_{i=0}^{n} R_i \sigma^i$$

- Introduce auxiliary variables Z_1, Z_2, …Z_n, satisfying

$$A_{n+1} U\sigma + Z_n = R_n$$

$$(A_n U - Z_n)\sigma + Z_{n-1} = R_{n-1}$$

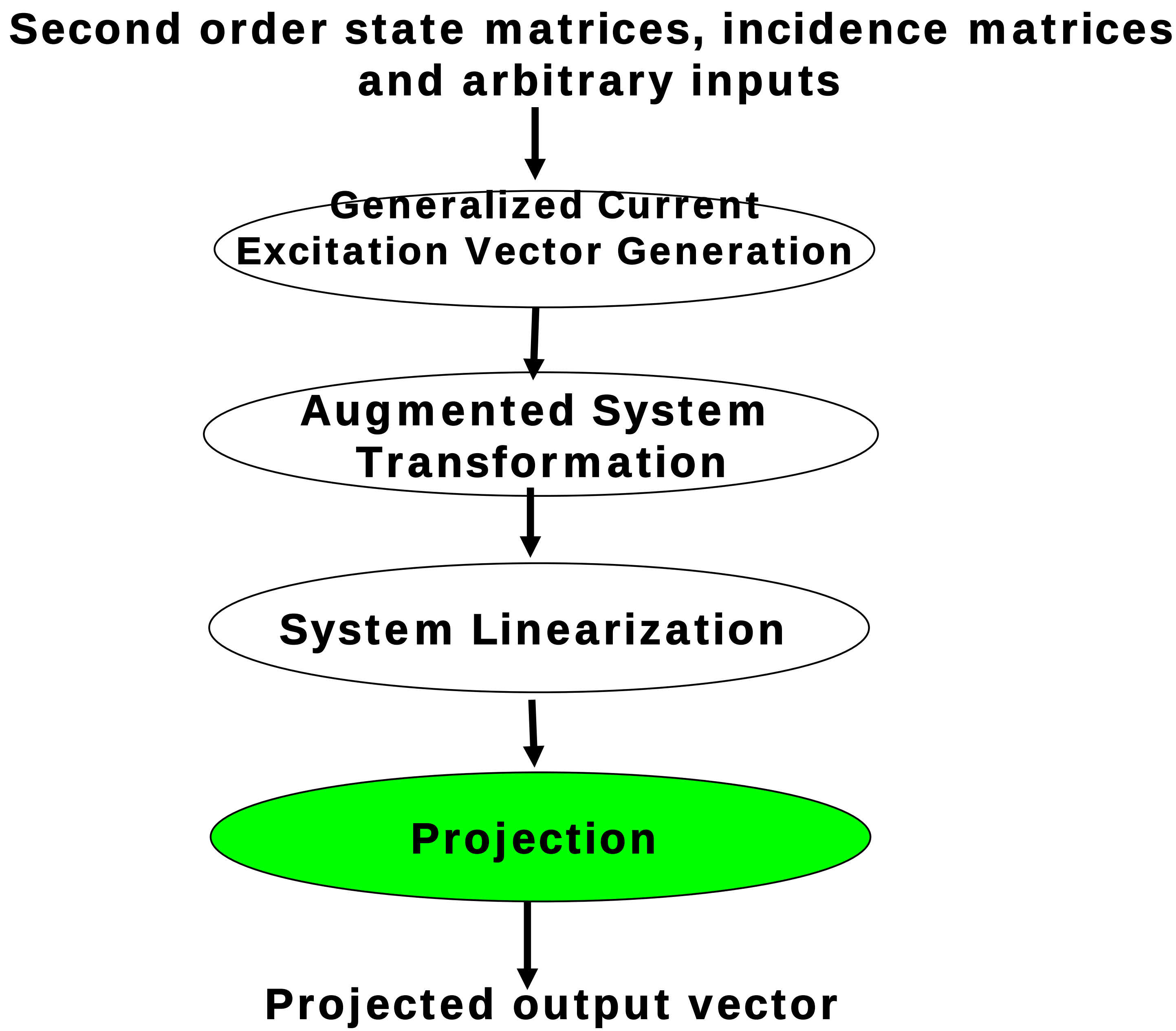
$$\ldots$$

$$(A_0 U - Z_1)\sigma + Z_1 = R_0$$

and we can obtain

$$(A_0 + A_1\sigma)U - \sigma Z_1 = R_0$$

SAMSON Algorithm: Overview



Projection

$$(I-\sigma A)\begin{bmatrix} V \\ D \end{bmatrix} = \begin{bmatrix} q_0 \\ p_0 \end{bmatrix}$$

