Robust Interconnect Communication Capacity Algorithm by Geometric Programming

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Speaker : Jin Sun

Talk Outline

- Motivation
- Proposed Methodology
  - A robust model for channel capacity considering parameter variations.
  - A robust capacity optimization procedure by Geometric Programming (GP)
- Numerical Results
- Conclusion

Motivation

- Network-on-chip (NoC) is one of the most important features in today's computer architecture.
  - Modeling and characterization of global interconnect is very important.

- The deep submicron (DSM) technologies make it possible to build high speed and high density global buses.
  - Shrinking feature size leads to severe random variations in circuit parameters.

- The goal of this paper is to optimize the interconnect capacity by Geometric Programming (GP) under parameters variations.

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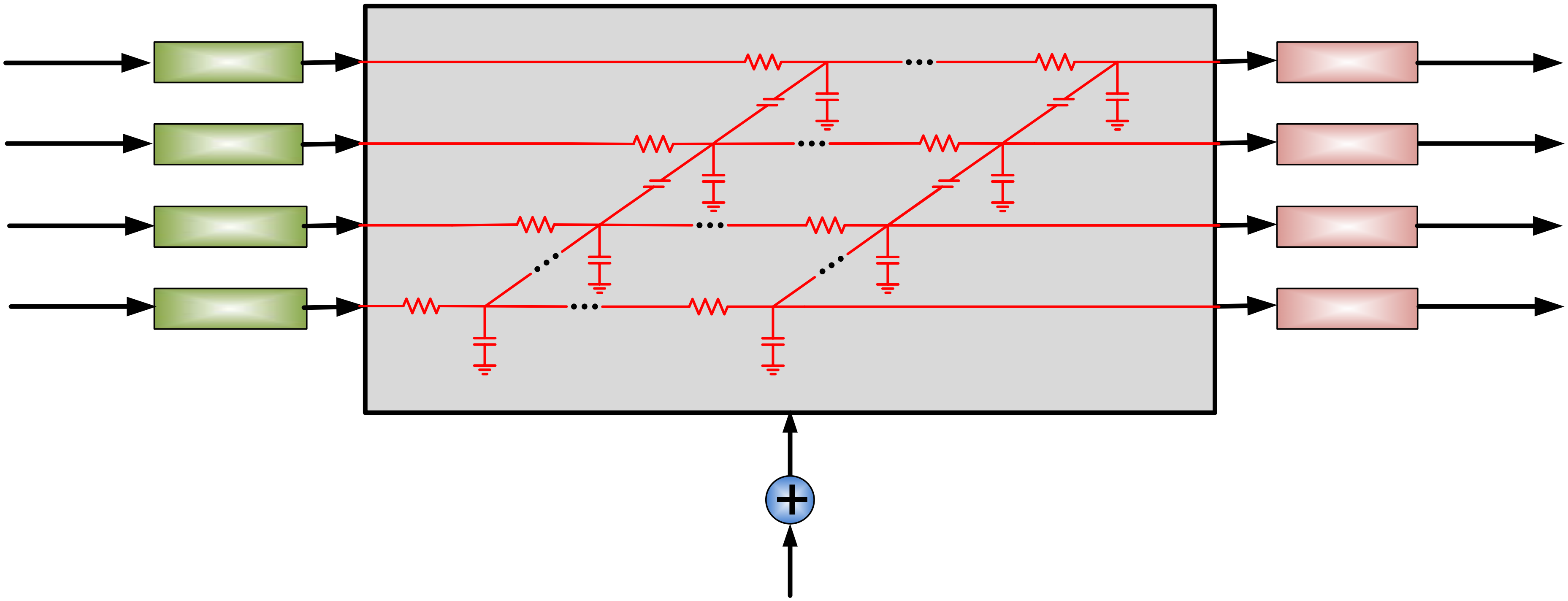
Global Bus Structure (1/2)

- The global bus is modeled as a RC network:



- The calculation of R, Cc and Cg follows PTM (Predictive Technology Model) [1].

