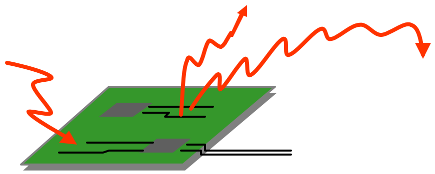

Geometrically Parameterized Interconnect Performance Models for Interconnect Synthesis

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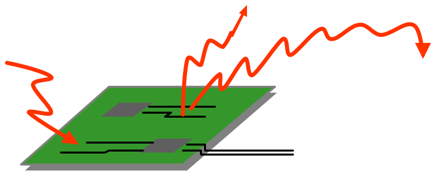
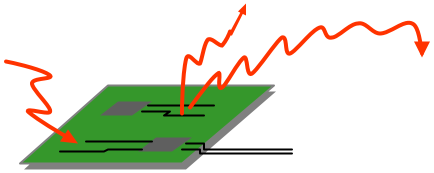


Motivation

- In interconnect design often would like to design for:
  - reliable functionality:
    - minimize capacitive cross-talk,
    - minimize inductive cross-talk,
    - minimize electromagnetic interference
  - high speed:
    - minimize resistance
    - minimize capacitance
  - low cost:
    - minimize area

- Need to explore tradeoff space and find optimal design!



Motivation (cont.)

- **The traditional design flow:**
  - **REPEAT**
    - **design all interconnect wires**
    - **extract accurately parasitics all at once**
  - **UNTIL noise and timing are within specs**

- **such procedure is not ideal for optimization!**
- **each iteration is very time consuming**

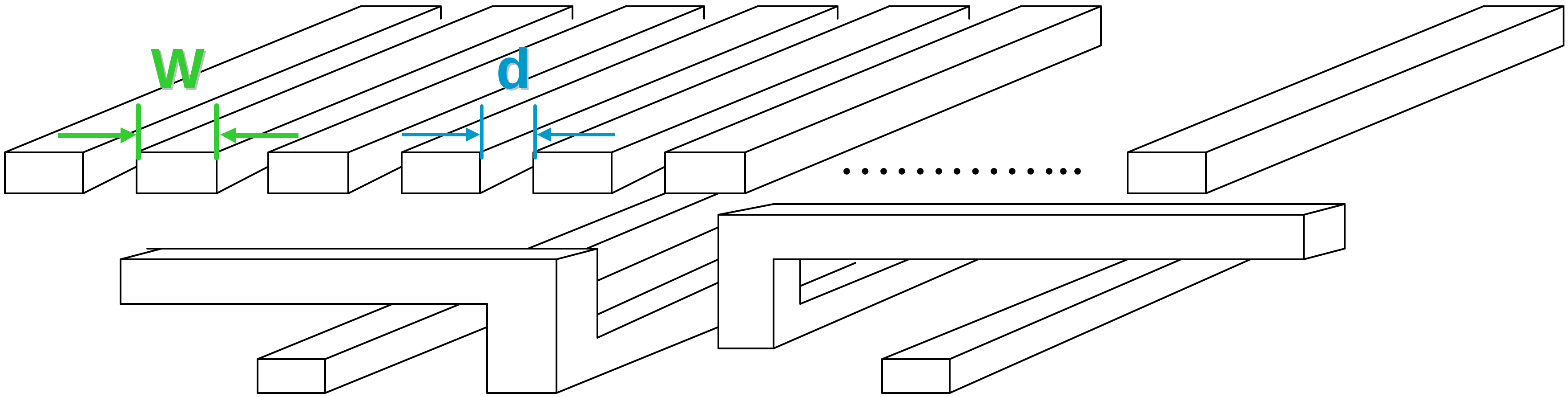
Alternative design methodologies

1. **Pre-characterize standard interconnect structures (e.g. busses):**
   - **using parasitic extraction and table lookup**
   - **or building parameterized and accurate low order models**