Moment space equation

- Find projection matrix using method similar to PRIMA
- But only take the first N rows (N is the number of the original variables)
- The moment calculation is efficient because the linearized system is sparse

Projection matrix Q

$$\hat{C} = Q^T C Q,\ \hat{G} = Q^T G Q,\ \hat{\Gamma} = Q^T \Gamma Q,\ \hat{V} = Q^T V$$

$$\hat{J}_{ex} = Q^T J_{ex}$$

Projected system equation

$$(\hat{G} + s\hat{C} + \frac{\hat{\Gamma}}{s})\hat{V}(s) = \hat{J}_{ex}$$

- Directly project on J_{ex} instead of B

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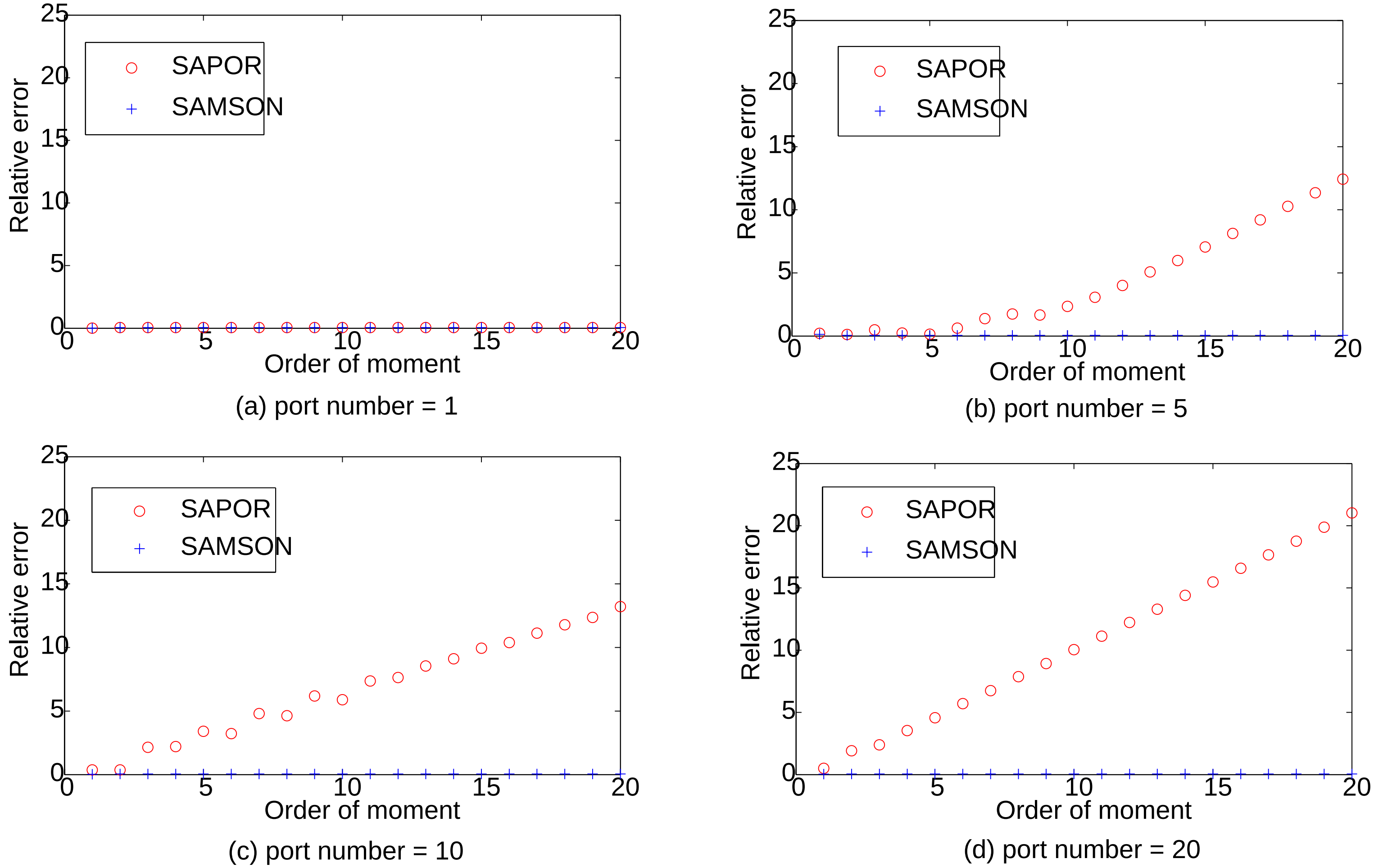
Experimental Setting