[1]. *Predictive Technology Model Website.* [Online]. Available: http://www.eas.asu.edu/~ptm

Input Sequence

Transmitter

Input

Global Bus Structure (2/2)

- Deterministic transfer function:

$$H(s) = (G + sC)^{-1} B$$

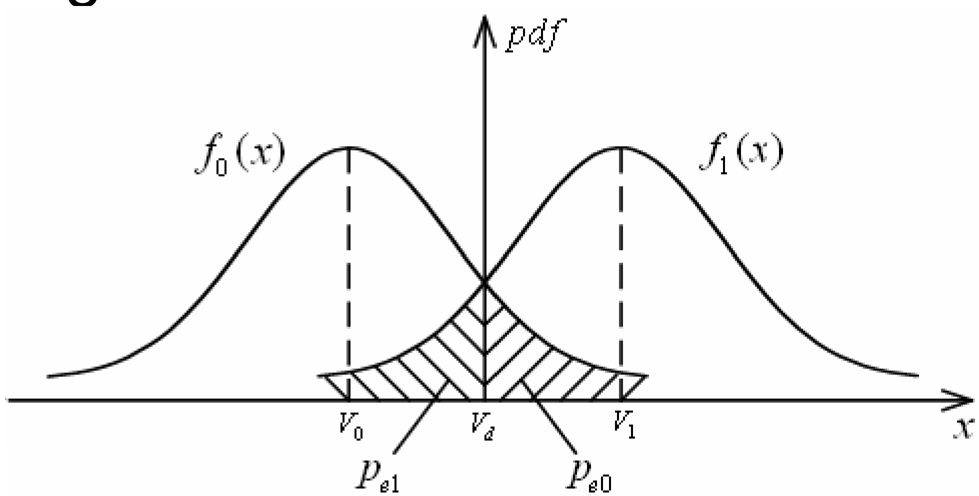
  - G: conductance matrix
  - C: capacitance matrix
  - B: input-output relationship

- Considering process variations, G and C matrices are random with deviations from their nominal values:

$$\mathring{H}(s) = (\mathring{G} + s\mathring{C})^{-1} B$$

Deterministic Capacity Model

- Output signal:



  - BER (Bit Error Rate):

$$p_{e1} = p(0|1) = P(V_1 < V_d) \quad p_{e0} = p(1|0) = P(V_0 > V_d)$$

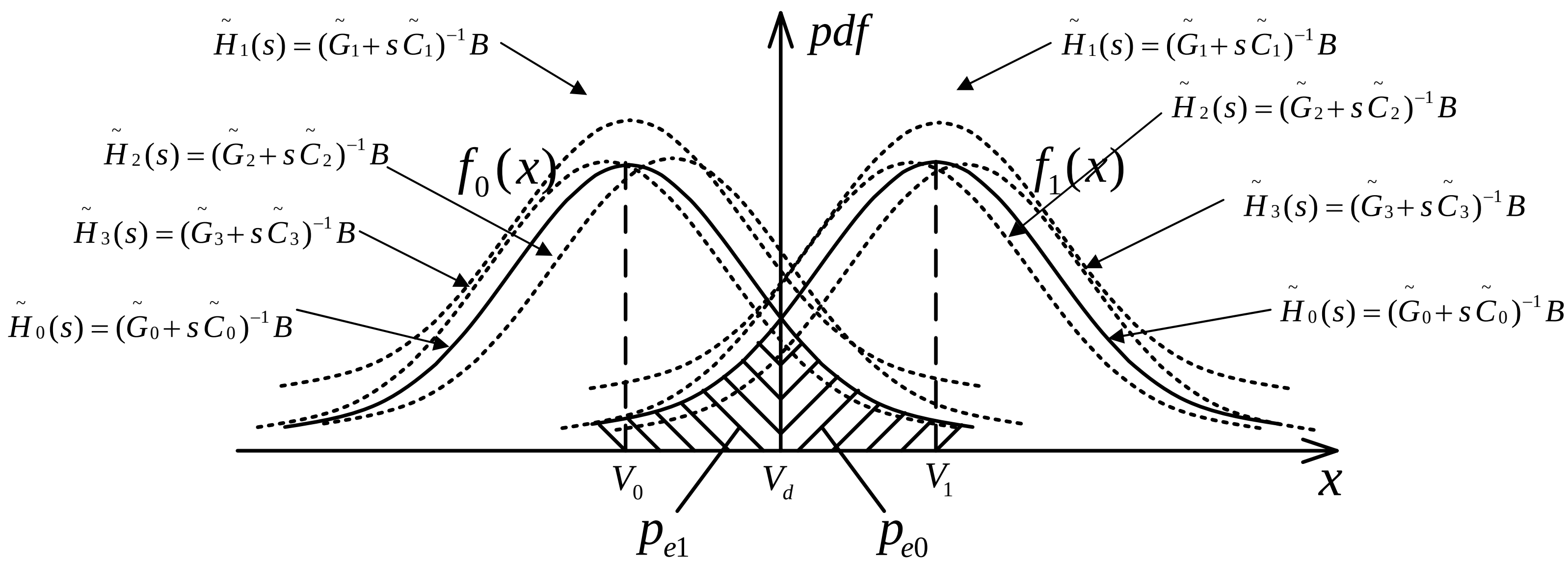
  - Channel Capacity:

$$C = \Gamma\left(p_{e1}, p_{e0}\right) @ \max I(X,Y)$$

  - The capacity is deterministic.