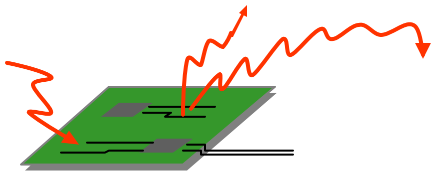
2. **And if the model construction is fast enough can also:**
   - **build the interconnect structure model "on the fly" during layout**
   - **accounting for any topology in surrounding topologies already committed to layout**
   - **then use optimizer to choose the best parameter for optimal tradeoff design.**

Example: an interconnect bus

- We construct a multi-parameter model of the bus parameterized in wire width W and separation d

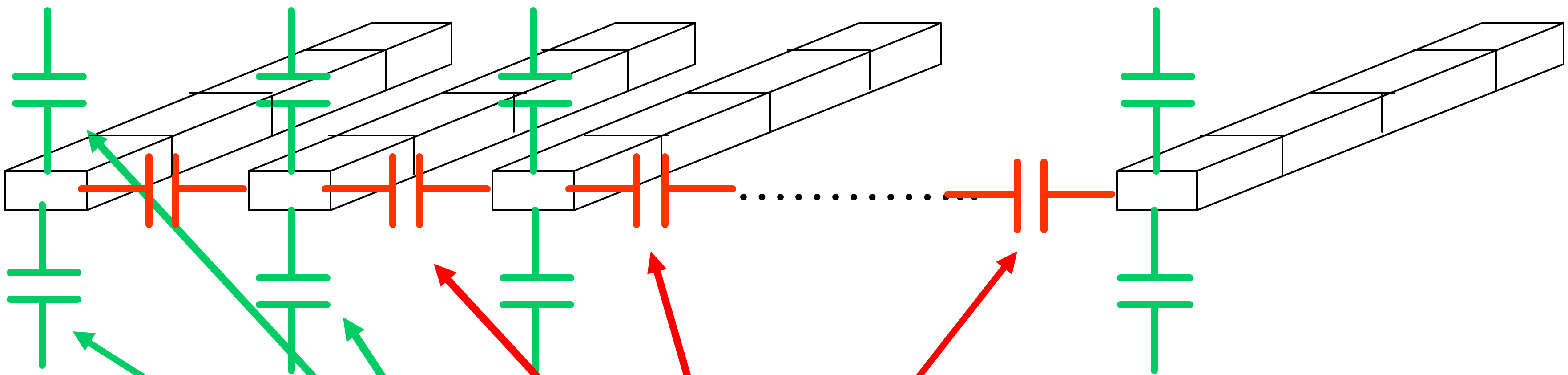


- accounting for surrounding topology



Parasitic extraction produces large state space models

- **E.g. subdividing wires in short sections and using for instance Nodal Analysis**

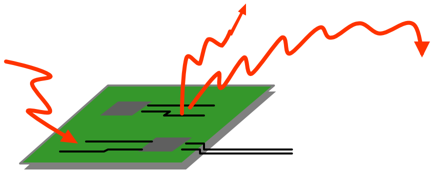


Conductance matrix

$$\left[sWC_{gnd} + s\frac{C_{side}}{d} + WG \right] x = bu$$

Large linear dynamical system

$$y = c^T x$$



Our goal

- **Given a large parameterized linear system:**

$$\left(sW\, C_{gnd} + \frac{s}{d}\, C_{side} + W\, G \right) x = b\, u$$

$$y = c^T x$$

- **construct a reduced order system with similar frequency response**

$$\left(sW\, \hat{C}_{gnd} + \frac{s}{d}\, \hat{C}_{side} + W\, \hat{G} \right) \hat{x} = \hat{b}\, u$$