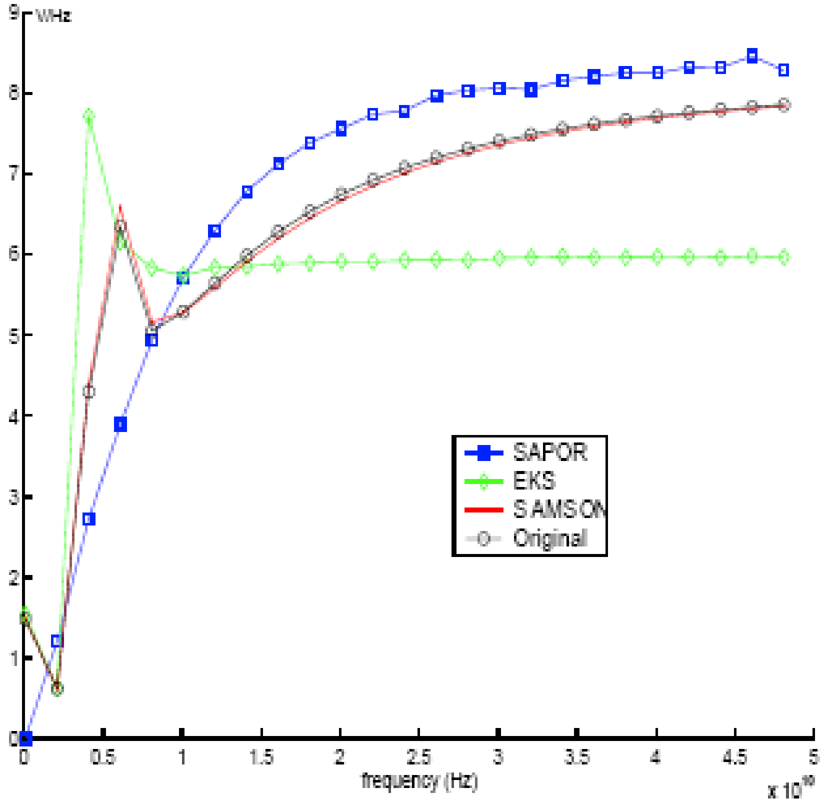
- Experiments are run on a PC with Intel Pentium IV 2.66G CPU and 1G RAM.
- All methods are implemented in MATLAB
- Time domain responses are calculated by IFFT (Inverse Fast Fourier Transformation with 1024 sampling points)
- The examples to be presented are from real industry applications (courtesy of Rio Design Automation)
  - Power planes and packages are modeled by RCS meshes
  - On chip power/ground grids are modeled by RC meshes
  - Vias and bumps are modeled by RC elements
  - PWL sources are generated from SPICE characterization of FPGA circuits

Moment Matching

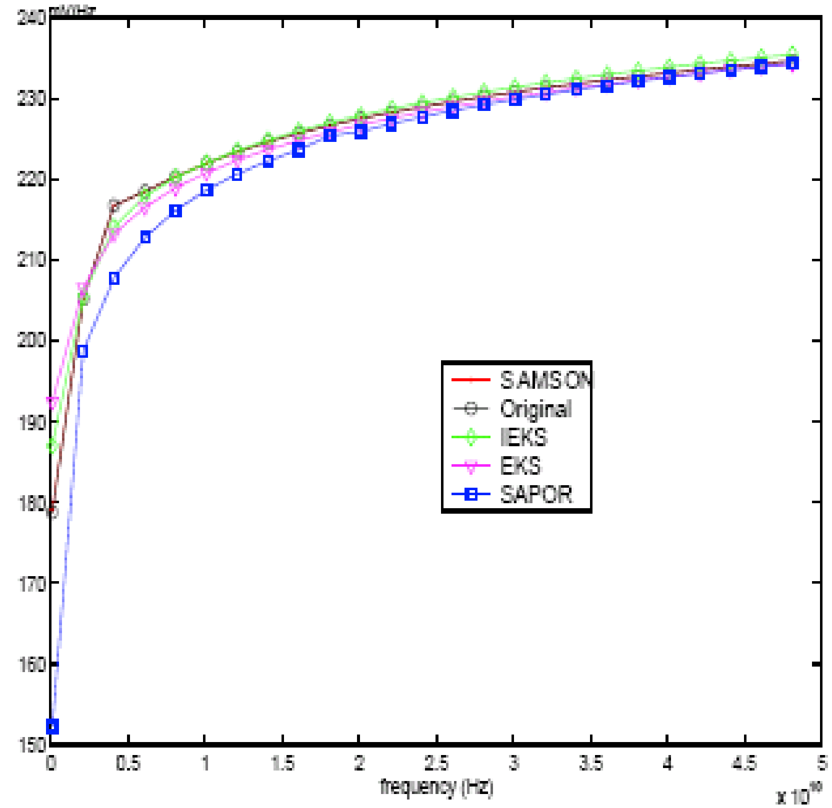


- With the increase of port number (from 1 to 20)
  - SAPOR (second order non-RHS) matches a decreasing number of moments
  - SAMSON always matches the same number of moments.