Robust Capacity Model

- Considering parameter variations:



  - *Pe*'s are variational:

$$p_e = f(w_i, d_i, t_i, h_i, W_{eff}, L_{eff})$$

  - *Pe*'s are not deterministic, so is the channel capacity:

$$C = \Gamma(w_i, d_i, t_i, h_i, W_{eff}, L_{eff})$$

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Robust Optimization Model

- Communication capacity optimization problem:

$$\begin{aligned} &\text{maximize} && C \\ &\text{Subject to} && p_e \le P_{spec} \\ & && X_{\min} \le X \le X_{\max} \\ &\text{variables} && X = (w, t, d, h, W_{eff}, L_{eff}) \end{aligned}$$

  - finds the optimal nominal parameter values.
  - maximizes the capacity with parameter variations.
  - a BER constraint specified by the designers.
  - upper and lower bounds for parameter variation.
- Standard GP formulation is required.

Standard GP Form

- Standard GP optimization problem:

$$\begin{aligned} &\text{minimize} && f_o(x) && \\ &\text{Subject to} && f_i(x) \le 1, && i = 1, 2, \ldots, m \\ & && h_l(x) = 1, && l = 1, 2, \ldots, p \\ &\text{variables} && x && \end{aligned}$$

  - The objective $f_o(x)$ is posynomial.
  - The inequality constraints are posynomials.
  - The equality constraints are monomials.

$$d \cdot x_1^{a^{(1)}} x_2^{a^{(2)}} \cdots x_n^{a^{(n)}}$$

$$C = \frac{w^{0.017} l^{0.008}}{t^{0.355} h^{0.344}}$$

monomial

$$\sum_{k=1}^{K} d_k \cdot x_1^{a_k^{(1)}} x_2^{a_k^{(2)}} \cdots x_n^{a_k^{(n)}}$$

posynomial

Objective Formulation in GP

- Formulates the objective into GP form:

$$\text{maximize } C \Leftrightarrow \text{minimize } -C$$

- Converting a signomial objective into posynomial form:
  - separates positive items and negative items:

$$\text{minimize } -C = f_{01}(X) - f_{02}(X)$$

  - introduces two slack variables:

$$f_{01}(X) - f_{02}(X) \le u_1$$
$$f_{01}(X) \le u_2 \le f_{02}(X) + u_1$$

  - translates into GP forms:

$$\begin{aligned} &\text{minimize} && u_1 \\ &\text{Subject to} && u_2^{-1} f_{02}(X) + u_2^{-1} \ge 1 \\ &&& u_2^{-1} f_{01}(X) \le 1 \end{aligned}$$

Constraint Formulation in GP (1/3)

- Constraint function with parameter variations:

$$p_e = \hat{f}(X) \overset{posy\ fitting}{\approx} \sum_{k=1}^{K} c_k \cdot w^{a_k} d^{b_k} t^{m_k} h^{n_k} W_{eff}^{p_k} L_{eff}^{q_k} \leq P_{spec}$$

$$X \Rightarrow X + \delta X \quad \Longrightarrow \quad \hat{f}(X) \Rightarrow \hat{f}(X + \delta X)$$

- Use first-order Taylor expansion to further expand:

$$\hat{f}(X + \delta X) = \hat{f}(X) + \nabla \hat{f}(X) \cdot (X + \delta X - X)$$

$$= \hat{f}(X) + \sum_i \frac{\partial \hat{f}(X)}{\partial X_i} \delta X_i \leq T_{spec}$$