$$y = \hat{c}^T \hat{x}$$

Background: Classical Non-parameterized Model Order Reduction

- **Given a large parameterized linear system:**

$$s\,A\,x = x + b\,u$$

$$y = c^T x$$

500,000 x 500,000

- **construct a reduced order system with similar frequency response**

$$s\,\hat{A}\,\hat{x} = \hat{x} + \hat{b}\,u$$

$$y = \hat{c}^T \hat{x}$$

20 x 20

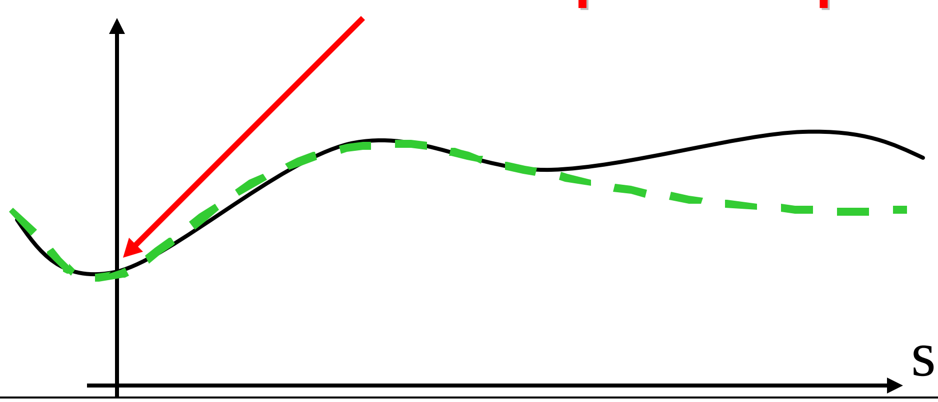
Background: Classical Non-parameterized Model Order Reduction (cont.)

$$sAx = x + bu \quad \Rightarrow \quad x = -(I - sA)^{-1} bu$$

Consider its Taylor series expansion:

$$x = \sum_{m=0}^{\infty} s^m \; A^m b \; u \quad \Rightarrow \quad x \in \operatorname{span}\{b, Ab, A^2 b, \cdots\}$$

- **idea for model order reduction: change base and use only the first few vectors of the Taylor series expansion: equivalent to match first derivatives around expansion point**

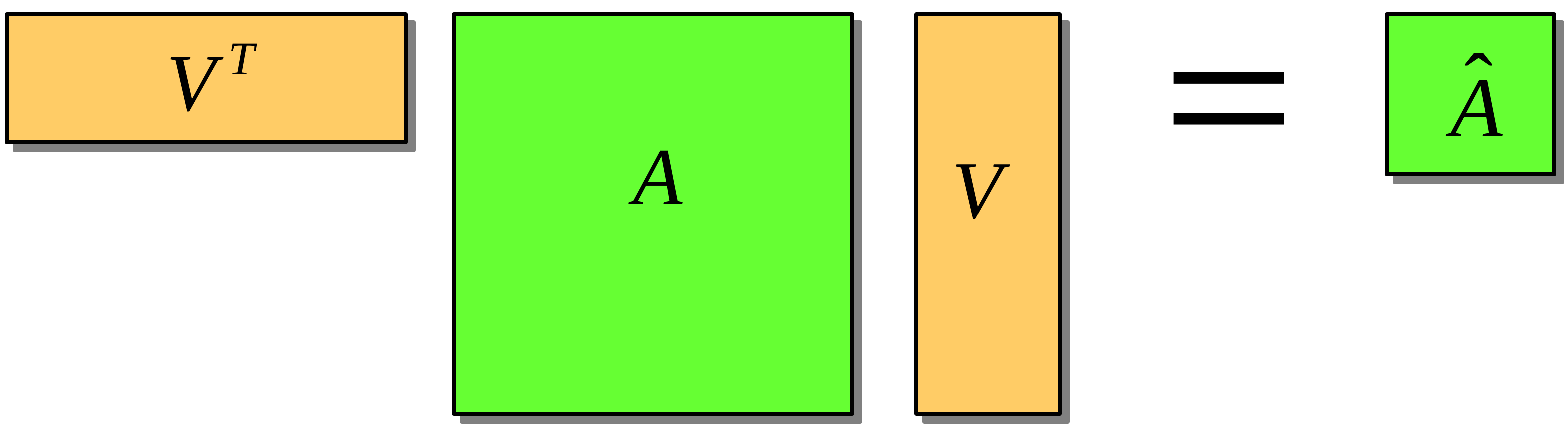


$$x = \underbrace{\begin{bmatrix} b & Ab & A^2 b \end{bmatrix}}_{V} \hat{x}$$

Parameterized Model Order Reduction (cont.)