Frequency Domain Accuracy Comparison



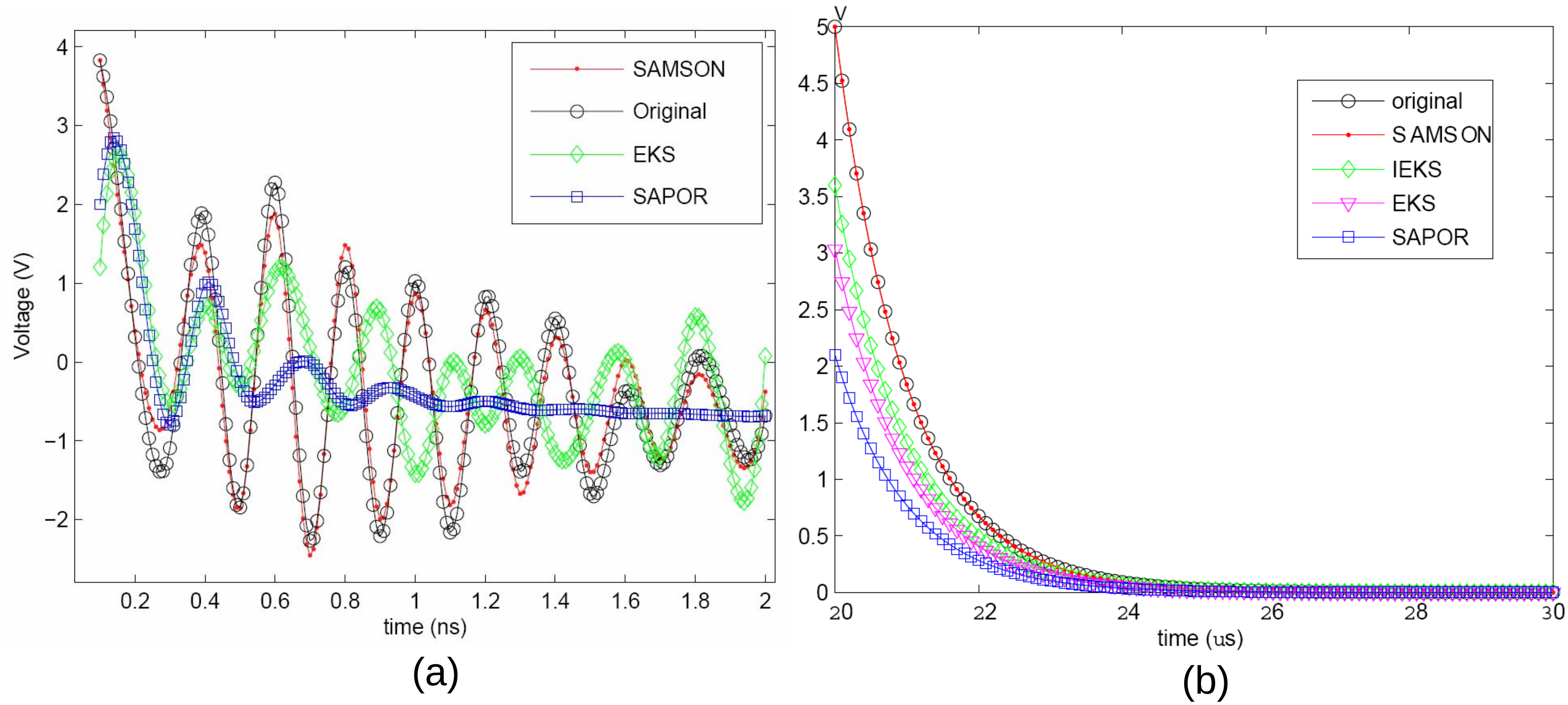
(a)



(b)