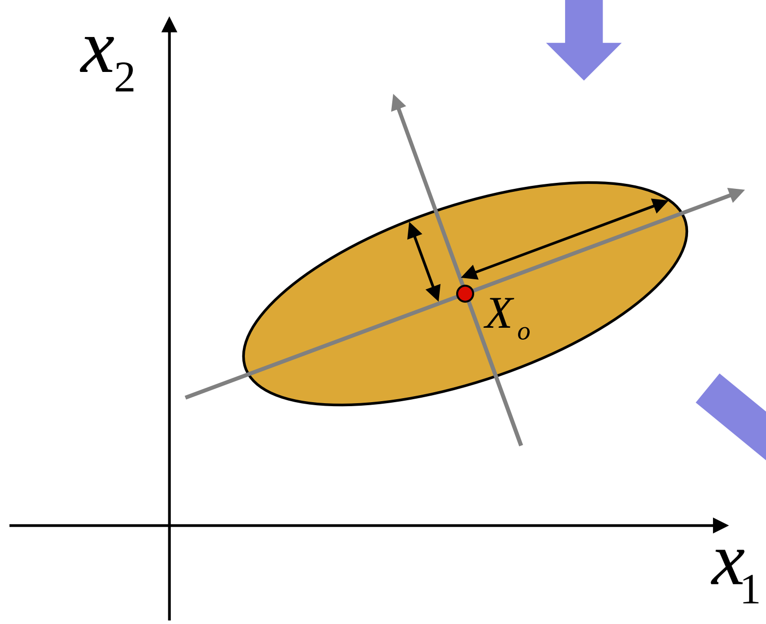
- We need to formulate the variational constraint into deterministic function.

Constraint Formulation in GP (2/3)

- Variational constraint function:

$$\hat{f}(X) + < \nabla \hat{f}(X), \delta X > \;\leq\; P_{spec}$$

$$\hat{f}(X) + \max\left\{ < \nabla \hat{f}(X), \delta X > \right\} \leq P_{spec}$$



- UE (Uncertainty Ellipsoid) :

$$\tilde{X} = \left\{ X_o + P^{1/2} u \;\middle|\; \|u\| \leq 1 \right\}$$

  - P is the covariance matrix.
  - $\|u\|$ is 2-norm of u.

$$\delta X = \tilde{X} - X_0 = P^{1/2} u$$

Constraint Formulation in GP (3/3)

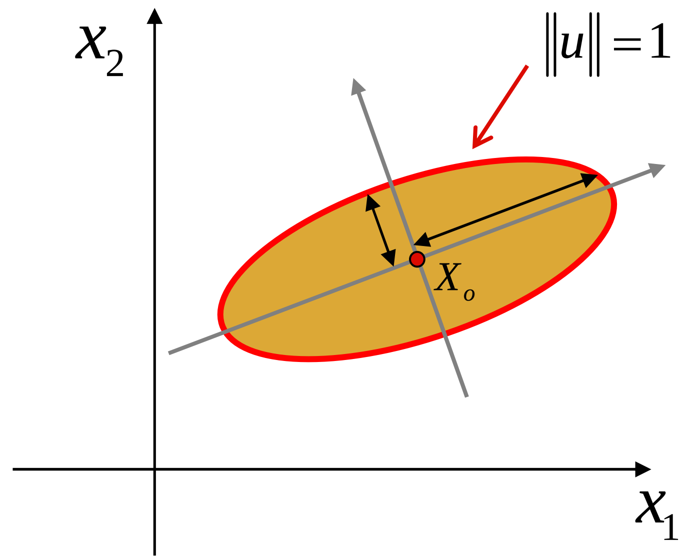
- How to get rid of u :

$$\hat{f}(X) + \max\left\{ < \nabla \hat{f}(X), P^{1/2}u > \right\} \le P_{spec}$$

Cauchy-Schwarz Inequality

$$< a,b > \; < \; \|a\| \cdot \|b\|$$

$$\hat{f}(X) + \left\| \nabla \hat{f}\, P^{1/2} \right\| \|u\| \le P_{spec}$$



- More slack variables to eliminate the $P^{1/2}$ item.