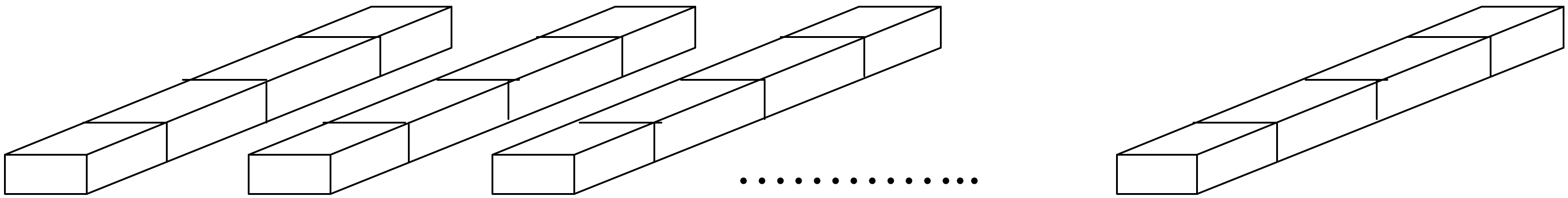
$$sAx = x + bu \qquad \Rightarrow \qquad s\overbrace{V^T A V}^{\hat{A}}\hat{x} = \hat{x} + \overbrace{V^T b}^{\hat{b}} u$$

$$y = c^T x \qquad \Rightarrow \qquad y = \underbrace{c^T V}_{\hat{c}^T}\hat{x}$$



Parameterized Model Order Reduction. Example: interconnect bus

- **Discretizing wires and using Nodal Analysis**



$$\left[sWC_{gnd} + s\frac{C_s}{d} + WG \right] x = bu$$

s_1 (green circle: sW), s_2 (red circle: $\frac{1}{d}$), s_3 (blue circle: W)

$$y = c^T x$$

Parameterized Model Order Reduction

- **In general:** $\left[s_1A_1+\ldots+s_pA_p-I\right]x=bu$

$$y=c^Tx$$

$$x=-\left[I-\left(s_1A_1+\cdots+s_pA_p\right)\right]^{-1}bu=\sum_{m=0}^{\infty}\left(s_1A_1+\cdots+s_pA_p\right)^m b\ \ u$$

- **It is a p-variables Taylor series expansion**

$$x\in span\left\{\underbrace{b,A_1b,A_2b,\cdots,A_pb}, A_1^2b,\left(A_1A_2+A_2A_1\right)b,\cdots\right\}$$

$$\begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} V \end{bmatrix} \begin{bmatrix} x_r \end{bmatrix}$$

Once again change basis and project state onto the first few vectors of the Taylor series expansion, in order to match the first derivatives with respect to all parameters

Parameterized Model Order Reduction (cont.)

$$\left[sWC_g + s\frac{C_s}{d} + WG\right]x = bu \quad \Rightarrow \quad \Big[sW\overbrace{V^T C_g V}^{\hat{C}_g} + s\frac{\overbrace{V^T C_s V}^{\hat{C}_s}}{d} + W\overbrace{V^T G V}^{\hat{G}}\Big]\hat{x} = \overbrace{V^T b}^{\hat{b}}u$$