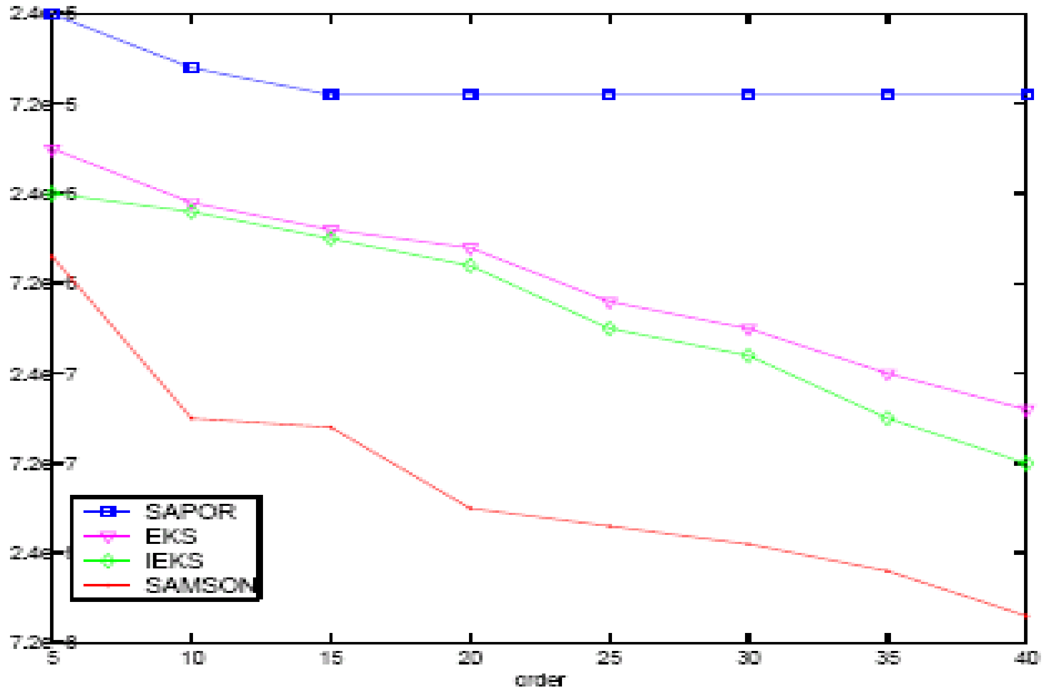
- ***(a) Frequency domain comparison between SAPOR, EKS, SAMSON and original with attenuated sine waveforms . (b) Frequency domain comparison between SAMSON, Original, IEKS, EKS and SAPOR with PWL sources; All circuits are reduced to the same order.***
  - Only SAMSON is identical to the original
  - EKS outperforms SAPOR due to the RHS MOR nature

Time Domain Accuracy Comparison



- ***(a) Time domain comparison between SAPOR, EKS, SAMSON and original with attenuated sine waveforms . (b) Time domain comparison between SAMSON, Original, IEKS, EKS and SAPOR with PWL sources; All circuits are reduced to the same order.***
  - Again only SAMSON is identical to the original
  - EKS outperforms SAPOR due to the RHS MOR nature

Scalability and Accuracy



- *Average time domain waveform error of SAMSON, EKS, IEKS and SAPOR with respect to the reduced order*
- *Different sizes of circuits from 200-70000 nodes are used. For each circuit 30% ports have independent PWL sources.*
- ***SAMSON*** has the fastest waveform convergence
  - 33X more accurate than EKS and IEKS at order 40
  - 48 X more accurate than SAPOR
  - The non-RHS method, SAPOR, does not converge

Runtime Comparison

# of nodes	# of sources	Cir Sim Time (s)	Reduction + Simulation Time (s)		
			EKS	IEKS	SAMSON
192	50	0.18	0.11+0.00	0.09+0.00	0.08+0.00
768	100	106	10.4+0.4	10.2+0.4	7.6+0.4
2048	200	362	20.6+0.8	20.4+0.8	15.8+0.8
11520	800	1164	66.1+3.2	65.2+3.2	47.3+3.2
69380	4000	N/A	384+92	381+92	295+92

- *Comparison of the reduction and simulation time under the same accuracy of up to 50 GHz on an RC mesh with 11,520 nodes and 800 ports*
- ***SAMSON*** runs the fastest
  - 25X faster than direct simulation
  - faster than EKS and IEKS
  - but have a similar trend

Conclusions and Future work

- SAMSON is an RHS MOR method
  - Compare with SAPOR and other non-RHS methods, it is more accurate
  - Can handle a large number of ports
- SAMSON can deal with all kinds of input sources accurately without frequency domain shifting or incremental orthonormalization
  - Numerically more stable, more efficient and more accurate in the whole frequency range, particularly at DC (s=0)
- SAMSON is based upon generalized second order Arnoldi method
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Thank you!