Resulting Model in Standard GP Form

Minimize u_1

Subject to

$$e^{-c_0-1} w^{-a_0 c_0} d^{-b_0 c_0} t^{-m_0 c_0} h^{-n_0 c_0} g W^{-p_0 c_0} L^{-q_0 c_0} u_1^{-1} u_2^{c_0+1} \le 1$$

$$c_1 g w^{a_1} d^{b_1} t^{m_1} h^{n_1} W^{p_1} L^{q_1} + r_1 + r_2 \le P_{spec}$$

$$\phi_1^T P \phi_1 r_1^{-2} \le 1$$

$$\phi_2^T P \phi_2 r_2^{-2} \le 1$$

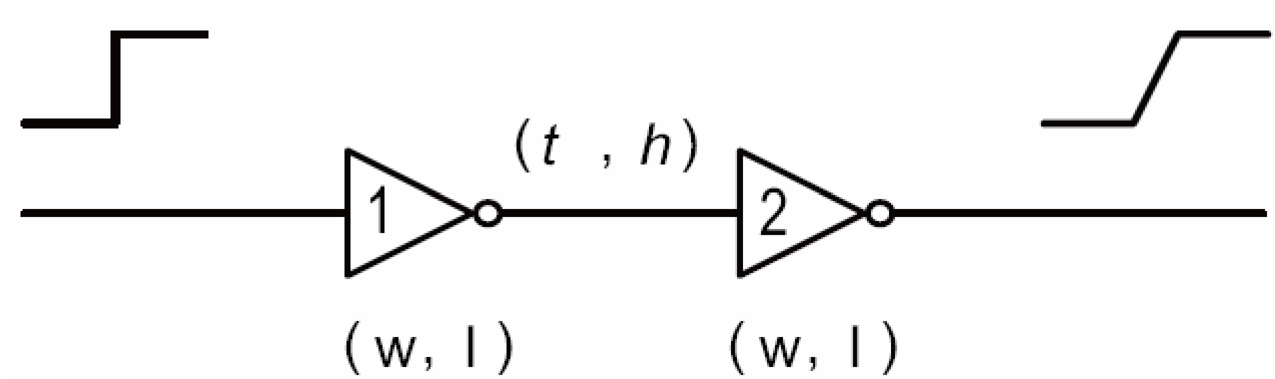
$$X_{min} \le X \le X_{max}$$

variables $X = (w, t, d, h, W, L), u_1, u_2, r_1, r_2$

Robust GP formulation for r_e constraint

Robust GP formulation for objective

A Simple Example (1/2)



- For this circuit, the channel capacity is:

$$C = \frac{w^{0.017} l^{0.008}}{t^{0.355} h^{0.344}}$$

- Thus, the optimization problem is formulated as:

$$\text{Minimize} \quad -\frac{w^{0.017} l^{0.008}}{t^{0.355} h^{0.344}}$$

$$\text{Subject to} \quad \frac{0.1415 w^{0.6021}}{t^{0.4089} h^{0.2307} l^{0.0319}} + \frac{0.1415 w^{0.5506}}{t^{0.4129} h^{0.2339} l^{0.0435}} \le T_{spec}$$

A Simple Example (1/2)

- Formulated into standard GP form:

$$\text{Minimize} \quad u_1$$

$$\text{Subject to} \quad e^{-2}\frac{t^{0.355}h^{0.344}u_2^2}{w^{0.017}l^{0.008}u_1} \le 1$$

$$\frac{0.1415w^{0.6021}}{t^{0.4089}h^{0.2307}l^{0.0319}} + \frac{0.1415w^{0.5506}}{t^{0.4129}h^{0.2339}l^{0.0435}}$$

$$+ r_1 + r_2 \le T_{spec}$$

$$\phi_1^T P \phi_1 r_1^{-2} \le 1$$

$$\phi_2^T P \phi_2 r_2^{-2} \le 1$$

  - Four slack variables u_1, u_2 and r_1, r_2 are introduced.
  - Two slack vectors ϕ_1 and ϕ_2 are introduced.
  - Covariance matrix is required.

Posynomial Fitting Result (1/3)

- The relationship between communication capacity C and the geometric parameters.

$$C = \sum_{k_1=1}^{3} c_{k_1} \cdot w^{a_{k_1}} t^{b_{k_1}} d^{m_{k_1}} h^{n_{k_1}} W^{p_{k_1}} L^{q_{k_1}}$$

	c_{k1}	a_{k1}	b_{k1}	m_{k1}	n_{k1}	p_{k1}	q_{k1}
$k_1=1$	0.058	0.8906	1.0534	0.7870	0.9452	0.2087	-0.2747
$k_1=2$	0.058	0.9089	0.8496	0.8109	0.7328	0.1895	-0.2747
$k_1=3$	0.058	1.1451	0.8021	0.6964	0.7611	0.1688	-0.2747

Posynomial Fitting Result (2/3)