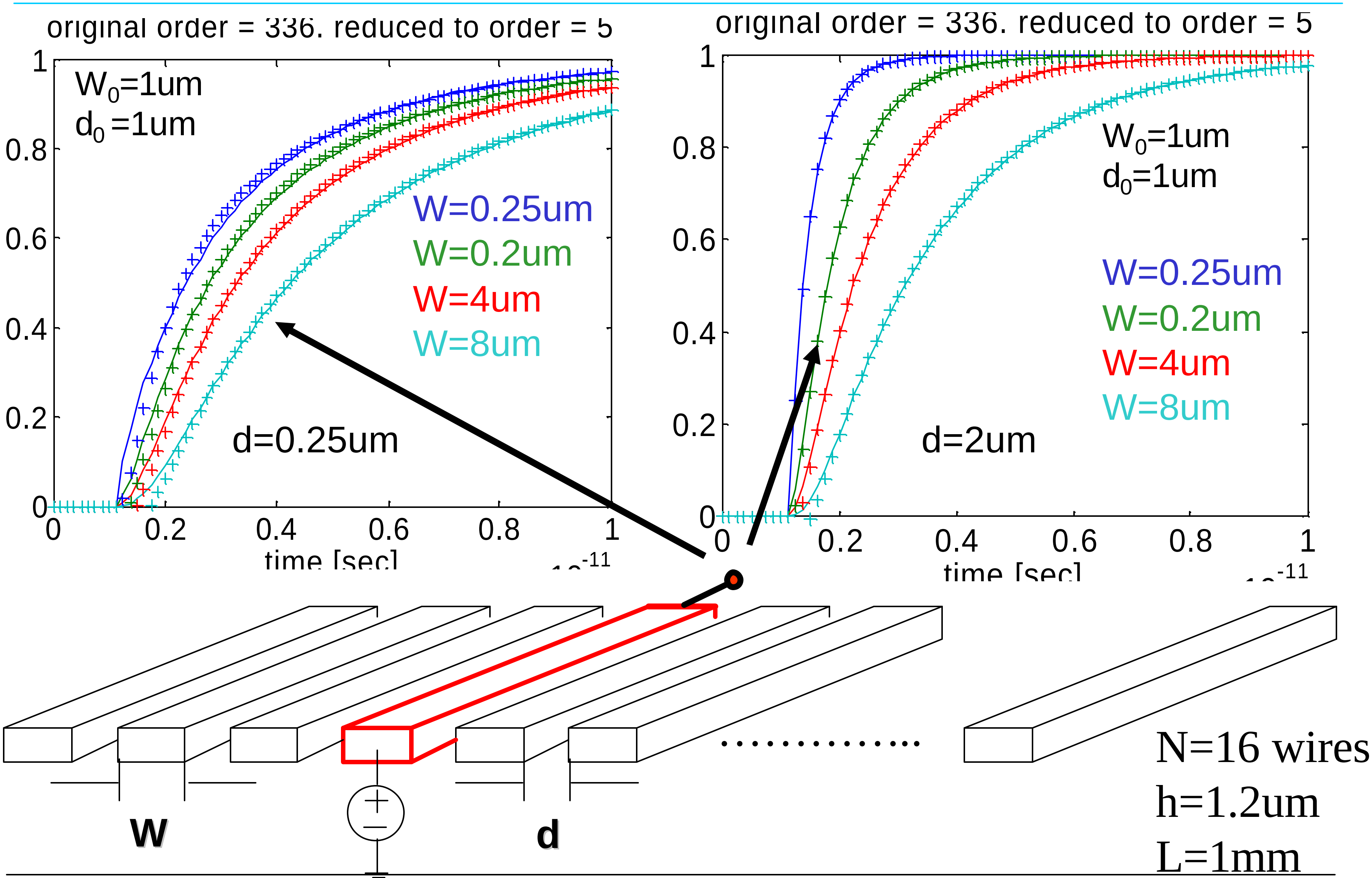
$$y = c^T x \qquad\qquad y = \underbrace{c^T V}_{\hat{c}}\hat{x}$$

$$V^T \; C_g \; V = \hat{C}_g$$

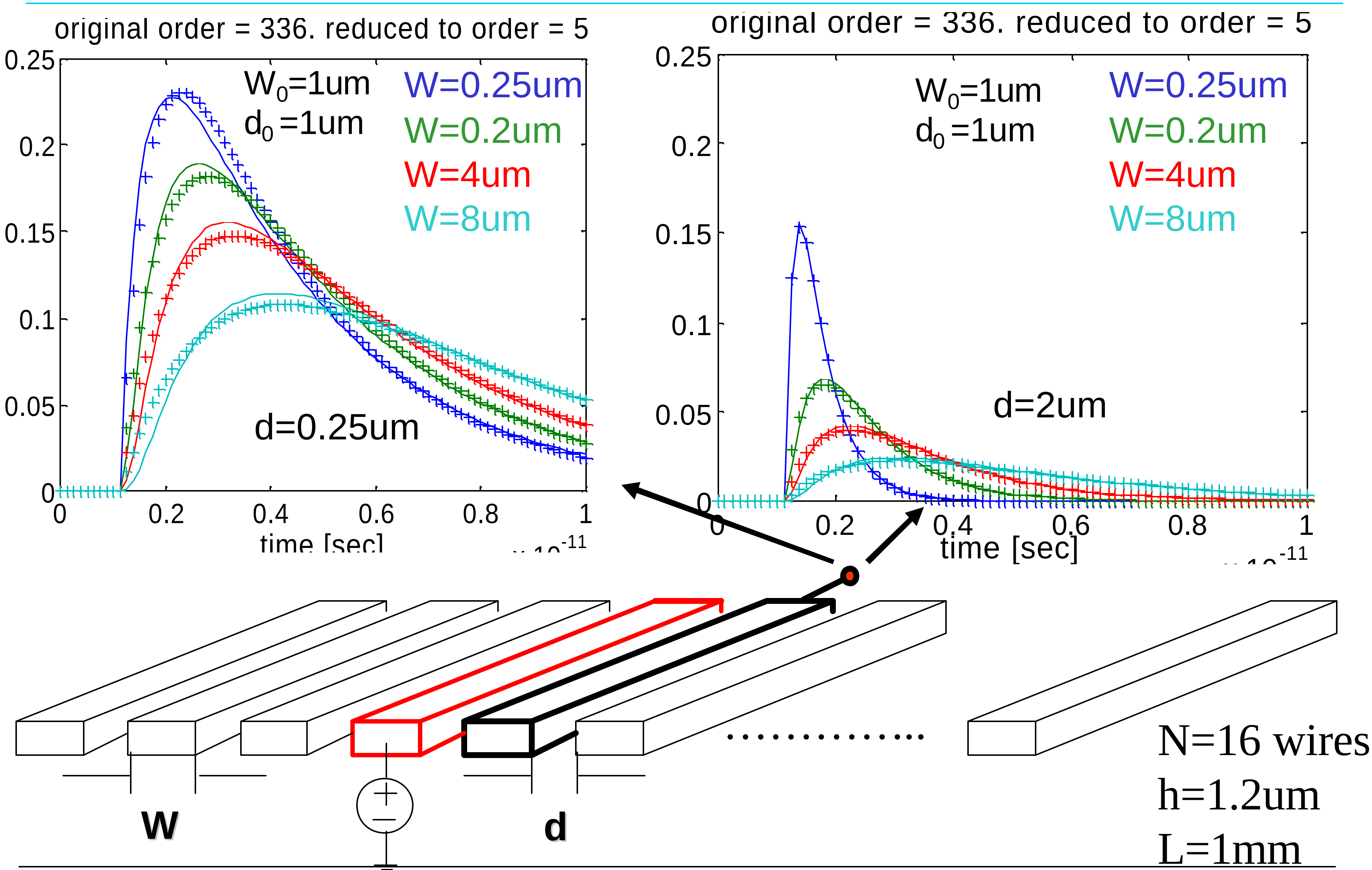
$$V^T \; C_s \; V = \hat{C}_s$$

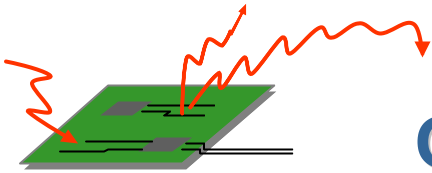
$$V^T \; G \; V = \hat{G}$$

Example: model step responses for different W and d



Example: model crosstalk responses for different W and d



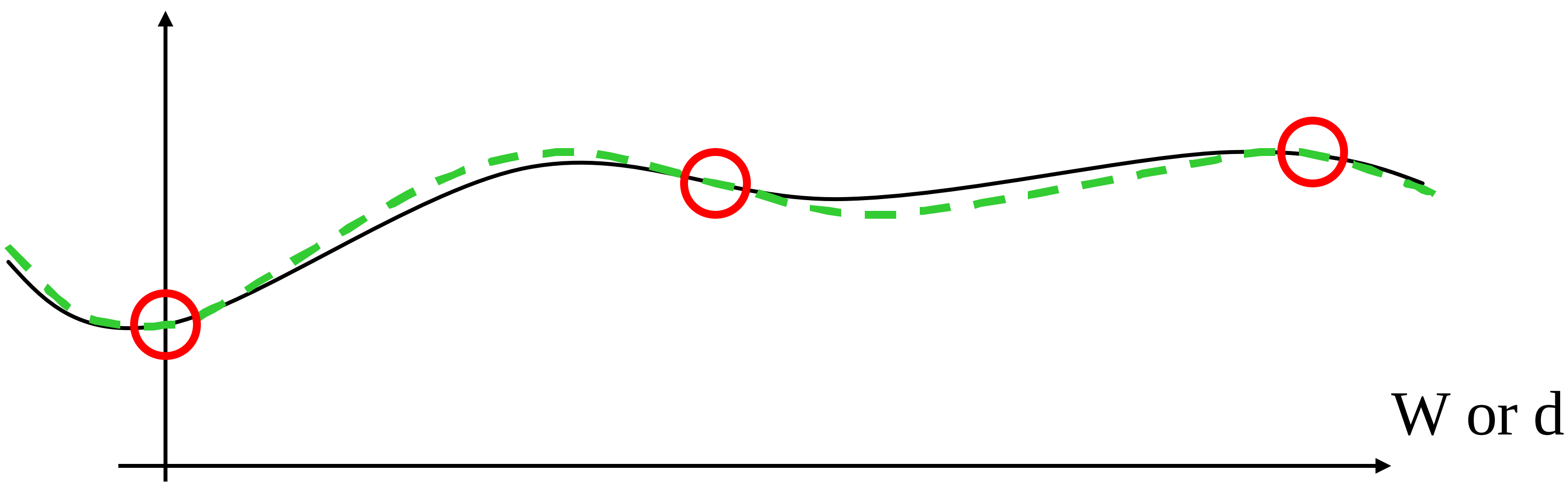


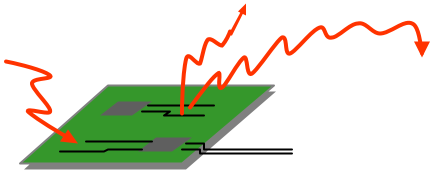
Open research issues and limitations

- So far only account for resistance and capacitance. Still need to verify if can account also for inductance.
- No good a priori error bounds available for moment matching reduced order modeling techniques
  - i.e. for a given accuracy, don't know how to pick order theoretically a priori.
  - however, practically, we do know how to construct the model incrementally increasing its order reusing all previous computation until we meet desired accuracy.

Open research issues and limitations (cont.)

- **Model order grows as $O(p^m)$ where p = # parameters and m = # derivatives matched for each parameter**
  - **however model order is linear in # of parameters when matching only one derivative per parameter (m = 1) and still produces good accuracy in our experiments.**
  - **furthermore, for higher accuracy instead of increasing # of matched derivatives, can instead match multiple points (or combine the two approaches)**





Conclusions

- **Parameterized low order model of interconnect structures can help interconnect synthesis and optimization**
- **Presented a technique for parameterized modeling:**
  - **based on Krylov subspace congruence transformation**
  - **requires only matrix-vector products: fast model construction**
  - **produced models capture accurately the behavior of the original system**
  - **and have low order: can be instantly evaluated for any parameter value for instance in an optimization procedure.**
- **Shown example result: bus interconnect parameterized in wire widths and separation.**