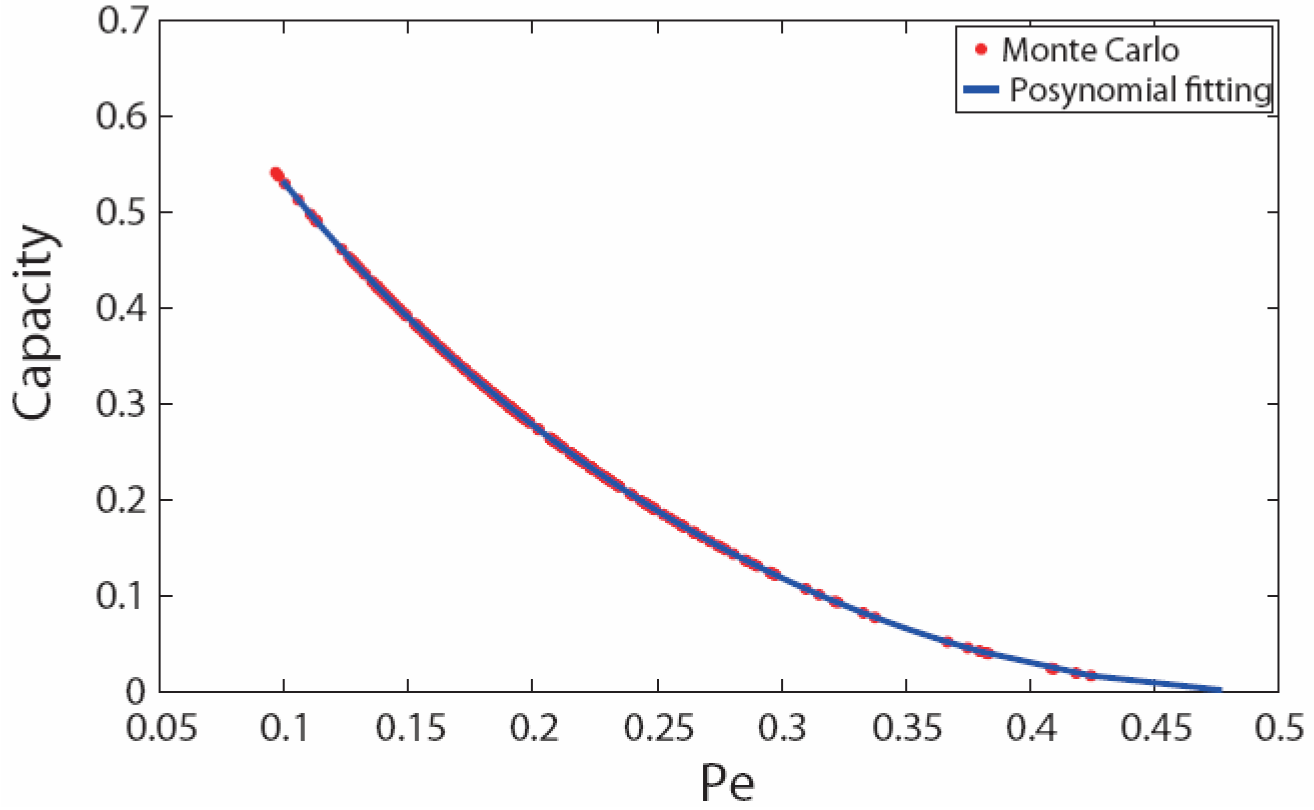
- The relationship between error probability and the geometric parameters:

$$p_e = \sum_{k_2=1}^{3} c_{k_2} \cdot w^{a_{k_2}} t^{b_{k_2}} d^{m_{k_2}} h^{n_{k_2}} W^{p_{k_2}} L^{q_{k_2}}$$

	C_{k2}	a_{k2}	b_{k2}	m_{k2}	n_{k2}	p_{k2}	q_{k2}
$k_2=1$	0.0787	-0.2242	-0.1554	0.4345	-0.2424	1.1642	-0.4942
$k_2=2$	0.0787	0.2970	-0.0985	-0.0257	-1.1651	-1.0235	0.1727
$k_2=3$	0.0787	-0.5210	0.9972	-0.3799	0.7566	-1.7016	0.3541

Posynomial Fitting Result (3/3)

- The comparison of capacity generated by Monte-Carlo simulation and that by posynomial fitting:



Fitting Results on ISCAS Circuits

	C432	C499	C880	C1335	C1908	C2670	C3540	C5315	C6288	C7552
c_1	0.0542	0.0543	0.0572	0.0557	0.0170	0.0170	0.0377	0.0190	0.0737	0.0533
c_2	0.0542	0.0543	0.0572	0.0557	0.0170	0.0170	0.0377	0.0190	0.0737	0.0533
c_3	0.0542	0.0543	0.0572	0.0557	0.0170	0.0170	0.0377	0.0190	0.0737	0.0533
a_1	-0.4912	-0.4041	-0.0830	-0.4376	-1.0179	-1.0745	-0.7625	-1.3418	-0.3262	-0.5552
a_2	-0.5390	-0.3684	0.0617	-0.4795	-1.2549	-0.8886	0.7386	-1.2385	-0.3937	-0.3263
a_3	-0.4652	-0.3783	0.0678	-0.4024	-1.4898	-1.0841	-0.7493	-0.9636	-0.2482	-0.6264
b_1	-0.4504	-0.7878	-0.0411	-0.5337	-0.9958	-1.6494	-0.3728	-1.1074	-0.3651	-0.5600
b_2	-0.3959	-0.7702	-0.0410	-0.5271	-0.8728	-1.1704	-0.8717	-1.0696	-0.3577	-0.4677
b_3	-0.5719	-0.5081	-0.0285	-0.5121	-1.2031	-0.8219	-0.7846	-1.1500	-0.4263	-0.4636
m_1	-0.7463	-0.6669	-0.0709	-0.5507	-1.0752	-0.6566	-0.5917	-1.1810	-0.6731	-0.6031
m_2	-0.2827	-0.4014	0.0734	-0.5965	-1.1607	-1.1059	-0.5992	-1.3141	-0.5602	-0.5790
m_3	-0.6069	-0.5354	0.0778	-0.4431	-0.8472	-1.0403	-0.7751	-0.9822	-0.6747	-0.4310
n_1	-0.5430	-0.5580	-0.0620	-0.3671	-1.2120	-0.9397	-0.8879	-1.1503	-0.8414	-0.3874
n_2	-0.4874	-0.4542	-0.0689	-0.6759	-0.7573	-0.9212	-0.5013	-1.2699	-0.7494	-0.4835
n_3	-0.4150	-0.3072	-0.0436	-0.5372	-0.9588	-1.3188	-0.6621	-1.2014	-0.7699	-0.5852
p_1	-0.2069	-0.2744	0.5739	-0.0988	-0.5881	-0.5185	-0.3982	-0.6557	-1.3331	-0.1252
p_2	-0.1660	-0.5393	0.5484	-0.1084	-0.7120	-0.5732	-0.3573	-0.8963	-1.2701	-0.0992
p_3	-0.1968	-0.3483	0.4895	-0.1386	-0.8703	-0.6156	-0.5313	-1.1825	-0.6941	-0.1162
q_1	0.7785	0.7017	0.9947	0.7937	2.0873	2.2756	0.7580	2.3855	3.7865	0.6102
q_2	0.9914	0.8046	1.2464	0.8307	2.1327	2.0087	1.1471	3.1161	2.5294	0.8028
q_3	1.0955	0.8565	1.2215	0.5926	1.5581	1.6363	1.5164	1.6179	3.5287	0.9548

Optimization Results on ISCAS Circuits

	C432	C499	C880	C1335	C1908	C2670	C3540	C5315	C6288	C7552
*d**	1.2999	1.3000	0.7000	1.2999	1.2548	1.2438	1.0426	1.0545	1.2231	1.2165
*w**	1.2999	1.3000	1.2999	1.2360	1.2446	1.2519	1.2312	1.2492	1.2124	1.2184
*t**	1.2652	1.3000	0.7044	1.2747	1.2520	1.2040	1.0717	1.2322	1.2249	1.2341
*h**	1.2737	1.3000	1.2793	1.2791	1.1653	1.2560	1.2097	1.0246	1.2702	1.7680
*W**	0.7000	1.3000	0.7000	1.1449	1.1741	1.2006	1.2680	1.2397	1.2409	1.0906
*L**	1.2999	0.7000	0.7083	0.7000	0.7159	0.7114	0.6907	0.7816	0.7963	0.7008

Conclusion

- A theoretical optimization framework of global interconnect channel capacity considering geometric parameter variations.
  - A statistical interconnect communication capacity model under parameter variations.
  - A GP (Geometric Programming) based capacity optimization methodology.

Thank you!

Q&A